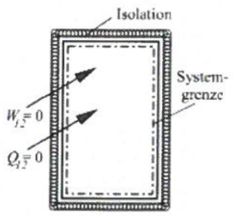
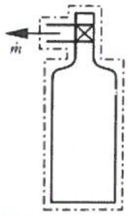
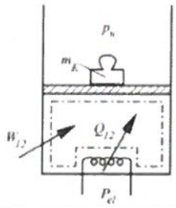
Grundlagen *to*

Systeme

Abgeschlossenes System	Offenes System	Geschlossenes System	Homogenes System
$\dot{m} = \dot{W} = \dot{Q} = 0$	$\dot{m} = \dot{W} = \dot{Q} \neq 0$	$\dot{m} = 0;\ \vert\ \dot{W} = \dot{Q} \neq 0$	1 Phase
			Bsp.: Flüssiges Wasser
			Heterogenes System
			2+ Phasen
			Bsp.: Luft

Zustandsgrößen

Definition	Intensiv	Extensiv	Spezifisch	Molar
$\oint dZ = 0$	$Z \not\propto m$	$Z \propto m$	$\frac{Z}{m}$	$\frac{Z}{n} = \frac{Z}{m} \cdot M$
Bsp.: p, T, V, m, U	p, T, μ_i	$V, U, N, E, S, H, F, G, m, \rho, n$	Bsp.: $v = \frac{V}{m} = \frac{1}{\rho}$	Bsp.: $V_m = \frac{V}{n} = v \cdot M$

Prozessgrößen

Definition	Spezifisch
$\oint dZ \neq 0$	$\frac{P}{m}$
W, Q	Bsp.: $q_{12} = \frac{Q_{12}}{m}$

Temperaturskalen

Fahrenheit	Rankine	Kelvin
$t[°F] = \frac{9}{5} \cdot t[°C] + 32$	$t[°Ra] = \frac{9}{5} \cdot t[°C] + 491,68$	$t[°K] = t[°C] + 273,15$

Energie

Kinetische	Potentielle	Gesamt
$E_{\text{kin}} = \frac{1}{2} \cdot m \cdot c^2$	$E_{\text{pot}} = mgh$	$E_{\text{ges}} = E_{\text{kin}} + E_{\text{pot}} + U$

Arbeit

Wärme	Volumenänderung
$\delta W = \vec{F} \cdot d\vec{s}$	$\delta W_V = -pdV$

Allgemeine Form von Bilanzen

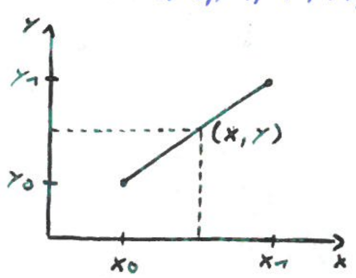
$$\frac{dZ_{System}}{dt} = \underbrace{\sum_j \left[(K_{Konvektion})_j\right]_{\substack{\text{über}\\ \text{Systemgrenze}}}}_{\text{Makroskopische Bewegung}} + \underbrace{\sum_k \left[(D_{Diffusion})_k\right]_{\substack{\text{über}\\ \text{Systemgrenze}}}}_{\text{Mikroskopische Bewegung}} + \underbrace{\sum_l \left[(F_{Feld})_l\right]_{\substack{\text{auf ganzes}\\ \text{Systemvolumen wirkend}}}}_{\text{Feldeinflüsse}} + \underbrace{\sum_m \left[(S_{\text{Quellen und Senken}})_m\right]_{\substack{\text{im}\\ \text{System}}}}_{\text{Prozesse im System}}$$

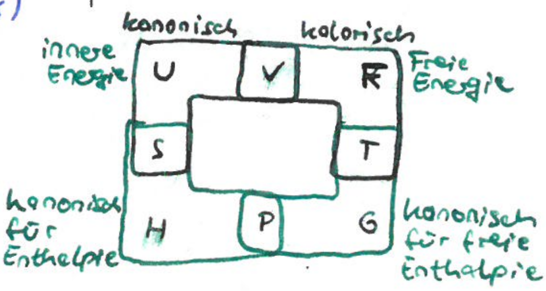
1. Hauptsatz

$$\frac{d}{dt}\left\{U + m\left(\frac{c^2}{2} + gz\right)\right\}_{System} = \sum_j \left[\dot{m}_j \left(h + \frac{c^2}{2} + gz\right)_j\right]_{\substack{\text{über}\\ \text{Systemgrenze}}} + \sum_l \left[(\dot{Q})_l\right]_{\substack{\text{über}\\ \text{Systemgrenze}}} + \sum_i \left[(\dot{W}_t)_i\right]_{\substack{\text{über}\\ \text{Systemgrenze}}} - \left(p\frac{dV}{dt}_{System}\right)$$

Interpolation	Totales Differential	Massenstrom
$y = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0$	$dA = \left(\frac{\partial A}{\partial B}\right)_C dB + \left(\frac{\partial A}{\partial C}\right)_B dC$	$\dot{m}_i = \rho_i \dot{V}_i = \rho_i A_i c_i = \frac{A_i c_i}{v_i}$

tof1 (y_0, y_1, x_0, x_1, x)





Lateinische Zeichen

A = Fläche [m^2]
An = Anergie [J]
c = Geschwindigkeit [m s^{-1}]
c_S = Schallgeschwindigkeit [m s^{-1}]
C_v = Wärmekapazität bei konstantem Volumen [J K^{-1}]
C_p = Wärmekapazität bei konstantem Druck [J K^{-1}]
c_v = Spez. Wärmekapazität bei konstantem Volumen [J kg^{-1} K^{-1}]
c_p = Spez. Wärmekapazität bei konstantem Druck [J kg^{-1} K]
E = Energie [J]
e = Spezifische Energie [J kg^{-1}]
$Ex = -W_{ex}$ = Exergie [J]
F = Kraft [J m^{-1}]
$F = U - TS$ = Freie Energie [J]
$f = u - Ts$ = Spezifische freie Energie [J kg^{-1}]
f = Fugazität [Pa]
$G = H - TS$ = Freie Enthalpie [J]
$g = h - Ts$ = Spezifische freie Enthalpie [J kg^{-1}]
g = Erdbeschleunigung [m s^{-2}]
$H = U + pV$ = Enthalpie [J]
$h = u + pv$ = Spezifische Enthalpie [J kg^{-1}]
ΔH_R = Molare Reaktionsenthalpie [J mol^{-1}]
K = Konstante des Massenwirkungsgesetzes [−]
M = Molmasse [kg mol^{-1}]
m = Masse [kg]
$\dot{m}$ = Massenstrom [kg s^{-1}]
m' = Masse der flüssigen Phase [kg]
m'' = Masse der gasförmigen Phase [kg]
$Ma = c/c_S$ = Machzahl [−]
n = Molzahl/Soffmenge[mol]
n = Polytropenexponent [−]
P = Leistung [W]
P_t = Technische Leistung [W]
P = Druck [Pa]
Q = Wärme [J]
$\dot{Q}$ = Wärmestrom [W]
q = Spezifische Wärme [J kg^{-1}]
r = Spezfische Verdampfungsenthalpie [J kg^{-1}]
R bzw. R_j = Spezifische Gaskonstante des Stoffes j [J kg^{-1} K^{-1}]
R_m = Universelle Gaskonstante [J mol^{-1} K^{-1}]
S = Entropie [J K^{-1}]
s = Spezifische Entropie[J kg^{-1} K^{-1}]
T = Temperatur [K]
t = Zeit [s]
t = Temperatur (Celsiusskala) [°C]
T_s = Sättigungstemperatur [K]
U = Innere Energie [J]
u = Spezifische innere Energie [J kg^{-1}]
V = Volumen [m^3]
v = Spezifisches Volumen [m^3 kg^{-1}]
V_m = Molares Volumen [m^3 mol^{-1}]
W = Arbeit [J]
w = Spezifische Arbeit [J kg^{-1}]
W_V = Volumenänderungsarbeit [J]
w_{el} = Elektrische Arbeit [J]
W_w = Wellenarbeit [J]
W_{diss} = Dissipationsarbeit [J]
W_t = Technische Arbeit [J]
$W_{V,irrev}$ = Arbeitsverlust durch Irreversibilitäten [J]
$x = m''/(m' + m'')$ = Dampfanteil [−]
$x = m_{H_2O}/m_L$ = Wassergehalt[−]
Z = Allgemeine extensive Zustandsgröße [Z]
z = Allgemeine spezifische Zustandsgröße [Zkg^{-1}]

Griechische Zeichen

β = Isobarer Ausdehnungskoeffizient [K^{-1}]
γ = Isochorer Spannungskoeffizient [K^{-1}]
δ_T = Isothermer Drosselkoeffizient [m^3 kg^{-1}]
δ_h = Isenthalper Ausdehnungskoeffizient [K s^2 m kg^{-1}]
ε = Leistungsziffer [−]
ε = Verdichtungsverhältnis [−]
η_{th} = Thermischer Wirkungsgrad [−]
η_{mech} = Mechanischer Wirkungsgrad [−]
$\eta_{carnot} = 1 - \frac{T_0}{T_{max}}$ = Carnot-Wirkungsgrad [−]
κ = Adiabaten- oder Isentropenexponent [−]
λ = Reaktionslaufzahl [−]
μ_i = Chemisches Potential [J mol^{-1}]
ν_i = Stöchioetrische Koeffizienten [−]
$\xi_i = m_i/m$ = Massenanteil [−]
π = Druckverhältnis [−]
ρ = Dichte [kg m^{-3}]
τ = Temperaturverhältnis [−]
φ = Relative Feuchte [−]
φ = Einspritzverhältnis [−]
χ = Isothermer Kompressibilitätskoeffizient [m^2 N^{-1}]
ψ = Dissipationsenergie [J]
ψ = Spezifische Dissipationsenergie [J kg^{-1}]
ψ = Druksteigerungsverhältnis [−]
$\psi_i = n_i/n$ = Molanteil [−]

Indizes

ab = abgeführt
$Carnot$ = Carnot
$im\ System$ = Prozess im System
$irrev$ = irreversibel
K = kritische Größen
K = Kältemaschine
KG = Kühlgrenze
kin = kinetisch
m = molare Größe
max = maximal
min = minimal
opt = optimal
p = bei konstantem Druck
pm = partielle molare Größe
pot = potenziell
$prod$ = produzierte Größe, Quellterm
rev = reversibel
S = Sättigungsgrößen
$System$ = Zustandsgröße eines Systems
$über\ Systemgrenze$ = Transfer einer Größe über die Systemgrenze
v = bei konstantem Volumen
WP = Wärmepumpe
zu = zugeführt
$ZÜ$ = Zwischenüberhitzung
0 = auf den Kühlraum bezogen
0 = Ruhe- bzw. Totalgrößen

$[J] = \frac{kg \cdot m^2}{s^2}$

$[mm^2] = [1 \cdot 10^{-6} m^2]$

$[Pa] = \frac{N}{m^2} = \frac{kg}{m s^2}$

$[1 \frac{l}{s}] = [\frac{1}{1000} \frac{m^3}{s}]$

$[R] = \frac{J}{kg K}$

$[N] = \frac{kg m}{s^2}$

$[\frac{kJ}{min} : 60 = kW]$

Differentialquotienten

Chemisches Potential

$$\mu_i = \left(\frac{\partial U}{\partial n_i}\right)_{S,V,\ n_j \neq n_i}$$

$$Z_{pm,i} = \left(\frac{\partial Z}{\partial n_i}\right)_{T,p,\ n_j \neq n_i}$$

Gibbssche Fundamentalgleichung

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,n_j} = T(S, V, n_1, n_1, ..., n_K)$$

$$-p = \left(\frac{\partial U}{\partial V}\right)_{S,n_j} = p(S, V, n_1, n_1, ..., n_K)$$

$$\mu_i = \left(\frac{\partial U}{\partial n_i}\right)_{S,V,\ n_j \neq n_i} = \mu_i(S, V, n_1, n_2, ..., n_K)$$

Enthalpie

$$T = \left(\frac{\partial H}{\partial S}\right)_{p,n_j}$$

$$V = \left(\frac{\partial H}{\partial p}\right)_{S,n_j}$$

$$\mu_i = \left(\frac{\partial H}{\partial n_i}\right)_{S,p,\ n_j \neq n_i}$$

Freie Energie

$$-S = \left(\frac{\partial F}{\partial T}\right)_{V,n_j}$$

$$-p = \left(\frac{\partial F}{\partial V}\right)_{T,n_j}$$

$$\mu_i = \left(\frac{\partial F}{\partial n_i}\right)_{T,V,\ n_j \neq n_i}$$

Freie Enthalpie

$$-S = \left(\frac{\partial G}{\partial T}\right)_{p,n_j}$$

$$V = \left(\frac{\partial G}{\partial p}\right)_{T,n_j}$$

$$\mu_i = \left(\frac{\partial G}{\partial n_i}\right)_{T,p,\ n_j \neq n_i}$$

$$= G_{pm,i}$$

$$= H_{pm,i} - TS_{pm,i}$$

Maxwellsche Beziehungen

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j} \qquad \left(\frac{\partial T}{\partial V}\right)_{S,n_j} = -\left(\frac{\partial p}{\partial S}\right)_{V,n_j}$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j} \qquad \left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j}$$

$$\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,\ n_j \neq n_i} \qquad \left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,\ n_j \neq n_i}$$

Van-der-Waals-Gas

$$\left(\frac{\partial p}{\partial v}\right)_{T_K} = \frac{2a}{v_K^3} - \frac{RT_K}{(v_K - b)^2} = 0$$

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_{T_K} = -\frac{6a}{v_K^4} + \frac{2RT_K}{(v_K - b)^3} = 0$$

Spezifische Wärmekapazität

Konstantes Volumen

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = \frac{R}{\kappa - 1}$$

Konstanter Druck

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = R\frac{\kappa}{\kappa - 1}$$

Zusammenhang

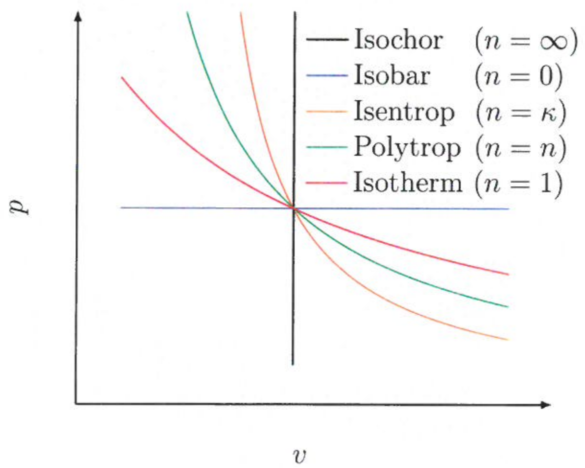
$$\kappa = \frac{c_p}{c_v} \quad \Big| \quad R = c_p - c_v$$

$pv^n = konst$

Diagramme

$c_p = c_v + \frac{T v \beta^2}{\varkappa}$

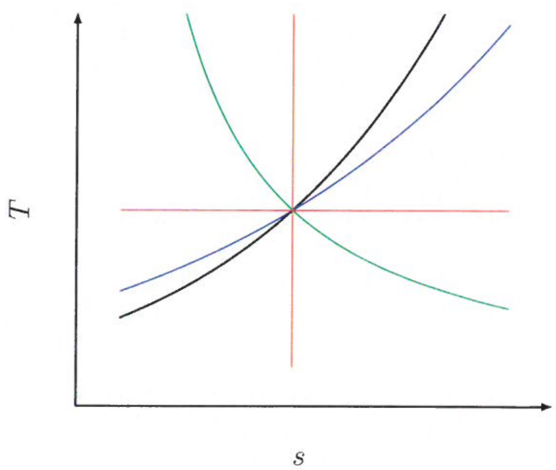
p-v-Diagramm



$n < \kappa$: Polytrope Steigung < Isentrope Steigung

$n > \kappa$: Polytrope Steigung > Isentrope Steigung

T-s-Diagramm



$n < \kappa$: Polytrope nach rechts gekrümmt

$n > \kappa$: Polytrope nach links gekrümmt

Isochor $(n = \infty)$
Isobar $(n = 0)$
Isentrop $(n = \kappa)$
Polytrop $(n = n)$
Isotherm $(n = 1)$

Hauptsätze der Thermodynamik

0. Hauptsatz	1. Hauptsatz	2. Hauptsatz	3. Hauptsatz
Thermodynamisches Gleichgewicht:	Abgeschlossenes System:	Entropieänderung eines Systems:	Systemunabhängig:
$T_A = T_B$ $T_A = T_B \wedge T_B = T_C \Longleftrightarrow T_A = T_C$	$E_{\text{ges}} = \text{konst.}$	$dS_{System} = \frac{\delta Q_{rev}}{T} + dS_{prod}$	$\lim\limits_{T\to 0\text{K}} S = S_0 := 0\frac{\text{J}}{\text{K}}$

Anwendung der Hauptsätze

$Q_{12} = H_2 - H_1$

1. Hauptsatz

Geschlossenes Instationäres System

Allg.: $$\frac{dU_{System}}{dt} = \boxed{\sum_j \dot{Q}_j + \sum_k \dot{W}_k}$$

Einfach: $$dU = \delta Q - pdV$$
$$dh = \delta q + \delta w_{diss,12} + vdp$$

Offenes Stationäres System

$$0 = \dot{m}\left(h_1 + \frac{c_1^2}{2} + gz_1\right) - \dot{m}\left(h_2 + \frac{c_2^2}{2} + gz_2\right) + \dot{Q}_{12} + \dot{W}_{t,12}$$

Enthalpie

$$H = U + pV$$
$$dH = dU + pdV + Vdp$$
$$dH = TdS + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$

Volumenänderungsarbeit

$$W_{V,12} = -\int_1^2 pdV$$

Technische Arbeit

$$W_{t,12} = p_2V_2 - p_1V_1 - \int_1^2 pdV = \int_1^2 Vdp$$
$$w_{t,12} = w_{\text{diss},12} + \int_1^2 vdp + \frac{c_2^2}{2} - \frac{c_1^2}{2} + gz_2 - gz_1$$

2. Hauptsatz

Entropieänderung

$$dS_{System} = dS_a + dS_{prod} = \frac{\delta Q_{rev}}{T} + \frac{\delta \Psi}{T}$$
$$dS_{prod} \overset{!}{>} 0$$

Dissipationsenergie

$$\Psi = \int_1^2 TdS_{prod}$$

Entropierate für Offenes System

$$\frac{dS_{System}}{dt} = \sum_j (\dot{m}_j \dot{s}_j)_{\substack{\text{über}\\ \text{Systemgrenze}}} + \sum_l \left(\frac{\dot{Q}_l}{T_l}\right)_{\substack{\text{über}\\ \text{Systemgrenze}}} + \left(\dot{S}_{prod}\right)_{\substack{\text{im}\\ \text{System}}}$$

Folgerungen aus den Hauptsätzen

Chemisches Potential

$$dU = \sum_{k=1}^{K} \left(\frac{\partial U}{\partial n_k}\right)_{\substack{S,V,\\ n_j \neq n_i}} dn_k = \sum_{k=1}^{K} \mu_k dn_k$$

Gibbssche Fundamentalgleichung

Mehrstoffsysteme

$$dU = TdS - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

Reinstoffsysteme

$$dU = TdS - pdV$$

Kalorische Zustandsgleichung

$$U = U(T, V, n_1, n_2, ..., n_K)$$

Thermische Zustandsgleichung

$$p = p(T, V, n_1, n_2, ..., n_K)$$

Thermodynamische Potentiale

$$U = U(S, V, n_1, n_2, ..., n_K)$$
$$H = H(S, p, n_1, n_2, ..., n_K)$$
$$F = F(T, V, n_1, n_2, ..., n_K)$$
$$G = G(T, p, n_1, n_2, ..., n_K)$$

Eulergleichung

$$U = TS - pV + \sum_{k=1}^{K} \mu_k n_k$$

Gibbs-Duhem Gleichung

$$0 = SdT - Vdp + \sum_{k=1}^{K} n_k d\mu_k$$
$$= S_m dT - V_m dp + \sum_{k=1}^{K} \psi_k d\mu_k$$

$F = U - TS \quad ; U = F + TS$

Freie Energie

$$dF = -SdT - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

Freie Enthalpie

$$dG = -SdT + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$
$$G = U + pV - TS$$

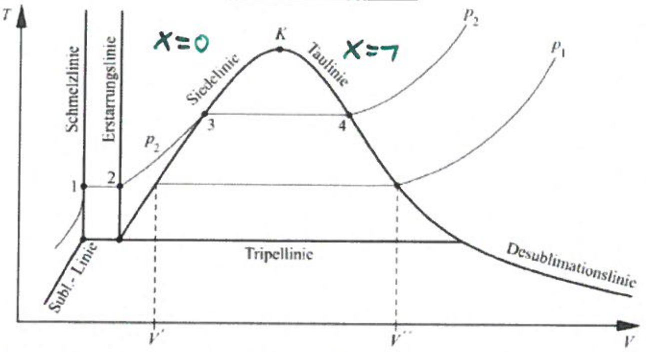
$g = h - Ts$

↳ Masse der Gasförmigen Phase

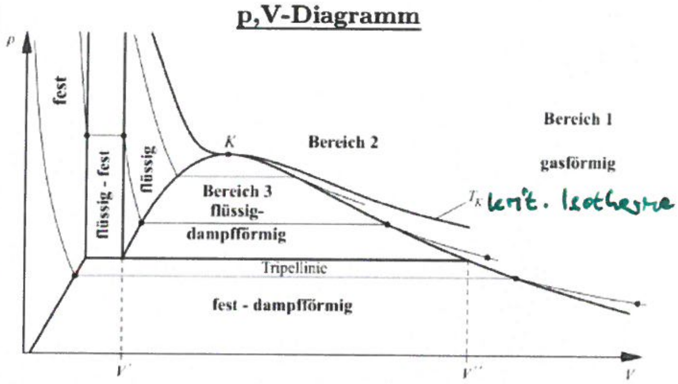
Dampfgehalt $x = \frac{m''}{m''+m'}$

Reale Stoffe

T,V-Diagramm



p,V-Diagramm



Nassdampfgebiet

Dampfgehalt

$$x = \frac{m''}{m''+m'} = \frac{m_{Dampf}}{m_{gesamt}}$$

Gibbssche Phasenregel

$$F = K + 2 - P$$

Clausius-Clapeyronsche Gleichung

$$\frac{dp}{dT} = \frac{s''-s'}{v''-v'} = \frac{1}{T}\frac{h''-h'}{v''-v'} = \frac{1}{T}\frac{r}{v''-v'}$$

Kritischer Punkt

$$\left(\frac{\partial p}{\partial v}\right)_{T_K} = 0$$

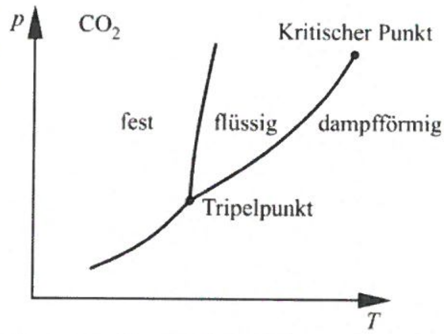
$$\left(\frac{\partial^2 p}{\partial v^2}\right)_{T_K} = 0$$

Zustandsgrößen

$$u''-u' = -p\,(v''-v') + T\,(s''-s')$$

$$\underbrace{v = v' + x\,(v''-v')}_{\text{Analog für u,h,s,...}}$$

p,T-Diagramm



Zustandsgleichungen

Ableitungen

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \qquad \left[\frac{1}{\mathrm{K}}\right]$$

$$\gamma = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V \qquad \left[\frac{1}{\mathrm{K}}\right]$$

$$\chi = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \quad \left[\frac{\mathrm{ms}^2}{\mathrm{kg}}\right]$$

$$\beta = p\gamma\chi$$

Kalorische

$$du = \left(\frac{\partial u}{\partial v}\right)_T dv + c_v\,(v,T)\,dT$$

$$dh = \left(\frac{\partial h}{\partial p}\right)_T dp + c_p\,(p,T)\,dT$$

Wärmekapazitäten

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

$$c_p = c_v + \frac{Tv\beta^2}{\chi}$$

Entropieänderung

$$ds = \left\{\frac{1}{T}\left(\frac{\partial u}{\partial v}\right)_T + \frac{p}{T}\right\} dv + \frac{c_v}{T} dT$$

Innere Energieänderung

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T - p = T\left(\frac{\partial p}{\partial T}\right)_v - p$$

$$\dot{Q} = \dot{m}\, c_p\,(T_{außen} - T_{innen})$$

$m = M \cdot n$ [mol]

$c_p = \frac{\kappa}{\kappa-1} R \quad [\frac{J}{kg\,K}]$

$c_v = \frac{1}{\kappa-1} R \quad [\frac{J}{kg\,K}]$

$c_n = \frac{n-\kappa}{n-1} c_v \quad [\frac{J}{kg\,K}]$

Ideales Gas

Thermische Zustandsgleichung

$p = \rho R T$

$\rho = \frac{m}{V}$

$$pV = mRT = \frac{m}{M} R_m T = n R_m T$$

$= \frac{m}{V} [\frac{kg}{m^3}]$

$$pv = RT = \frac{R_m}{M} T = n \frac{R_m}{m} T$$

Koeffizienten

$$\beta = \frac{1}{T}$$
$$\gamma = \frac{1}{T}$$
$$\chi = \frac{1}{p}$$

Innere Energie

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT = C_v(T) dT$$
$$\Rightarrow U_2 - U_1 = \underbrace{m c_v (T_2 - T_1)}_{c_v = \text{konst.}}$$

Enthalpie

$$H_2 - H_1 = m \int_{T_0}^{T} c_p\left(\tilde{T}\right) d\tilde{T} = \underbrace{m c_p (T_2 - T_1)}_{c_p = \text{konst.}}$$

Q_{12}

Gaskonstante

$$R = p \left(\frac{\partial v}{\partial T}\right)_p = c_p(T) - c_v(T)$$

$R = \frac{R_m}{M}$

$$R_m = 8,3143 \frac{\text{J}}{\text{mol K}}$$

$[\frac{J}{kg\,K}]$

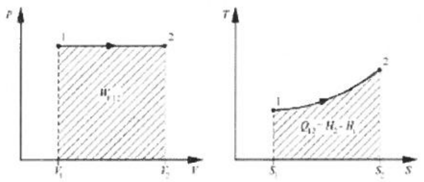
Entropieänderung

$$ds = \frac{R}{v} dv + \frac{c_v(T)}{T} dT \Longleftrightarrow s_2 - s_1 = R \ln\left(\frac{v_2}{v_1}\right) + \int_{T_1}^{T_2} \frac{c_v\left(\tilde{T}\right)}{\tilde{T}} d\tilde{T}$$

$$s_2 - s_1 = R \ln\left(\frac{v_2}{v_1}\right) + c_v \ln\left(\frac{T_2}{T_1}\right) = c_v \ln\left(\frac{p_2}{p_1}\right) + c_p \ln\left(\frac{v_2}{v_1}\right) = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

Zustandsänderungen

Isobar $p = \text{konst.}$

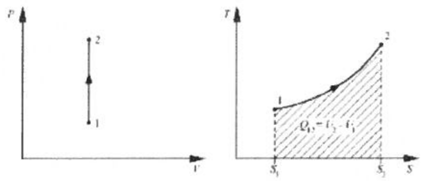


$$W_{V,12} = -p(V_2 - V_1)$$

$$Q_{12} = U_2 - U_1 + p(V_2 - V_1) = H_2 - H_1 = m \int_1^2 c_p(T) dT = \underbrace{m c_p (T_2 - T_1)}_{c_p = \text{konst.}}$$

$$S_2 - S_1 = \underbrace{m c_p \ln\left(\frac{T_2}{T_1}\right)}_{c_p = \text{konst.}}$$

Isochor $v = \text{konst.}$

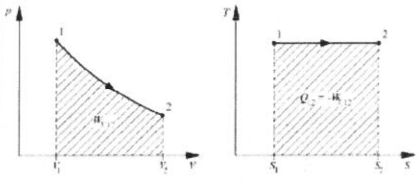


$$W_{V,12} = 0$$

$$Q_{12} = U_2 - U_1 = m \int_1^2 c_v(T) dT = \underbrace{m c_v (T_2 - T_1)}_{c_v = \text{konst.}}$$

$$S_2 - S_1 = \underbrace{m c_v \ln\left(\frac{T_2}{T_1}\right)}_{c_v = \text{konst.}}$$

Isotherm $T = \text{konst.}$



$$W_{V,12} = -Q_{12} = -mRT \ln\left(\frac{V_2}{V_1}\right)$$

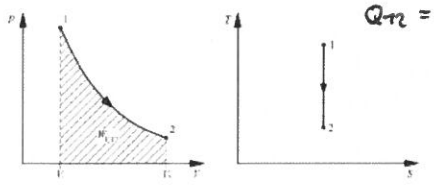
$$Q_{12} = T(S_2 - S_1)$$

$$S_2 - S_1 = \frac{Q_{12}}{T}$$

Reversibel Adiabat

$s = \text{konst.}$

$Q_{12} = 0$



$$\int_1^2 \frac{c_v(T)}{T} dT = R \ln\left(\frac{v_1}{v_2}\right)$$

$$T_2 v_2^{(\kappa-1)} = T_1 v_1^{(\kappa-1)}$$
$$p_2 v_2^{\kappa} = p_1 v_1^{\kappa}$$
$$\kappa = \frac{c_p}{c_v}$$

Polytrop

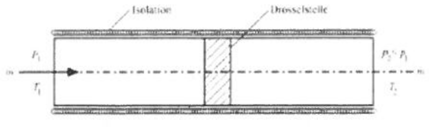
$$pv^n = \text{konst.}$$
$$Tv^{n-1} = \text{konst.}$$
$$Tp^{\frac{1-n}{n}} = \text{konst.}$$

$$W_{V,12} = -\frac{p_1 V_1}{n-1} \left[1 - \left(\frac{V_1}{V_2}\right)^{n-1}\right]$$

$$Q_{12} = m c_v \frac{n-\kappa}{n-1} (T_2 - T_1) = m c_n (T_2 - T_1)$$

Isobare:	$n = 0$
Isochore:	$n \to \infty$
Isotherme:	$n = 1$
Reversibel Adiabate:	$n = \kappa$

Adiabate Drosselung



$$dh = dT = 0$$

$$h_1 + \frac{c_1^2}{2} + g z_1 = h_2 + \frac{c_2^2}{2} + g z_2$$

$$s_2 - s_1 = R \ln\left(\frac{v_2}{v_1}\right) = R \ln\left(\frac{p_1}{p_2}\right)$$

Gemische Idealer Gase

Definitionen

Massenanteil	Molanteil	Partialdruck	Innere Energie	Thermische Zustandsgleichung
$\xi_i = \frac{m_i}{m}$	$\Psi_i = \frac{n_i}{n}$ ≙ Volumenanteile	$p_i = \Psi_i p$	$U_G = \sum_{k=1}^{K} c_{vk} m_k T$	$p_i V = m_i R_i T$ $p_i V = n_i R_m T$ $pV = m R_G T$
Mittlere Molmasse	**Zusammenhang**	**Wärmekapazitäten**	**Enthalpie**	**Mittlere Spezifische Gaskonstante**
$M_G = \frac{m}{n} = \frac{\sum_{k=1}^{K} M_k n_k}{n}$	$\xi_i = \frac{M_i n_i}{\sum_{k=1}^{K} M_k n_k} = \frac{M_i}{M_G}\Psi_i$	$c_{vG} = \sum_{k=1}^{K} c_{vk}\xi_k$ $c_{pG} = \sum_{k=1}^{K} c_{pk}\xi_k$	$H_G = \sum_{k=1}^{K} c_{pk} m_k T$	$R_G = \frac{1}{m}\sum_{k=1}^{K} m_k R_k = \sum_{k=1}^{K} \xi_k R_k$

Entropieerhöhung bei Vermischung

$$S_2 - S_1 = \frac{1}{T} p_I V_I \ln\left(\frac{V}{V_I}\right) + \frac{1}{T} p_{II} V_{II} \ln\left(\frac{V}{V_{II}}\right) \qquad \left[\frac{J}{K}\right]$$

$$= R_m \left[n \ln(n) - \sum_{k=1}^{K} n_k \ln(n_k) \right]$$ ← Die hier wenn möglich!

Van-der-Waals-Gas t4

Thermische Zustandsgleichung

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

↳ $\left[\frac{m^3}{kg}\right]$

$$\left(\bar{p} + \frac{3}{\bar{v}^2}\right)(3\bar{v} - 1) = 8\bar{T}$$

Reduzierte Variablen

→ im krit. Punkt

$$\bar{p} = \frac{p}{p_K} \quad \Big| \quad \bar{v} = \frac{v}{v_K} \quad \Big| \quad \bar{T} = \frac{T}{T_K}$$

Kritischer Punkt

$$a = 3 p_K v_K^2 \left[\frac{m^5}{kg\, s^2}\right]$$

$$b = \frac{v_K}{3} \left[\frac{m^3}{kg}\right]$$

$$\frac{3}{8} = \frac{p_K v_K}{R T_K}$$

$R = \frac{R_m}{M}$

Koeffizienten

Normal $\left[\frac{1}{K}\right]$

1 $$\beta = \frac{(v-b)Rv^2}{RTv^3 - 2a(v-b)^2}$$

2 $$\gamma = \frac{Rv^2}{RTv^2 - a(v-b)}$$

3 $$\chi = \frac{(v-b)^2 v^2}{RTv^3 - 2a(v-b)^2}$$

Reduziert

4 $$\beta T_K = \frac{8(3\bar{v}-1)\bar{v}^2}{24\bar{T}\bar{v}^3 - 6(3\bar{v}-1)^2}$$

5 $$\gamma T_K = \frac{\bar{v}^2}{\bar{T}\bar{v}^2 - \frac{3}{8}(3\bar{v}-1)}$$

6 $$\chi p_K = \frac{(3\bar{v}-1)^2 \bar{v}^2}{24\bar{T}\bar{v}^3 - 6(3\bar{v}-1)^2}$$

Kalorische Differenz

$$c_p - c_v = \frac{Tv\beta^2}{\chi} = \left[\left(\frac{\partial u}{\partial v}\right)_T + p\right]\left(\frac{\partial v}{\partial T}\right)_p$$

$$= \frac{R}{1 - \frac{2a(v-b)^2}{RTv^3}} = \frac{R}{1 - \frac{(3\bar{v}-1)^2}{4\bar{T}\bar{v}^3}}$$

Innere Energie

$$du = \frac{a}{v^2}dv + c_v(T)dT$$

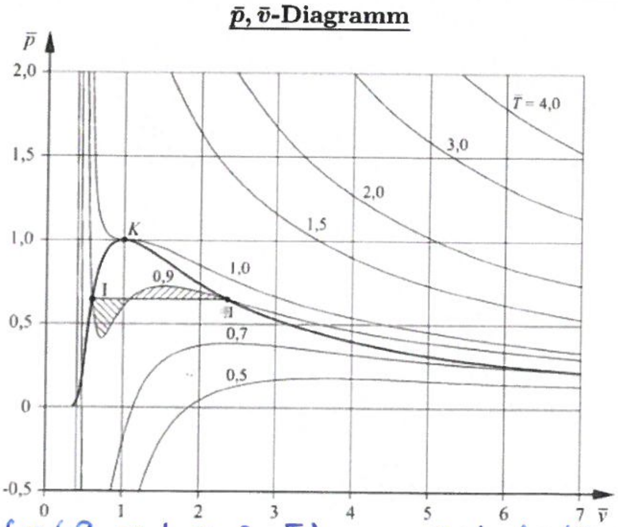
$$\underbrace{u_2 - u_1 = \left(\frac{a}{v_1} - \frac{a}{v_2}\right) + c_v(T_2 - T_1)}_{c_v = \text{konst.}}$$

Entropieänderung

$$ds = \left(\frac{a}{v^2} + p\right)\frac{1}{T}dv + \frac{c_v}{T}dT = \frac{R}{v-b}dv + \frac{c_v}{T}dT$$

$$\underbrace{s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2 - b}{v_1 - b}\right)}_{c_v = \text{konst.}}$$

$\bar{p}, \bar{v}$-Diagramm



$V_i = \frac{1}{p_i}$

Volumenänderungsarbeit

$$\int_{v'}^{v''} p\,dv = p(v'' - v') = \left(\frac{a}{v''} - \frac{a}{v'}\right) + RT\ln\left(\frac{v'' - b}{v' - b}\right)$$

Van-der-Waals-Typ-Gas

$$\left(p + \frac{a}{v^2 + cbv + db^2}\right)(v - b) = RT$$

Gleichung	a	b	c	d
Soave-Redlich-Kwong	$0,42748 \frac{R^2 T_K^2}{p_K}\left(1 + f_\omega\left(1 - \sqrt{\bar{T}}\right)^2\right)$ $f_\omega = 0,48 + 1,574\omega - 0,176\omega^2$	$0,08664 \frac{R T_K}{p_K}$	1	0
Peng-Robinson	$0,45724 \frac{R^2 T_K^2}{p_K}\left(1 + f_\omega\left(1 - \sqrt{\bar{T}}\right)^2\right)$ $f_\omega = 0,3746 + 1,542\omega - 0,2699\omega^2$	$0,0778 \frac{R T_K}{p_K}$	2	−1

1: t4f1(β, v, b, a, R, T)
2: t4f2(γ, R, v, T, a, b)
3: t4f3(χ, v, b, a, R, T)
4: t4f4(β, T_K, $\bar{T}$, $\bar{v}$)
5: t4f5(γ, T_K, $\bar{T}$, $\bar{v}$)
6: t4f6(χ, p_K, $\bar{T}$, $\bar{v}$)

Van-der-Waals-Gas

Zustandsänderungen

Isobar

$$q_{12} = u_2 - u_1 + p\,(v_2 - v_1) = h_2 - h_1$$
$$= \frac{a}{v_1} - \frac{a}{v_2} + \underbrace{c_v\,(T_2 - T_1)}_{c_v=\text{konst.}} + p\,(v_2 - v_1)$$

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2 - b}{v_1 - b}\right)$$

Isochor

$$q_{12} = u_2 - u_1 = \underbrace{c_v\,(T_2 - T_1)}_{c_v=\text{konst.}}$$

$$s_2 - s_1 = \underbrace{c_v \ln\left(\frac{T_2}{T_1}\right)}_{c_v=\text{konst.}}$$

Isotherm

$$q_{12} = -w_{V,12} + u_2 - u_1 = RT_1 \ln\left(\frac{v_2 - b}{v_1 - b}\right)$$

$$s_2 - s_1 = R\ln\left(\frac{v_2 - b}{v_1 - b}\right)$$

$$w_{V,12} = -\int_1^2 p\,dv = -\int_1^2 \left(\frac{RT_1}{v - b} - \frac{a}{v^2}\right) dv$$
$$= RT_1 \ln\left(\frac{v_1 - b}{v_2 - b}\right) - \frac{a}{v_2} + \frac{a}{v_1}$$

Adiabat

$$\int_1^2 \frac{c_v(T)}{T}\,dT = R\ln\left(\frac{v_1 - b}{v_2 - b}\right) \xRightarrow[\text{konst.}]{c_v} T(v-b)^{\frac{R}{c_v}} = \frac{1}{R}\left(p + \frac{a}{v^2}\right)(v-b)^{\frac{c_v + R}{c_v}} = \text{konst.}$$

Adiabate Drosselung

Drosselkoeffizienten

$$dh = c_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp = 0 \Rightarrow \underbrace{\left(\frac{\partial T}{\partial p}\right)_h}_{\delta_h} = -\frac{1}{c_p}\underbrace{\left(\frac{\partial h}{\partial p}\right)_T}_{\delta_T}$$

$$\delta_T = v + T\left(\frac{\partial s}{\partial p}\right)_T$$

$\left(\delta_h = \left(\frac{\partial T}{\partial p}\right)_h = \frac{\Delta T}{\Delta p}\right.$ Gilt je nach Aufgabe $\left.\right)$

$$\delta_h = -\frac{v}{c_p}(1 - \beta T)$$
$$= -\frac{v}{c_p}\left(\frac{RTv^3 - 2a(v-b)^2 - T(v-b)Rv^2}{RTv^3 - 2a(v-b)^2}\right)$$

$\delta_h < 0$: Temperaturerhöhung

$\delta_h > 0$: Temperaturverminderung

z.B. Werte für Zustand 2: $\delta_{h\,2\to 4}$ $\left[\frac{\text{K m s}^2}{\text{kg}}\right]$

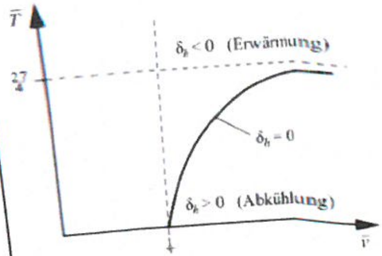
tufz($\delta_h, a, b, R, T, v, c_p$)

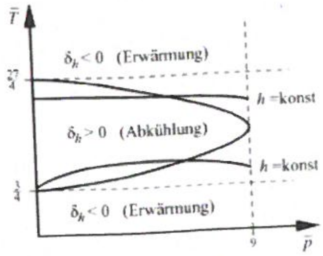
Inversionslinie

$$\frac{RT}{2} = \frac{a}{b}\frac{(v-b)^2}{v^2}$$

$$\frac{4}{27}\bar{T} = \frac{(3\bar{v} - 1)^2}{9\bar{v}^2}$$

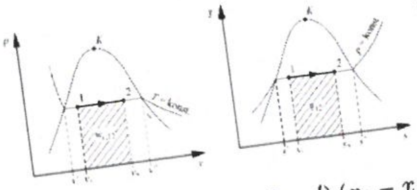
$$\bar{p} = 24\sqrt{3\bar{T}} - 12\bar{T} - 27$$





Zustandsänderungen im Nassdampfgebiet

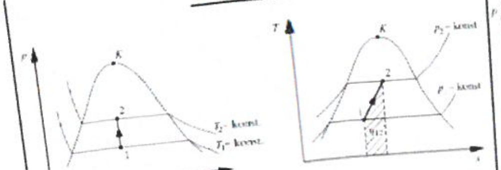
Isobar



$$q_{12} = T\,(s_2 - s_1) = T\,(s'' - s')\,(x_2 - x_1)$$

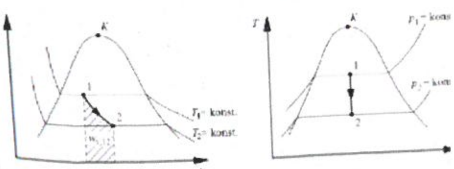
$$w_{V,12} = -\int_1^2 p\,dv = -p\,(v_2 - v_1)$$
$$= -p\,(v'' - v')\,(x_2 - x_1)$$

Isochor

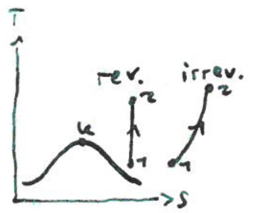


$$q_{12} = u_2 - u_1$$
$$= u' + x_2\,(u_2'' - u_2')$$
$$- u_1' - x_1\,(u_1'' - u_1')$$

Reversibel Adiabat



$$w_{V,12} = u_2 - u_1$$
$$= u_2' + x_2\,(u_2'' - u_2')$$
$$- u_1' - x_1\,(u_1'' - u_1')$$



9

$dU = 0 = \int q + \int w_v$

$\Rightarrow W_{ges} = -q_{ges}$

Zustandsbeziehungen

Ideales Gas

"Isentrop" $\Delta s = 0$

	Isotherm	Isobar	Isochor	Reversibel Adiabat	Polytrop
	$T = \text{konst.}$	$p = \text{konst.}$	$v = \text{konst.}$	$\delta q = 0$	$pv^n = \text{konst.}$
Beziehung zwischen den Zuständen 1 und 2	-	-	-	$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^{\kappa}$	$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^{n}$
	$\frac{p_1}{p_2} = \frac{v_2}{v_1}$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\kappa-1}$	$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1}$
	-	-	-	$\left(\frac{T_1}{T_2}\right)^{\frac{\kappa}{\kappa-1}} = \frac{p_1}{p_2}$	$\left(\frac{T_1}{T_2}\right)^{\frac{n}{n-1}} = \frac{p_1}{p_2}$
p, v	$\frac{p_2}{p_1} = \frac{v_1}{v_2}$	$p_2 = p_1$	$v_2 = v_1$	$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\kappa}$	$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{n}$
p, T	$T_2 = T_1$	$p_2 = p_1$	$\frac{p_2}{p_1} = \frac{T_2}{T_1}$	$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}}$	$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$
v, T	$T_2 = T_1$	$\frac{v_2}{v_1} = \frac{T_2}{T_1}$	$v_2 = v_1$	$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\kappa-1}$	$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$
q_{12}	$pv = RT$ $q_{12} = p_1 v_1 \ln\left(\frac{p_1}{p_2}\right)$	$q_{12} = c_p(T_2 - T_1)$ $\cdot m$ für Q_{12}	$q_{12} = c_v(T_2 - T_1)$	$q_{12} = 0$	$q_{12} = c_v \frac{n-\kappa}{n-1}(T_2 - T_1)$
$w_{V,12}$	$w_{V,12} = -q_{12}$	$w_{V,12} = -p_1(v_2 - v_1)$	$w_{V,12} = 0$	$w_{V,12} = \frac{p_1 v_1}{\kappa - 1}\left[\left(\frac{v_1}{v_2}\right)^{\kappa-1} - 1\right]$	$w_{V,12} = \frac{p_1 v_1}{n-1}\left[\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right]$
$s_2 - s_1$	$s_2 - s_1 = R \ln\left(\frac{p_1}{p_2}\right)$	$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right)$	$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right)$	$s_2 - s_1 = 0$	$s_2 - s_1 = c_v \frac{n-\kappa}{n-1} \ln\left(\frac{T_2}{T_1}\right)$

Van-der-Waals-Gas t_4

	Isotherm	Isobar	Isochor	Reversibel Adiabat
	$T = \text{konst.}$	$p = \text{konst.}$	$v = \text{konst.}$	$\delta q = 0$
Beziehung zwischen den Zuständen 1 und 2	$\frac{p_1 + \frac{a}{v_1^2}}{p_2 + \frac{a}{v_2^2}} = \frac{v_2 - b}{v_1 - b}$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v_2 - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_2^2}}{T_2}$	$p_{M1} v_{M1}^{\kappa_M} = p_{M2} v_{M2}^{\kappa_M}$ $T_1 v_{M1}^{\frac{R}{c_v}} = T_2 v_{M2}^{\frac{R}{c_v}}$
p, v	$p_2 = \frac{v_1 - b}{v_2 - b}\left(p_1 + \frac{a}{v_1^2}\right) - \frac{a}{v_2^2}$	$p_2 = p_1$	$v_2 = v_1$	$p_2 = -\frac{a}{v_2^2} + p_{M1}\left(\frac{v_{M1}}{v_{M2}}\right)^{\frac{R}{c_v}+1}$
p, T	$T_2 = T_1$	$p_2 = p_1$	$p_2 = \frac{T_2}{T_1} p_{M1} - \frac{a}{v_1^2}$	$p_2 = -\frac{a}{v_2^2} + p_{M1}\left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}+1}$
v, T	$T_2 = T_1$	$T_2 = T_1 \frac{v_2 - b}{v_1 - b} + \frac{a}{R}(v_2 - b)\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right)$ 8	$v_2 = v_1$	$T_2 = T_1 \left(\frac{v_1 - b}{v_2 - b}\right)^{\frac{R}{c_v}}$
$\left[\frac{kJ}{kg}\right]$ q_{12}	$q_{12} = RT_1 \ln\left(\frac{v_2 - b}{v_1 - b}\right)$	$q_{12} = \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) + p_1(v_2 - v_1)$ 9	$q_{12} = c_v(T_2 - T_1)$	$q_{12} = 0$
$w_{V,12}$	$w_{V,12} = -RT_1 \ln\left(\frac{v_2 - b}{v_1 - b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$	$w_{V,12} = -p_1(v_2 - v_1)$	$w_{V,12} = 0$	$w_{V,12} = \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$
$\left[\frac{J}{kgK}\right]$ $s_2 - s_1$	$s_2 - s_1 = R \ln\left(\frac{v_2 - b}{v_1 - b}\right)$	$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2 - b}{v_1 - b}\right)$	$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right)$	$s_2 - s_1 = 0$

$$p_M = p + \frac{a}{v^2} \quad \Big| \quad v_M = v - b \quad \Big| \quad \kappa_M = \frac{c_v + R}{c_v}$$

wichtig, nicht übersehen!

8: $t_4f8(T_2, T_1, v_2, v_1, a, b, R)$

9: $t_4f9(q_{12}, T_2, T_1, v_2, v_1, p_1, a, c_v)$

Exergie

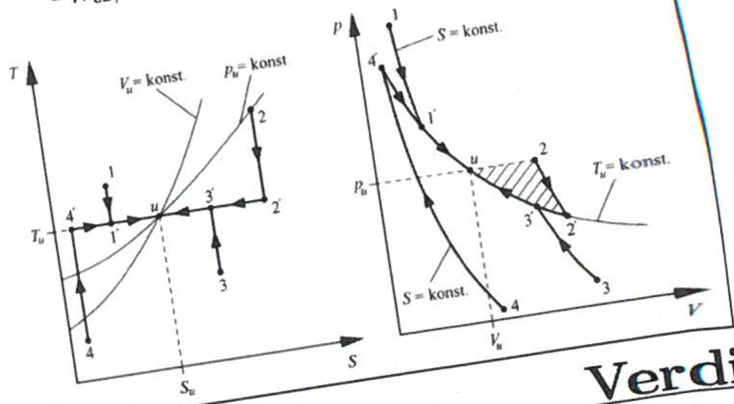
Definition

$$-\dot W_{ex} = -\frac{d}{dt}\left\{U + m\left(\frac{c^2}{2} + gz\right) + p_u V - T_u S\right\}_{\text{System}} + \sum_{j=1}^{K}\left[\dot m_j\left(h + \frac{c^2}{2} + gz + T_u s\right)_j\right]_{\substack{\text{über}\\ \text{Systemgrenze}}} + \sum_{l=1}^{N}\left(1 - \frac{T_u}{T_{\text{Wärmebehälter } l}}\right)\dot Q_{\text{Wärmebehälter}}$$

Geschlossenes Instationäres System

$$-\dot W_{ex,1u} = U_1 - U_u + p_u (V_1 - V_u) - T_u (S_1 - S_u)$$

$$-\dot W_{ex,2u} = m c_p T_u \left[\frac{T_2}{T_u} - 1 - \ln\left(\frac{T_2}{T_u}\right)\right]$$

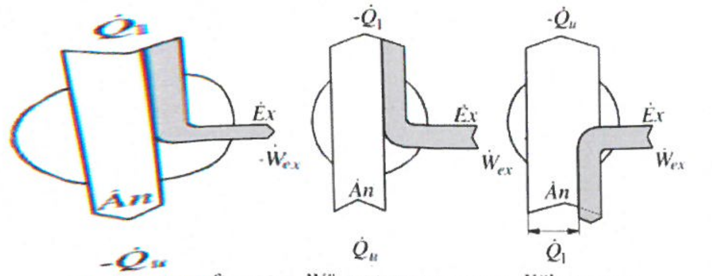


Offenes Stationäres System

$$-\dot W_{ex,1u} = \dot m \left[h_1 - h_u - T_u (s_1 - s_2)\right]$$

Geschlossenes Stationäres System

$$-\dot W_{ex} = \left(1 - \frac{T_u}{T_1}\right)\dot Q_1 = \eta_{th,Carnot}\,\dot Q_1$$



Wärmekraftprozess Wärmepumpenprozess Kälteprozess

Verdichter

Kolbenverdichter

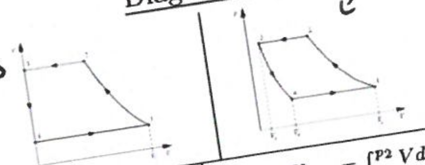
Schädlicher Raum

$$\mu = \frac{V_1 - V_4}{V_1 - V_3} \qquad \varepsilon_S = \frac{V_3}{V_1 - V_3} \qquad \mu = 1 - \varepsilon_S\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right]$$

ohne schädlichen Raum

mit schädlichem Raum

Diagramme



	$W_{t,12}$	Q_{12}
Allgemein	$W_{t,12} = \int_{p_1}^{p_2} V\,dp = p_2V_2 - p_1V_1 - \int_{V_1}^{V_2} p\,dV$	$Q_{12} = (H_2 - H_1) - W_{t,12}$
Adiabat	$W_{t,12} = H_2 - H_1 = mc_p (T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2V_2 - p_1V_1)$ 1	$Q_{12} = 0$
Reversibel Adiabat	$W_{t,12} = \frac{\kappa}{\kappa-1} p_1 V_1 \left(\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right) = mc_p (T_2 - T_1)$ 2	$Q_{12} = 0$
Irreversibel Adiabat (Als Polytrop, $n > \kappa$)	$W_{t,12} = \frac{\kappa}{\kappa-1} p_1 V_1 \left(\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right) = mc_p (T_2 - T_1)$ 3	$Q_{12} = 0$
Reversibel Polytrop	$W_{t,12} = \frac{n}{n-1}(p_2V_2 - p_1V_1) = \frac{n}{n-1} mR (T_2 - T_1) = \frac{n}{n-1} p_1 V_1 \left(\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right)$ 4	$Q_{12} = mc_n (T_2 - T_1) = \frac{n-\kappa}{(n-1)(\kappa-1)} p_1 V_1 \left(\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right.$; $c_n = \frac{n-\kappa}{n-1} c_v$
Isotherm	$W_{t,12} = p_1 V_1 \ln\left(\frac{p_2}{p_1}\right)$	$Q_{12} = -W_{t,12}$

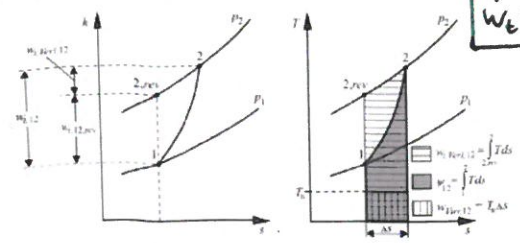
Bei schädlichem Raum: $V_1 - V_4$ statt V_1 Wichtig, nicht vergessen!

$\dot m = \rho_1$
$\dot W_t = n$

Turboverdichter

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1} = \underbrace{\frac{T_{2,rev} - T_1}{T_2 - T_1}}_{c_p = \text{konst.}}$$

$$\eta_{sT} = \frac{w_{t,12}}{w_{t,12,rev}} = \frac{h_1 - h_2}{h_1 - h_{2,rev}} = \underbrace{\frac{T_1 - T_2}{T_1 - T_{2,rev}}}_{c_p = \text{konst.}}$$



1: $t_2 f_1(W_{12}, p_1, p_2, V_1, V_2, k)$

2: $t_2 f_2(W_{12}, p_1, p_2, V_1, k)$

3: $t_2 f_3(W_{12}, p_1, p_2, V_1, k, n)$

4: $t_2 f_4(W_{12}, p_1, p_2, V_1, n)$

$\boxed{\Pi_{1-2} = \frac{p_2}{p_1}}$

Adiabat -> $q = 0$

Viertaktmotor: Zwei Umdrehungen pro Zyklus

$c_v = \frac{1}{\kappa - 1} R$

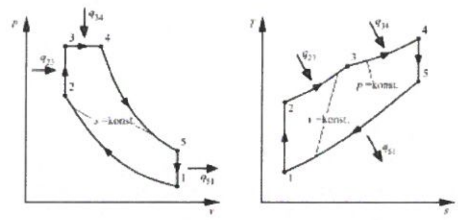
$|w| = -Q_{zu} - Q_{ab}$

Kreisprozesse F3

Wärmekraftprozesse

Seiliger-Prozess

$$\varepsilon = \frac{v_1}{v_2} \qquad \psi = \frac{p_3}{p_2} \qquad \varphi = \frac{v_4}{v_3}$$



$$\eta_{th} = 1 - \frac{|q_{ab}|}{q_{zu}} = 1 - \frac{u_5 - u_1}{u_3 - u_2 + h_4 - h_3}$$
$$= 1 - \frac{T_5 - T_1}{T_3 - T_2 + \frac{c_p}{c_v}(T_4 - T_3)}$$
$$= 1 - \frac{\varphi^\kappa \psi - 1}{\varepsilon^{\kappa-1}[\psi - 1 + \kappa\psi(\varphi - 1)]}$$

F3 f3 ($\eta_{th}, \kappa, \varphi, \psi, \varepsilon$)

$1 \to 2$: Reversibel Adiabate Verdichtung: $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\kappa-1} = \varepsilon^{\kappa-1}$

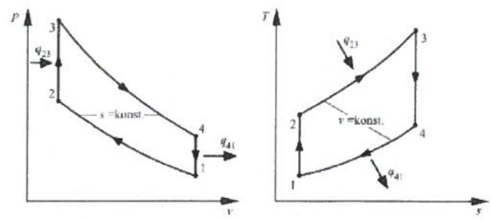
$2 \to 3$: Isochore Wärmezufuhr: $\frac{T_3}{T_2} = \frac{p_3}{p_2} = \psi$

$3 \to 4$: Isobare Wärmezufuhr: $\frac{T_4}{T_3} = \frac{v_4}{v_3} = \varphi$

$4 \to 5$: Reversibel Adiabate Entspannung: $\frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{\kappa-1} = \left(\frac{\varphi}{\varepsilon}\right)^{\kappa-1}$

$5 \to 1$: Isochore Wärmeabgabe: $\frac{T_5}{T_4} = \left(\frac{v_4}{v_1}\right)^{\kappa-1} = \left(\frac{\varphi}{\varepsilon}\right)^{\kappa-1}$

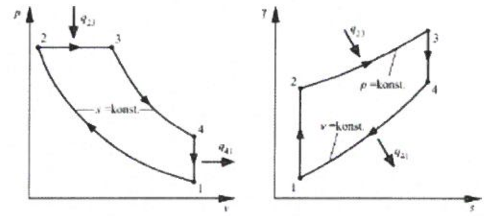
Otto-Prozess



$$\eta_{th} = 1 - \frac{|q_{41}|}{q_{23}} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$$

$\varphi = 1$ | $T_4 = T_3 \frac{1}{\varepsilon^{\kappa-1}}$

Diesel-Prozess



$$\eta_{th} = 1 - \frac{|q_{41}|}{q_{23}} = 1 - \frac{\varphi^\kappa - 1}{\varepsilon^{\kappa-1}\kappa(\varphi - 1)}$$

$\psi = 1$

Leistung

$$\dot{W} = W_t n_D = n m q_{zyk}$$

Otto/Diesel: $\frac{n_D}{2}$

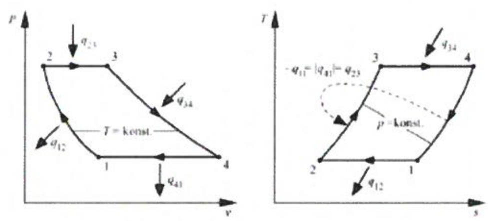
Drehzahl

$$n_D = \frac{\dot{m}}{m_A}$$

Wirkungsgrad

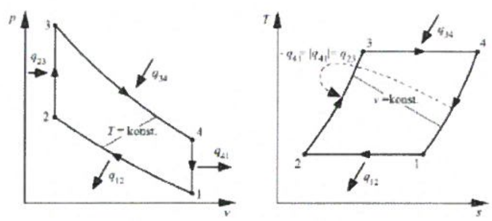
$$\left[\frac{\text{Nutzen}}{\text{Aufwand}}\right] \eta_{th} = \frac{P}{\dot{Q}_{ges}}$$

Ericsson-Prozess



$$\eta_{th} = 1 - \frac{|q_{12}|}{q_{34}} = 1 - \frac{RT_1 \ln\left(\frac{p_1}{p_2}\right)}{RT_3 \ln\left(\frac{p_4}{p_3}\right)} = 1 - \frac{T_1}{T_3}$$ 1

Stirling-Prozess



$$\eta_{th} = 1 - \frac{|q_{12}|}{q_{34}} = 1 - \frac{RT_1 \ln\left(\frac{v_1}{v_2}\right)}{RT_3 \ln\left(\frac{v_4}{v_5}\right)} = 1 - \frac{T_1}{T_3}$$ 2

Joule-Prozess

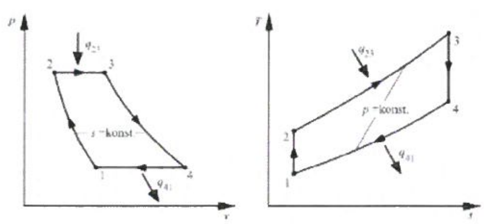
2 -> 3 Verdichten

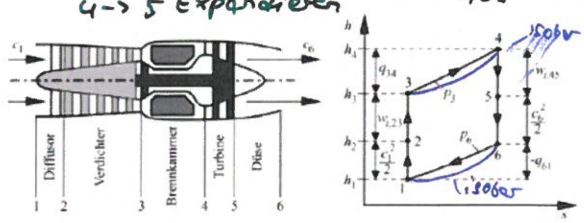
4 -> 5 Expandieren

$\to W_{t,23} = -W_{t,45}$

Π

$$\pi = \frac{p_2}{p_1} \qquad \pi^* = \frac{p_3}{p_1} \qquad \tau = \frac{T_3}{T_1}$$





$$\eta_{th} = 1 - \frac{|q_{41}|}{q_{23}} = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}}$$

$$\lambda = \frac{|w|}{c_p T_1} = \frac{T_3}{T_1} - \frac{T_2}{T_1} - \frac{T_4}{T_1} + 1 = \tau - \pi^{\frac{\kappa-1}{\kappa}} - \tau\pi^{\frac{1-\kappa}{\kappa}} + 1$$

$\pi_{opt} = \tau^{\frac{\kappa}{2(\kappa-1)}}$ | $\eta_{eff} = \eta_{th}\eta_g\eta_{mech}\eta_{el}$

$$\eta_{th} = 1 - \frac{|q_{61}|}{q_{34}} = 1 - \frac{h_6 - h_1}{h_4 - h_3} = 1 - \frac{T_1}{T_3} = 1 - \left(\frac{p_1}{p_3}\right)^{\frac{\kappa-1}{\kappa}} = \frac{|q_{06}|}{q_{zu}} + 1$$

$$\eta_{th} = \frac{|w|}{q_{zu}} = \frac{h_5 - h_6 - (h_2 - h_1)}{q_{zu}} = \frac{c_6^2 - c_1^2}{2 q_{zu}}$$

$$\eta_{TW} = \frac{\dot{m}(c_6 - c_1)c_1}{\dot{m} q_{zu}} = \frac{2c_1}{c_6 + c_1}\eta_{th} = \eta_V \eta_{th}$$

1: F3 f1 (η_{th}, R, T_1, T_3, p_1, p_2, p_3, p_4)

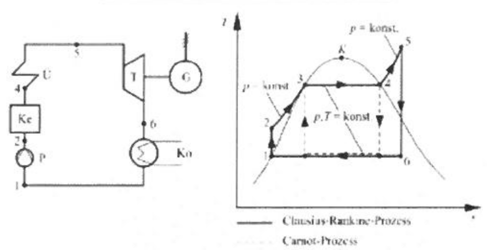
2: F3 f2 (η_{th}, R, T_1, T_3, v_1, v_2, v_4, v_5)

$F = \dot{m} \cdot (c_{Ende} - c_{Anfang})$

Dampfkraftprozesse

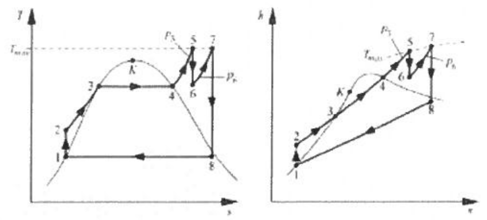
Clausius-Rankine-Prozess

Ohne Zwischenüberhitzung



$$\eta_{th} = 1 - \frac{|q_{61}|}{q_{23} + q_{34} + q_{45}} = 1 - \frac{h_6 - h_1}{h_5 - h_2}$$

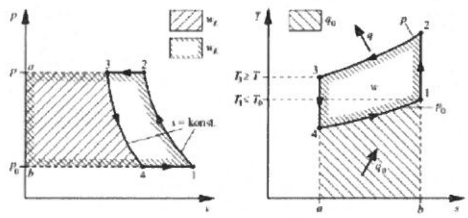
Mit Zwischenüberhitzung



$$\eta_{th,ZÜ} = 1 - \frac{|q_{81}|}{q_{23} + q_{34} + q_{45} + q_{67}} = 1 - \frac{h_8 - h_1}{h_5 - h_2 + h_7 - h_6}$$

Kälteprozesse

Kaltluftprozess

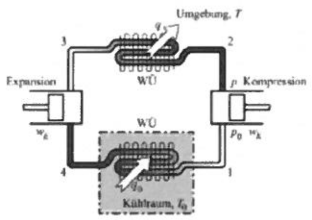


$$\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} = \frac{T_2}{T_1} = \frac{T_3}{T_4} > \frac{T}{T_0}$$

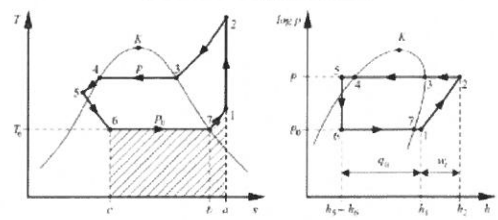
$$q_0 = c_p\,(T_1 - T_4) \quad \Big| \quad q = c_p\,(T_3 - T_2)$$

$$w = w_K + w_E = -\,(q + q_0)$$

$$\varepsilon_{K,Kaltluft} = \frac{1}{\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} - 1}$$



Kaltdampfprozess

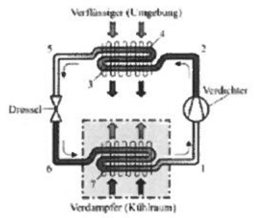


$$\dot{Q}_0 = \dot{m}\,(h_1 - h_6)$$

$$q_0 = q_{61} = q_{67} + q_{71}$$

$$\varepsilon_{K,Kaltdampf} = \frac{q_0}{|q| - q_0} = \frac{q_0}{w_t} = \frac{h_1 - h_6}{h_2 - h_1}$$

$1 \rightarrow 2:$ Reversibel Adiabat

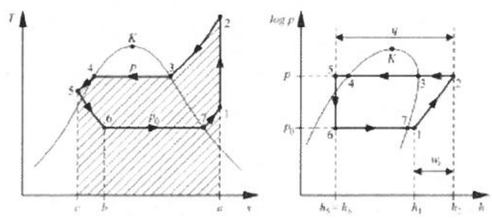


Leistungszahl

$$\varepsilon_K = \frac{\dot{Q}_0}{P}$$

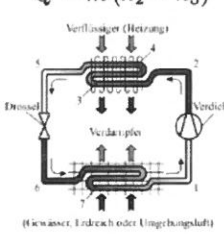
$$\varepsilon_{K,Carnot} = \frac{\dot{Q}_{0,Carnot}}{P_{Carnot}} = \frac{T_0}{T - T_0}$$

Wärmepumpe

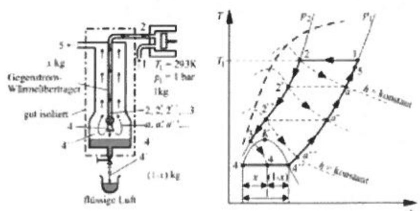


$$\varepsilon_{WP} = \frac{|q|}{|q| - q_0} = \frac{|q|}{w_t} = \frac{h_2 - h_5}{h_2 - h_1} = 1 + \varepsilon_K = 1 + \frac{\dot{Q}_0}{\dot{W}}$$

$$\dot{Q} = \dot{m}\,(h_2 - h_5)$$



Luftverflüssiger nach Linde



$$(1 - x) = \frac{h_5 - h_2}{h_5 - h_{4'}} \leq \frac{h_1 - h_2}{h_1 - h_{4'}} \left[\frac{\text{kg Flüssigkeit}}{\text{kg Ansaugluft}}\right]$$

$$h_2 = (1 - x)h_{4'} + xh_5$$

Enddruck	200	250	350	bar
Flüssigkeitsanteil $(1 - x)$	0,093	0,105	0,118	$\frac{\text{kg Flüssigkeit}}{\text{kg Ansaugluft}}$
Spez. Verdichterarbeit	436	455	486	$\frac{\text{kJ}}{\text{kg Ansaugluft}}$

$p_u = 1\text{bar} \mid T_u = 28\text{K}$

$[\frac{J}{kg\,s}]$

Eindimensionale Strömungsvorgänge t1

$\dot m = \frac{A\cdot S}{S_2 - S_1}$ $\quad \rho = \frac{p}{RT} = \frac{\dot m}{A\cdot c}$

$\dot m = \rho \dot V$

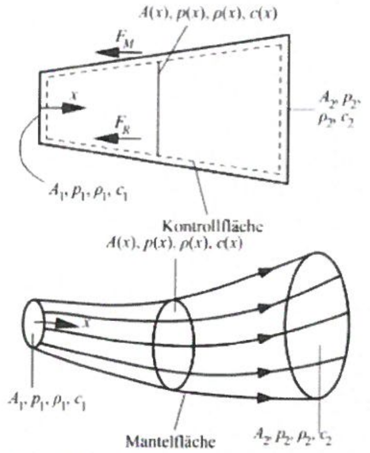
Stationäre Fadenströmung

Kontinuitätsgleichung

$$\dot m = \rho_1 c_1 A_1 = \rho_2 c_2 A_2 = \text{konst.}$$

$$\frac{d\rho}{\rho} + \frac{dc}{c} + \frac{dA}{A} = 0$$

$$\rho c dc + dp = 0$$



Schallgeschwindigkeit

$Ma = \frac{v}{c} = \frac{v}{\sqrt{\kappa R T}}$

$$c_S = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_S} \approx \sqrt{\left(\frac{\Delta p}{\Delta \rho}\right)_S} = v\sqrt{-\left(\frac{\partial p}{\partial v}\right)_S} = v\sqrt{\left(\frac{\partial p}{\partial T}\right)_v^2 \frac{T}{c_v} - \left(\frac{\partial p}{\partial v}\right)_T}$$

Ideales Gas

$$c_S = \sqrt{\kappa R T} = \sqrt{\kappa \frac{p}{\rho}}$$

Van-der-Waals-Gas

$$c_S = \sqrt{\left(\frac{R}{c_v} + 1\right)\left(v^2 \frac{RT}{(v-b)^2}\right) - \frac{2a}{v}}$$

Machzahl

$Ma \ll 1$	Inkompressible Unterschallströmung
$Ma < 0{,}2$	Unterschallströmung, Inkompressible Betrachtung Zulässig
$0{,}2 < Ma < 1$	Kompressible Unterschallströmung
$Ma \approx 1$	Transsonische Strömung
$Ma > 1$	Überschallströmung
$Ma \gg 1$	Hyperschallströmung (Überlicherweise $Ma > 5$)

Adiabate Strömungsvorgänge

Ruhezustand

T_0 manchmal T_t "T_{total}"

$$\frac{T_0}{T} = 1 + \frac{\kappa - 1}{2} Ma^2 \quad t1f5(T_0, T, Ma, \kappa)$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\kappa}{\kappa-1}} = \left(1 + \frac{\kappa-1}{2}Ma^2\right)^{\frac{\kappa}{\kappa-1}} \quad t1f6(p_0, p, Ma, \kappa)$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\kappa-1}} = \left(1 + \frac{\kappa-1}{2}Ma^2\right)^{\frac{1}{\kappa-1}} \quad t1f7(\rho_0, \rho, Ma, \kappa)$$

$$h_0 = h + \frac{c^2}{2} \Leftrightarrow T_0 = T + \frac{c^2}{2c_p}$$

Kritischer Zustand

[*]

$$\frac{T^*}{T_0} = \frac{2}{\kappa + 1} \quad t1f2(T^*, T_0, \kappa)$$

$$\frac{p^*}{p_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \quad t1f3(p^*, p_0, \kappa)$$

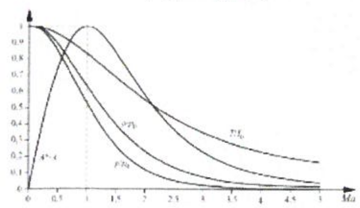
$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}} \quad t1f4(\rho^*, \rho_0, \kappa)$$

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\frac{2}{\kappa+1}\left(1 + \frac{\kappa-1}{2}Ma^2\right)\right]^{\frac{\kappa+1}{2(\kappa-1)}} \quad t1f1(A, A^*, Ma, \kappa)$$

$= \frac{H}{H^*}$ "Höhe Messtrecke zum Kanal"

Machzahl

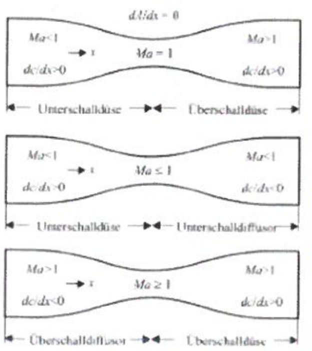
Zustandsgrößen als Funktion der Machzahl für ein ideales Gas ($\kappa = 1{,}4$)

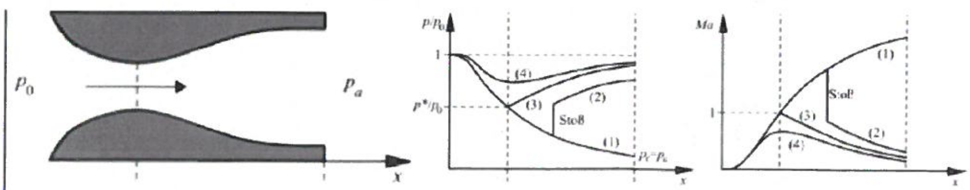


$$(Ma^2 - 1)\frac{dc}{c} = \frac{dA}{A}$$

$$\frac{d\rho}{\rho} = -Ma^2 \frac{dc}{c}$$

Lavaldüse





1: Schalldurchgang im engsten Querschnitt	$p_e = p_a$
2: Verdichtungsstoß nach engstem Querschnitt	$p_e < p_a$
3: Verdichtungsstoß im engsten Querschnitt	$p_e \ll p_a$
4: Unterschallströmung	$p_e \ll p_a$

Leistungsbilanz in der Brennkammer: $\dot Q_{21} = \dot m (h_{t3} - h_{t2})$

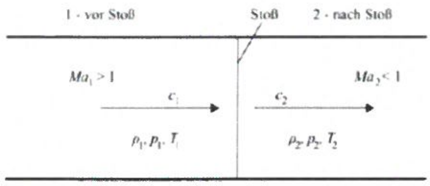
"Zugeführte Wärmeleistung"

e ≙ Exit

i ≙ Inlet

t1 14

Senkrechter Verdichtungsstoß



$$s_1 < s_2$$
$$h_{01} = h_{02}, T_1 < T_2$$
$$p_{01} > p_{02}, p_1 < p_2$$
$$\rho_{01} > \rho_{02}, \rho_1 < \rho_2$$

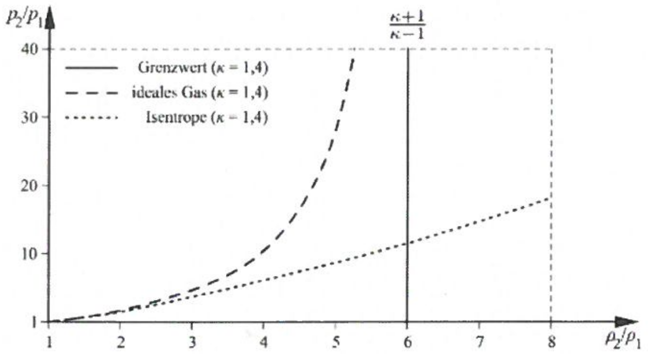
$$\rho_1 c_1 = \rho_2 c_2$$
$$\rho_1 c_1^2 + p_1 = \rho_2 c_2^2 + p_2$$
$$h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2}$$

Hugoniotkurve & -Gleichung

Ideales Gas

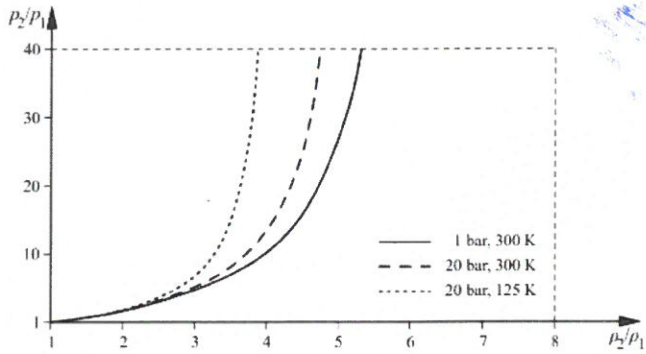
1 2

$$x\left[\frac{p_2}{p_1} = \frac{\frac{\kappa+1}{\kappa-1}\frac{\rho_2}{\rho_1} - 1}{\frac{\kappa+1}{\kappa-1} - \frac{\rho_2}{\rho_1}}\right] y \Leftrightarrow \frac{\rho_2}{\rho_1} = \frac{(\kappa+1)\frac{p_2}{p_1} + \kappa - 1}{(\kappa-1)\frac{p_2}{p_1} + \kappa + 1}$$



Reales Gas

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) = (p_2 - p_1)\frac{1}{2}(v_1 + v_2)$$



Stoßbeziehungen Ideales Gas

3 $$x\left[\frac{p_2}{p_1} = \frac{2\kappa Ma_1^2 - \kappa + 1}{\kappa + 1}\right.$$

4 $$x\left[\frac{\rho_2}{\rho_1} = \frac{(\kappa+1)Ma_1^2}{2 + (\kappa-1)Ma_1^2}\right.$$

5 $$x\left[\frac{T_2}{T_1} = \frac{\left(2\kappa Ma_1^2 - \kappa + 1\right)\left(2 + (\kappa-1)Ma_1^2\right)}{(\kappa+1)^2 Ma_1^2}\right.$$

6 $$Ma_2 = \sqrt{\frac{(\kappa-1)(Ma_1^2 - 1) + \kappa + 1}{2\kappa\left(Ma_1^2 - 1\right) + \kappa + 1}}$$

Verlustfreie Totalgrößen	Verlustbehaftete Totalgrößen
h_0, T_0	p_0, ρ_0

Entropiezunahme

Ideales Gas

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right) = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$
$$= s_{02} - s_{01} = c_p \ln\left(\frac{T_{02}}{T_{01}}\right) - R\ln\left(\frac{p_{02}}{p_{01}}\right) = -R\ln\left(\frac{p_{02}}{p_{01}}\right)$$

Van-der-Waals-Gas

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2 - b}{v_1 - b}\right) \quad \left[\frac{J}{kgK}\right]$$

1: $t_1 f_{12}(\overbrace{p_1, p_2}^{x}, \overbrace{\rho_1, \rho_2}^{y}, k)$

2: $t_1 f_{13}(p_1, p_2, \rho_1, \rho_2, k)$

3: $t_1 f_8(\overbrace{p_2, p_1}^{x}, Ma, k)$

4: $t_1 f_9(\overbrace{\rho_2, \rho_1}^{x}, Ma, k)$

5: $t_1 f_{10}(\overbrace{T_2, T_1}^{x}, Ma, k)$

6: $t_1 f_{11}(Ma_2, Ma_1, k)$

$m_{L1} = \frac{m_{f,1}}{1+x_1}$

Feuchte Luft

Konzentrationsmaße

Wassergehalt

$$x = \frac{m_{H_2O}}{m_L} = x_D + x_W + x_E$$

Beziehungen

$$p_{L,D}V = m_{L,D}R_{L,D}T$$
$$p = p_L + p_D$$

Relative Feuchte

$$\varphi = \frac{p_D}{p_s}$$

$\varphi = 0$ Für Trockene Luft

$\varphi = 1$ Für Gesättigte Luft

Ungesättigte Luft

$$x = x_D = \frac{m_D}{m_L} = \frac{R_L p_D}{R_D p_L} = \frac{R_L p_D}{R_D (p - p_D)} = 0{,}622\frac{p_D}{p - p_D}$$

Gesättigte Luft

$$x_s = \frac{m_{D_1 max}}{m_L} = 0{,}622\frac{p_s}{p - p_s}$$

Dichte

$\dot m = \dot V \rho$

Ungesättigte Luft

$$\rho = \frac{m_{ges}}{V} = \rho_L + \rho_D = \frac{p_L}{R_L T} + \frac{p_D}{R_D T} = \frac{p}{R_L T} + \left(\frac{1}{R_D} - \frac{1}{R_L}\right)\frac{p_D}{T} = \frac{p}{R_{ges}T} = \frac{(1+x)p}{(R_L + xR_D)T}$$

Gesättigte Luft

$$\rho = \frac{(1+x)p}{(R_L + x_s R_D)T}$$

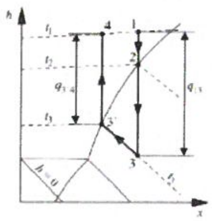
Enthalpie

$$h = c_{pL}t + \underbrace{x_D\left(c_{pD}t + r_D\right)}_{\text{Dampf}} + \underbrace{x_W c_W t}_{\text{Wasser}} + \underbrace{x_E\left(c_E t - r_E\right)}_{\text{Eis}}$$

$t > 0°C$: $x_W = x - x_s$

$t < 0°C$: $x_E = x - x_s$

Wärmeübertragung

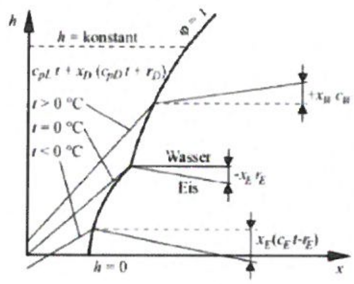


Abgeschiedene Wassermenge:

$$x_W = x_1 - x_{3'}$$

h,x-Diagramm

Geradwinklig

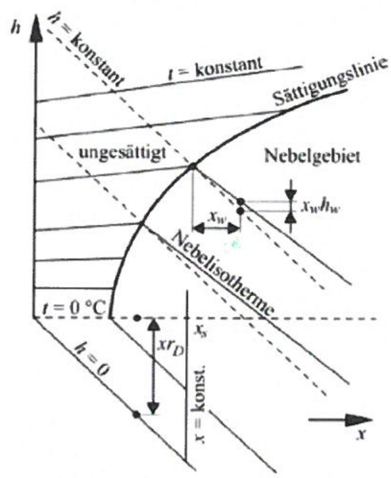


Ungesättigt: $\left(\frac{\partial h}{\partial x}\right)_t = h_D = c_{pD}t + r_D$

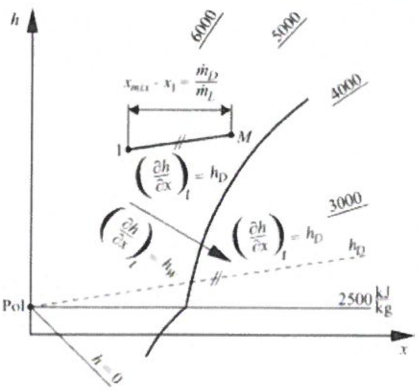
Nebelgebiet: $\left(\frac{\partial h}{\partial x}\right)_t = h_W = c_W t \qquad (t \geq 0°C)$

$\left(\frac{\partial h}{\partial x}\right)_t = h_E = c_E t - r_E \qquad (t \leq 0°C)$

Schiefwinklig

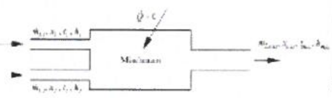


Dampf- & Wassereinspritzung



$$\dot m_{L1}x_1 + \dot m_{H_2O} = \dot m_{L1}x_{mix}$$
$$\dot m_{L1}h_1 + \dot m_{H_2O}h_{H_2O} = \dot m_{L1}h_{mix}$$
$$h_{H_2O} = \frac{h_{mix} - h_1}{x_{mix} - x_1} = \left(\frac{\partial h}{\partial x}\right)_t$$
$$\frac{\Delta h}{\Delta x} = c_w t_{ein} = h_D$$

Mischung

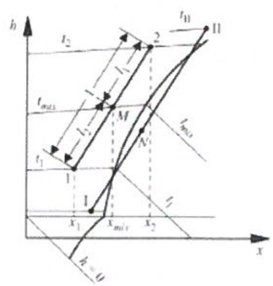


Luftbilanz: $\dot m_{L1} + \dot m_{L2} = \dot m_{Lmix}$

Wasserbilanz: $\dot m_{L1}x_1 + \dot m_{L2}x_2 = \dot m_{Lmix}x_{mix}$

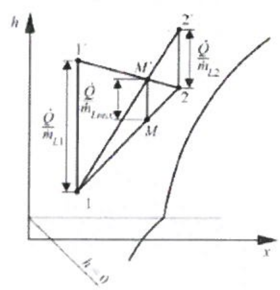
Energiebilanz: $\dot m_{L1}h_1 + \dot m_{L2}h_2 = \dot m_{Lmix}h_{mix}$ 1. HS

Adiabat



$$l_1 = \frac{\dot m_{L1}}{\dot m_{L1} + \dot m_{L2}} = \frac{x_2 - x_{mix}}{x_2 - x_1}$$
$$l_2 = \frac{\dot m_{L2}}{\dot m_{L1} + \dot m_{L2}} = \frac{x_{mix} - x_1}{x_2 - x_1}$$
$$x_{mix} = l_1 x_1 + l_2 x_2$$
$$h_{mix} = l_1 h_1 + l_2 h_2$$

Wärmezufuhr



$$h_{1'} - h_1 = \frac{\dot Q}{\dot m_{L1}}$$
$$h_{2'} - h_2 = \frac{\dot Q}{\dot m_{L2}}$$
$$h_{mix'} - h_{mix} = \frac{\dot Q}{\dot m_{Lmix}}$$

$\frac{\dot m_{LU}}{\dot m_{LU} + \dot m_{LH}} = \frac{\bar l_U}{\bar l_U + \bar l_H}$

$\frac{\dot m_{LU}}{\dot m_{LH}} = \frac{l_U}{l_H}$ (Bei Mischgeraden)

Chemische Reaktionen

Stöchiometrische Beziehung

$$\nu_1 B_1 + \nu_2 B_2 + \cdots + \nu_i B_i + \cdots + \nu_K B_K = \sum_{k=1}^{K} \nu_k B_k = 0$$

$$\frac{dn_1}{\nu_1} = \frac{dn_2}{\nu_2} = \cdots = \frac{dn_i}{\nu_i} = \frac{dn_K}{\nu_K} = d\lambda = \text{konst.}$$

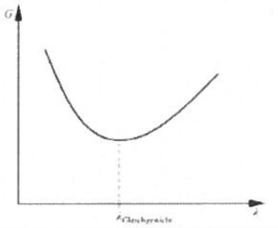
Chemisches Potential

$$\mu_i = \left(\frac{\partial U}{\partial n_i}\right)_{\substack{S,V,\\ n_j \neq n_i}} = \left(\frac{\partial H}{\partial n_i}\right)_{\substack{S,p,\\ n_j \neq n_i}} = \left(\frac{\partial F}{\partial n_i}\right)_{\substack{T,V,\\ n_j \neq n_i}} = \left(\frac{\partial G}{\partial n_i}\right)_{\substack{T,p,\\ n_j \neq n_i}}$$

Chemisches Gleichgewicht

Allgemein

$$\sum_{k=1}^{K} \mu_k dn_k = \sum_{k=1}^{K} \mu_k \left(\nu_k d\lambda\right) = \sum_{k=1}^{K} \mu_k \nu_k = 0$$



Reinstoff

Allgemein

$$\mu = \left(\frac{\partial G}{\partial n}\right)_{T,p} = \left(\frac{\partial \left(n G_m\right)}{\partial n}\right)_{T,p} = G_m$$

$$G(p,T) = G\left(p^+, T\right) + \int_{p^+}^{p} V d\tilde{p}$$

$$\mu(p,T) = \mu\left(p^+, T\right) + \frac{1}{n} \int_{p^+}^{p} V d\tilde{p}$$

Ideales Gas

$$\mu(p,T) = \mu\left(p^+, T\right) + R_m T \ln\left(\frac{p}{p^+}\right)$$

Reales Gas

$$\mu(p,T) = \mu\left(p^+, T\right) + R_m T \ln\left(\frac{f}{p^+}\right)$$

Ideales Gemisch

$$\mu_i = \mu_{0i}\left(p^+, T\right) + R_m T \ln\left(\frac{p_i}{p^+}\right) = \mu_{0i}(p,T) + R_m T \ln\left(\psi_i\right)$$

$$K(p,T) = \prod_{k=1}^{K} \left(\frac{p_k}{p}\right)^{\nu_k} = \prod_{k=1}^{K} \psi_k^{\nu_k} = \exp\left(-\frac{1}{R_m T} \sum_{k=1}^{K} \nu_k \mu_{0k}(p,T)\right)$$

$$= \exp\left(-\frac{1}{R_m T} \sum_{k=1}^{K} \nu_k G_{m,k}(p,T)\right)$$

$$K'(T) = K(p,T) p^{\sum \nu_k}$$

Reales Gemisch

$$\mu_i = \mu_{0i}(p,T) + R_m T \ln\left(\frac{f_i}{p}\right)$$

$$K'(T) = \prod_{k=1}^{K} f_k^{\nu_k}$$

Prinzip des kleinsten Zwanges

Druckänderung

$$\frac{1}{K}\left(\frac{\partial K}{\partial p}\right)_T = \left(\frac{\partial \ln(K)}{\partial p}\right)_T = -\frac{\sum \nu_k}{p}$$

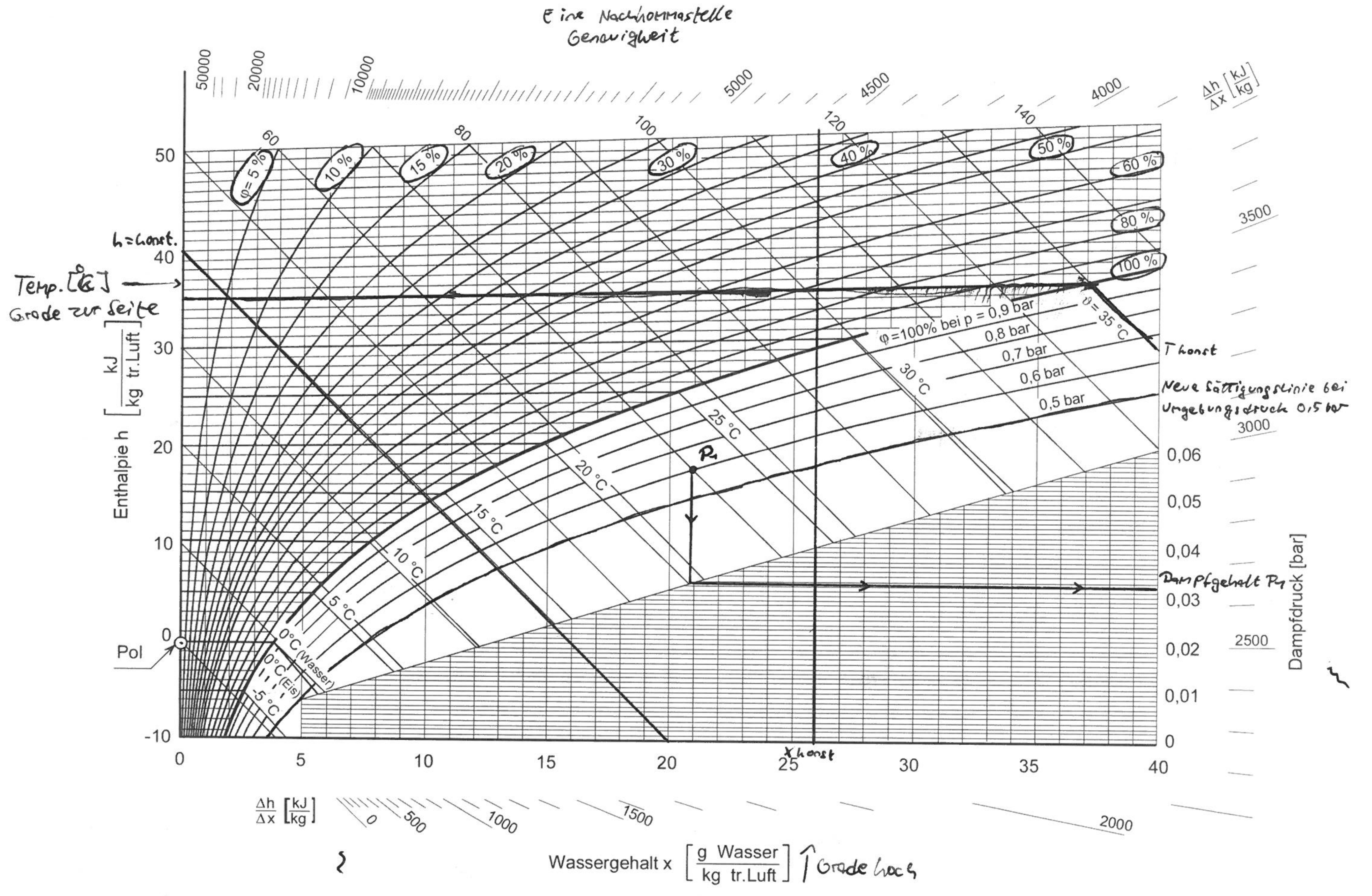
$$K(p_2, T) = K(p_1, T)\left(\frac{p_1}{p_2}\right)^{\sum \nu_k}$$

Temperaturänderung

$$\left(\frac{\partial \ln(K)}{\partial T}\right)_p = \frac{1}{R_m T^2} \sum_{k=1}^{K} \nu_k G_{m,k} + \frac{1}{R_m T} \sum_{k=1}^{K} \nu_k S_{m,k}$$

$$= \frac{1}{R_m T^2} \sum_{k=1}^{K} \nu_k H_{m,k} = \frac{\Delta H_R}{R_m T^2}$$

$$K\left(p, T_2\right) = K\left(p, T_1\right) \exp\left(\frac{\Delta H_R}{R_m}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right)$$



Eine Nachkommastelle Genauigkeit
$\frac{\Delta h}{\Delta x}$ [kJ/kg]
50000
20000
10000
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
60
80
100
120
140
φ = 5 %
10 %
15 %
20 %
30 %
40 %
50 %
60 %
80 %
100 %
h=konst.
Temp. [°C]
Grade zur Seite
Enthalpie h [kJ / kg tr.Luft]
φ=100% bei p = 0,9 bar
0,8 bar
0,7 bar
0,6 bar
0,5 bar
ϑ = 35 °C
30 °C
25 °C
20 °C
15 °C
10 °C
5 °C
0 °C (Wasser)
0 °C (Eis)
-5 °C
T konst
Neue Sättigungslinie bei Umgebungsdruck 0,5 bar
P1
Dampfgehalt P1
Pol
x konst
Dampfdruck [bar]
Wassergehalt x [g Wasser / kg tr.Luft]
Grade hoch

Thermo VÜ 26.7.23

Mischung Distickstoff (N_2O) und Sauerstoff (O_2)

$V = 10\,l$ $\quad T_1 = 5°C$ $\quad T_3 = 18°C$ $\quad \frac{p_{O_2}}{p_{N_2O}} = 3$ $\quad p_2 = 5\,bar$

$T_2 = T_1$

a) ges: p_{O_2}, p_{N_2O}, R_{N_2O}

$\frac{p_{O_2}}{p_{N_2O}} = 3 \qquad p_2 = p_{O_2} + p_{N_2O} \Rightarrow \frac{p_2 - p_{N_2O}}{p_{N_2O}} = 3$

$\rightarrow p_{N_2O} = \frac{1}{4}\, p_2 = 1{,}25\,bar$

$\hookrightarrow p_{O_2} = 3{,}75\,bar$

$R_{N_2O} = \frac{R_M}{M_{N_2O}} = 188{,}9\,\frac{J}{kgK}$

b) ges: m_{O_2}, m_{N_2O}, ξ_{O_2}, ξ_{N_2O}

$p_i V = m_i R_i T$

$m_{N_2O} = \frac{p_{N_2O} V}{R_{N_2O} T} = 2{,}379 \cdot 10^{-2}\,kg$

$R_{O_2} = 259{,}8\,\frac{J}{kgK}$

$m_{O_2} = \frac{p_{O_2} V}{R_{O_2} T} = 5{,}189 \cdot 10^{-2}\,kg$

$\left.\begin{array}{l} \xi_{O_2} = \frac{m_{O_2}}{m} = 0{,}6857 \\ \xi_{N_2O} = \frac{m_{N_2O}}{m} = 0{,}3143 \end{array}\right\} \overset{!}{=} 1$ ✓

c) ges: V_{O_2}, V_{N_2O}

ψ berechnen: $\quad V_i = V \cdot \psi_i$

$p_i = p \cdot \psi_i \quad \Leftrightarrow \quad \psi_i = p_i / p$

$\left.\begin{array}{l} \psi_{N_2O} = \frac{p_{N_2O}}{p} = 0{,}25 \quad ; \quad \psi_{O_2} = \frac{p_{O_2}}{p} = 0{,}75 \\ V_{N_2O} = V \cdot \psi_{N_2O} = 2{,}5\,l \quad ; \quad V_{O_2} = V \cdot \psi_{O_2} = 7{,}5\,l \end{array}\right\} \overset{!}{=} 10$ ✓

d) ges: $S_2 - S_1$

Entropieänderung für die Vermischung zweier Gase

$S_2 - S_1 = \frac{1}{T} p\, V_{N_2O} \cdot \ln\left(\frac{V}{V_{N_2O}}\right) + \frac{1}{T} p\, V_{O_2} \ln\left(\frac{V}{V_{O_2}}\right)$ (Gl. 5.47)

$S_2 - S_1 = R_M \left[n \ln(n) - \sum_{k=1}^{k} n_k \ln(n_k) \right]$

$S_2 - S_1 = 10{,}77\,\frac{J}{kg}$

e) ges: ξ_{O_2}, ξ_{N_2O}

$Q_{23}: m_G \cdot c_{v,G} (T_{G,3} - T_{G,2})$ mit $T_{G,2} = T_1$

$c_{v,G} = \xi_{O_2} c_{v,O_2} + \xi_{N_2O} c_{v,N_2O} = \frac{m_{O_2}}{m_G} c_{v,O_2} + \frac{m_{N_2O}}{m_G} c_{v,N_2O}$

$$Q_{23} = (m_{O_2} c_{v,O_2} + m_{N_2O} c_{v,N_2O})(T_{G,3} - T_{G,2})$$

$$= (m_{O_2} c_{v,O_2} + (m_G - m_{O_2}) c_{v,N_2O})(T_{G,3} - T_{G,2})$$

$$m_{O_2} = \frac{Q_{21}}{(T_{G,3} - T_{G,2})(c_{v,O_2} - c_{v,N_2O})} - \frac{m_G c_{v,N_2O}}{(c_{v,O_2} - c_{v,N_2O})} = 1{,}740 \cdot 10^{-2}\ kg$$

$$\xi_{O_2} = \frac{m_{O_2}}{m_G} = 0{,}1962 \quad ; \quad \xi_{N_2O} = 1 - \xi_{O_2} = 0{,}8038$$

f) ges: $\frac{p_{N_2O}}{p_{O_2}}$

$$\xi_{N_2O} = \frac{M_{N_2O}}{M_G} \cdot \varphi_{N_2O} = \frac{M_{N_2O} \cdot p_{N_2O}}{M_G \cdot p_G} \rightsquigarrow \xi_{O_2} = \frac{M_{O_2} \cdot p_{O_2}}{M_G \cdot p_G}$$

$$\xi_{N_2O} = \frac{M_{N_2O}}{M_{O_2}} \cdot \frac{p_{N_2O}}{p_{O_2}}$$

$$\Leftrightarrow \frac{p_{N_2O}}{p_{O_2}} = \frac{\xi_{N_2O}}{\xi_{O_2}} \cdot \frac{M_{O_2}}{M_{N_2O}} = 2{,}979$$

Jetzt: $\frac{p_{N_2O}}{p_{O_2}} \approx 3$ Vorher: $\frac{p_{O_2}}{p_{N_2O}} = 3$

$\rightarrow$ Dr. Alber hat das Partialdruckverhältnis falsch vorgegeben.

4.4. Aufgabe - therm. Zustandsgleichung eines realen Mediums

$$pv = RT + Ap + Bp^2$$

a) ges: Gleichungen der Inversionslinien der Form $p(v)$, $T(v)$, $p(T)$

Joule-Thompson-Inversionslinie: $\delta_h = 0$

$$\delta_H = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{v}{c_p}(1 - \beta T) = 0$$

$$\underline{1 - \beta T \overset{!}{=} 0} \qquad \beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p : \text{isobarer Ausdehnungskoeff.}$$

$\quad\quad\quad\quad \hookrightarrow$ Aus therm. ZGL bestimmbar

Implizite Methode

$$\frac{\partial}{\partial T}\{pv - RT - Ap - Bp^2\}_p = 0$$

$$\Leftrightarrow p\left(\frac{\partial v}{\partial T}\right)_p - R = 0$$

$$\Leftrightarrow \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \rightarrow \beta = \frac{1}{v}\frac{R}{p} \text{ in } 1 - \beta T = 0$$

$$\Rightarrow 1 - \frac{RT_{inv}}{v_{inv} p_{inv}} = 0 \Leftrightarrow \left[\frac{RT_{inv}}{v_{inv} p_{inv}} = 1\right]$$

Inversionslinie in der Form $p(v)$

$\hookrightarrow T_{inv}$ muss eleminiert werden!

aus Therm ZGL folgt: $T = \frac{1}{R}(pv - Ap - Bp^2)$

eingesetzt in (*)

$$1 - \frac{pv - Ap - Bp^2}{pv} = 0 \Leftrightarrow pv = pv - Ap - Bp^2 \Rightarrow 0 = -A - Bp$$

$$\Leftrightarrow p_{inv} = -\frac{A}{B} \quad [p(v)]$$

9. Vortragsübung Thermo 15.12.22

[3.1] Carnot-Prozess mit einem Van-der-Waals-Gas

1 → 2 isotherme Expansion $T_1 = T_2$; $Q_{12} > 0$; $W_{12} < 0$

2 → 3 reversibel adiabate Entspannung $S_3 = S_2$; $Q_{23} = 0$; $W_{23} < 0$

3 → 4 isotherme Verdichtung $T_4 = T_3$; $Q_{34} < 0$; $W_{34} > 0$

4 → 1 reversibel adiabate Verdichtung $S_4 = S_1$; $Q_{41} = 0$; $W_{41} > 0$

Van-der-Waals-Gas

$(p + \frac{a}{v^2})(v - b) = RT$ → therm. Zustandsgleichung, die Realgaseffekte berücksichtigt
bzw.
$(\bar p + \frac{3}{\bar v^2})(3\bar v - 1) = 8\bar T$ → dim.lose Gleichung mit universellem Charakter mit:

$\bar p = \frac{p}{p_k}$; $\bar v = \frac{v}{v_k}$; $\bar T = \frac{T}{T_k}$ (kritisch)

a) ges: T_k, p_k, v_k, $\bar p \bar v$-Diagramm
Zusammenhang der Größen im kritischen Pkt. und den Konstanten a, b (4.39)

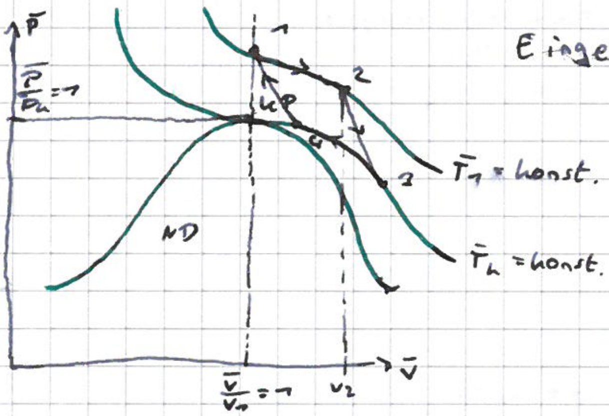
$a = 3 p_k v_k^2$ (1) $b = \frac{v_k}{3}$ (2) $\frac{p_k v_k}{R T_k} = \frac{3}{8}$ (3)

aus (2): $v_k = 3b = 6{,}600 \cdot 10^{-3} \frac{m^3}{kg}$
aus (1): $p_k = \frac{a}{3 v_k^2} = 4{,}706 \cdot 10^5$ Pa
aus (3): $T_k = \; = \frac{p_k v_k}{R} \frac{3}{8} = 300$ K

Zustand 1: $\bar T_1 = 1{,}5$
$\bar v_1 = 1{,}0$

Zustand 2: $\bar T_2 = \bar T_1 = 1{,}5$
$\bar v_2 = 2 \cdot \bar v_1$

Zustand 3: $\bar T_3 = \bar T_k = 1{,}0$



Eingeschl. Fläche ≙ W

b)

Zustand	$\bar v$	$\bar p$	$\bar T$
1	1,0	3,000	1,5
2	2,0	1,650	1,5
3	5,958	0,390	1,0
4	2,589	0,736	1,0

ges: $\bar p_1$, $\bar p_2$

$(\bar p + \frac{3}{\bar v^2})(3\bar v - 1) = 8\bar T$ → $\bar p_1 = \frac{8\bar T}{3\bar v_1 - 1} - \frac{1}{\bar v_1^2} = 3{,}000$
→ $\bar p_2 = 1{,}650$

c) ges: $\bar{v}_3$, $\bar{p}_3$ $(\bar{T}_3 = 1{,}0)$

$2 \to 3$ rev. adiabat, d.h. $s_3 - s_2 \stackrel{!}{=} 0$ (4.5)

$\to$ Entropiedifferenz für ein Van der Waals-Gas: $s - s_0 = c_v \cdot \ln\left(\frac{T}{T_0}\right) + R \cdot \ln\left(\frac{v-b}{v_0-b}\right)$

$\hookrightarrow = 0$, weil rev. adiabat

$0 = c_v \cdot \ln\left(\frac{T_3}{T_2}\right) + R \cdot \ln\left(\frac{v_3 - b}{v_2 - b}\right)$

$0 = c_v \cdot \ln\left(\frac{\bar{T}_3 \cdot T_k}{\bar{T}_2 \cdot T_k}\right) + R \cdot \ln\left(\frac{\bar{v}_3 \cdot v_k - b}{\bar{v}_2 \cdot v_k - b}\right)$

$v_3 = \frac{1}{v_k}\left(\left(\frac{\bar{T}_2}{\bar{T}_3}\right)^{c_v/R}(\bar{v}_2 \cdot v_k - b) + b\right) = 5{,}958$

$\bar{p}_3 = \frac{8\bar{T}_3}{3\bar{v}_3 - 1} - \frac{3}{\bar{v}_3^2} = 0{,}390$

d) ges: $\bar{v}_4$, $\bar{p}_4$

$4 \to 1$: rev. adiabat, gleich wie in c)

$\bar{v}_4 = \frac{1}{v_k}\left(\left(\frac{\bar{T}_1}{\bar{T}_4}\right)^{c_v/R}(\bar{v}_1 \cdot v_k - b) + b\right) = 2{,}583$

p_4 = Analog zu c) $= 0{,}736$

e) ges: q_{12} 2. HS für rev. ZÄ $ds = \frac{\delta q_{rev}}{T}$

$1 \to 2$: isotherm $\to T = \text{konst}$

$\delta q_{rev} = T\,ds \quad \Big| \int_1^2$

$q_{12} = T_1(s_2 - s_1) = T_1\left(c_v \underbrace{\cancel{\ln\left(\frac{T_2}{T_1}\right)}}_{=0} + R\ln\left(\frac{v_2 - b}{v_1 - b}\right)\right) = T_1 \cdot R \cdot \ln\left(\frac{v_2 - b}{v_1 - b}\right)$

$q_{12} = \bar{T}_1 \cdot T_k \cdot R \cdot \ln\left(\frac{\bar{v}_2 \cdot v_k - b}{\bar{v}_1 \cdot v_k - b}\right) = 113{,}8\ \frac{kJ}{kg}$

f) ges: therm. Wirkungsgrad η_{th}, w_{ges}

$\eta_{th,carnot} = \frac{\text{abgeführte Arbeit}}{\text{zugeführte Wärme}} = \frac{-w_{ges}}{q_{12}}$

$\eta_{th} = 1 - \frac{T_3}{T_2} = 1 - \frac{\bar{T}_3 T_k}{\bar{T}_2 T_k} = \frac{1}{3}$

$$\left[\begin{array}{l} \oint u = 0 = \oint w + \oint q \\ 0 = w_{12} + w_{23} + w_{34} + w_{41} \\ \quad + q_{12} + q_{34} \\ -w_{ges} = q_{12} + q_{34} \end{array}\right]$$

$w_{ges} = -\eta_{th} \cdot q_{12} = -37{,}95\ \frac{kJ}{kg}$

32. Aufgabe – künstlicher Kreisprozess

Ideales Gas:

$1 \to 2$ polytrope Verdichtung mit $n = 1{,}76$

$2 \to 3$ Isotherme Entspannung

$3 \to 4$ ~~adiabate Abkühlung~~ isochore Abkühlung

$4 \to 5$ adiabate Abkühlung

$5 \to 1$ isobare ZÄ

$p_1 = 1$ bar $\quad p_2 = 10$ bar

$v_1 = 0{,}83\ \frac{m^3}{kg}$ $\quad v_3 = 1{,}6\ \frac{m^3}{kg}$

$T_5 = $ ~~...~~ $857{,}5$ K

$c_p = 1005\ \frac{J}{kgK}$; $k = 1{,}4$; $n = 1{,}76$

b) ges: h(t) in 10k Schritten

Herleitung aus a) kann für beliebige Temp. angenommen werden

$\rightarrow \rho(0°C) \cdot V(0°C) = \rho(t) \cdot V(t)$ (3)

$\hookrightarrow V(t) = \Delta V(t) + V(0°C)$ (4)

$\hookrightarrow V(t) = \frac{\pi}{4} d_k^2 \cdot h(t)$ (5)

$\hookrightarrow$ gesucht

$\rho(t) = \frac{\rho(0°C)}{1+Et+Ft^2}$ (6)

(6) in (3) : $\rho(0°C) \cdot V(0°C) = \frac{\rho(0°C) \cdot V(t)}{1+Et+Ft^2}$ (7)

(4) in (7) : $V(0°C) = \frac{\Delta V(t) + V(0°C)}{1+Et+Ft^2}$

$\longrightarrow \Delta V(t) = V(0°C)(Et+Ft^2) = \frac{\pi}{4} d_k^2 \cdot h(t)$

$h(t) = \frac{V(0°C)(Et+Ft^2)}{\frac{\pi}{4} \cdot d_k^2}$

t (°C)	10	20	30	40	50	60	70	80	90	100
h (t)	1,997	3,985	5,982	7,980	9,979	11,98	13,98	15,99	17,99	20
c) $h_{neu}(t)$	2	4	6	8	10	12	14	16	18	20

$\hookrightarrow$ Größte Abweichung bei t = 50

ges: t(h = 10 cm), bzw Δt_{max}

$h(t) = \frac{V(0°C)(Et+Ft^2)}{\frac{\pi}{4} \cdot d_k^2}$ $\rightarrow$ auflösen nach t für h = 10 cm

~~$t_1 = -23560°C$~~ ; $t_2 = 50,107°C$

$t(h = 10\,cm) = 50,107°C$ $\rightarrow$ 0,21% Abweichung

d) Welcher Messfehler stellt sich ein?

System I: 100°C $\rightarrow$ kein Messfehler

System II: bei 100°C $\rightarrow$ Es wäre der gesamte Bereich von 0°C bis 100°C mit Quecksilber gefüllt

Aber: Masse muss auch bei T = 20°C konstant bleiben

$m_{II}(100°C) \overset{!}{=} m_{II}(20°C)$

$\rho(100°C) \cdot \frac{\pi}{4} \cdot d_k^2\, h(100°C) \overset{!}{=} \rho(20°C) \cdot \frac{\pi}{4} \cdot d_k^2 \cdot h^*$

mit $\rho(t) = \frac{\rho(0°C)}{1+Et+Ft^2}$

$\longleftrightarrow \frac{\rho(0°C)}{1+E\cdot 100°C + F\cdot(100°C)^2} \cdot 0,2m = \frac{\rho(0°C)}{1+E\cdot 20°C + F\cdot(20°C)^2} \cdot h^*$

$\longleftrightarrow h^* = 19,71\,cm$

aus c) : $h(t) = \frac{V(0°C)(Et+Ft^2)}{\frac{\pi}{4} \cdot d_k^2}$ $\Big\}$ $t(h^*) = 98,56°C$

$\longrightarrow$ Fehler von 1,44 %

Thermo Vortragsübung 20.10.22

Dichtefunktion der Atmosphäre

geg: isotherm, T = konst

$$\underbrace{\frac{\rho(z)}{\rho_0}}_{\text{Dimensionslos}} = \exp\left(\frac{-g\cdot z}{R\cdot T_0}\right) \qquad 1J = 1Nm = \frac{kg\cdot m^2}{s^2}$$

$$\rho_0 = \rho(z=0) = \frac{p_0}{R\cdot T_0} \rightarrow \text{ideales Gas}$$

$$\rho_0 = 1{,}225\,\frac{kg}{m^3}$$

a) $\frac{\rho(z)}{\rho_0} = \exp\left(\frac{-g\cdot z}{R\cdot T_0}\right)$

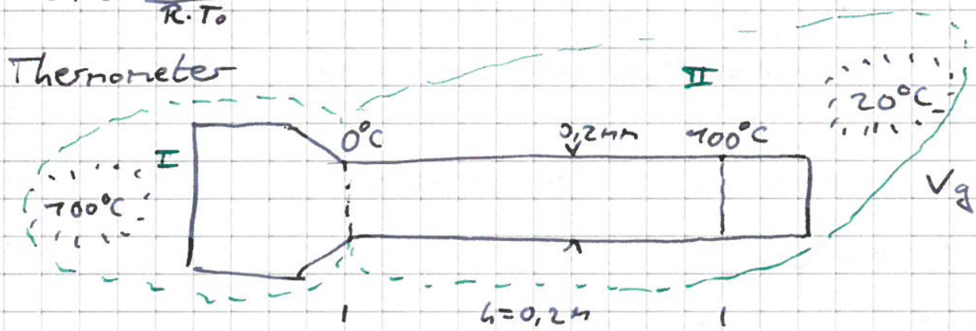
z (km)	0	5	10	20
$\frac{\rho(z)}{\rho_0}$	1	0,55	0,31	0,09

~~b) geg: $\frac{\rho(z^*)}{\rho_0} = \frac{1}{e}$~~ ges: $\frac{\rho(z^*)}{\rho_0} \overset{!}{=} \frac{1}{e}$

$$\frac{1}{e} = e^{-1} = \exp\left(\frac{-g\cdot z}{R\cdot T_0}\right) \qquad |\cdot \ln$$

$$-1 = \frac{-g\cdot z}{R\cdot T_0}$$

Thermometer



V_g = konst

$$\frac{\rho(0)}{\rho(t)} = 1 + Et + Ft^2 \qquad (t \text{ in } °C)$$

$E = 1{,}8182\cdot 10^{-4}\ 1/°C$
$F = 0{,}78\cdot 10^{-8}\ 1/°C$

a) ges: $V(0°C)$

Thermometer ist geschlossenes System $\rightarrow m = \text{konst}$
Bei Erwärmung von 0°C auf 100°C steigt die Quecksilbersäule um $h = 0{,}2\,m$

$$\rightarrow \Delta V = V(100°C) - V(0°C) = \frac{\pi}{4} d_k^2 \cdot h = 6{,}283\cdot 10^{-9}\ m^3$$

aus m = konst : $m(0°C) \overset{!}{=} m(100°C)$

$$\rho(0°C)\cdot V(0°C) = \rho(0°C)\cdot V(100°C)$$

$$\text{mit } \rho(t) = \frac{\rho_0}{1+Et+Ft^2} \qquad (1)$$

$$\rho(100°C) = \frac{\rho(0°C)}{1+E\cdot 100°C + F(100°C)^2} \qquad (2)$$

(2) in (1)

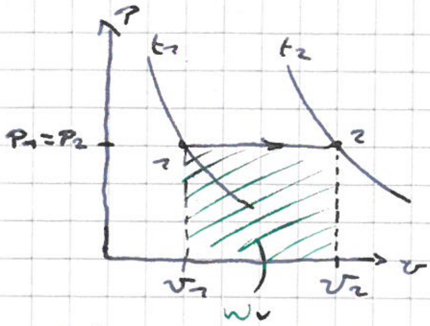
$$\rho(0°C)\cdot V(0°C) = \frac{\rho(0°C)}{1+E\cdot 100°C + F\cdot(100°C)^2}\cdot V(100°C)$$

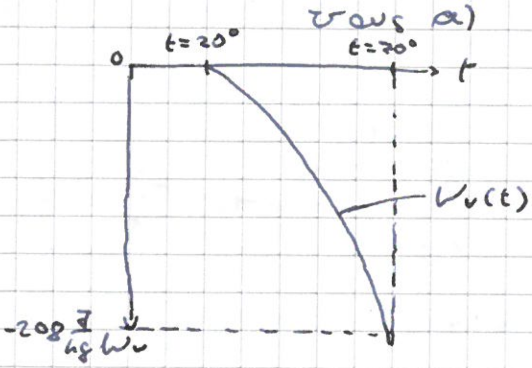
$\hookrightarrow \Delta V - V(0°C)$

$$V(0°C) = \frac{\Delta V}{E\cdot 100°C + F\cdot(100°C)^2} = 3{,}447\cdot 10^{-7}\ m^3$$

c) ges: spez. Volumenänderungsarbeit ~~~~ w_v

$$w_v = -\int_{v_1}^{v} p\,dv = -p(v - v_1)$$ $p =$ konst, v aus a)





d) ges: spez. Wärme q.

für ein geschlossenes System gilt 1. HS
$$dU = \delta Q + \delta W$$

In integraler Form + spezifisch

$$u_2 - u_1 = q_{12} + w_{12}$$ (hier nur Volumenänderungsarbeit) $p =$ konst.

$$u_2 - u_1 = q_{12} - \int_1^2 p\,dv$$

$$u_2 - u_1 = q_{12} - p(v_2 - v_1)$$

$$\Rightarrow q_{12} = u_2 - u_1 + pv_2 - pv_1$$

$$q_{12} = \underbrace{(u_2 + pv_2)}_{=h_2} - \underbrace{(u_1 + pv_1)}_{=h_1}$$

$$q_{12} = h_2 - h_1$$ Gilt nur bei geschlossenen System, Isobarer ZÄ, nur w_v

$$h_2 - h_1 = \int_1^2 c_p\,dT \quad (c_p = \text{konst.})$$
$$= c_p(T_2 - T_1)$$

$$\rightarrow q_{12} = c_p(T_2 - T_1)$$

e) 3D-Diagram auf Ilias hochgeladen.

Thermo Vortragsübung 7.11.22 ③

12. Aufgabe

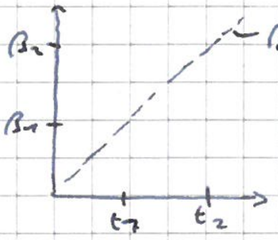
$① \xrightarrow[p = \text{konst}]{\text{isobar}} ②$ $\qquad c_p = \text{konst}$

geg: $p_1 = 100\ \text{bar}$
$T_1 = 20°C$
$\rho = 1002{,}808\ \frac{kg}{m^3}$
$\beta_1 = 2{,}146 \cdot 10^{-4}\ \frac{1}{K}$

$p_2 = p_1$
$T_2 = 70°C$
$v_2 = 0{,}0010183\ \frac{m^3}{kg} = \frac{1}{\rho_2}$
$\beta_2 = ?$

ges: $\beta = \beta(t_1, t_1)$ als lin. Gleichung : Einheit C

Ansatz: $\beta(t) = \beta_1 + C(t - t_1)$



β_1, t_1 bekannt
Steigung C unbekannt.

Bestimmung der Einheit von C:
- jeder Summand muss die gleiche Einheit besitzen

$[\beta_1] = 1/K\ ;\ [C(\underbrace{t - t_1}_{\text{in °C}})] \rightarrow [C(\underbrace{T_2 - T_1}_{\text{in K}})] = [C] \cdot K \overset{!}{=} \frac{1}{K}$

$\rightarrow [C] = \frac{1}{K^2}$

$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$

totales Differential von $v(p,T)$

$dv = \left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T \underbrace{\cancel{dp}}_{=0,\ \text{da } p = \text{konst.}}$

$\longleftrightarrow \frac{dv}{dT} = \left(\frac{\partial v}{\partial T}\right)_p$

$\beta(t) = \beta_1 + C\ (t - t_1) = \frac{1}{v}\frac{dv}{dT} \qquad | \cdot dT \quad | \int_1^2$

$\int_1^2 (\beta_1 + C(t - t_1))\, dt = \int_1^2 \frac{1}{v}\, dv$

$\left[\beta_1 t + C\left(\frac{t^2}{2} - t_1 t\right)\right]_1^2 = \left[\ln(v)\right]_1^2$

$\left(\beta_1 t_2 + C\left(\frac{t_2^2}{2} - t_1 t_2\right)\right) - \left(\beta_1 t_1 + C\left(\frac{t_1^2}{2} - t_1^2\right)\right) = \ln\left(\frac{v_2}{v_1}\right)$

$\beta_1 (t_2 - t_1) + C\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right) = \ln\left(\frac{v_2}{v_1}\right)$

$C = \frac{2}{(t_2 - t_1)^2} \cdot \ln\left(\frac{v_2}{v_1}\right) - \frac{2}{(t_2 - t_1)}\beta_1 = 8{,}767 \cdot 10^{-6}\ \frac{1}{K^2}$

$\beta(t) = \beta_1 + 8{,}767 \cdot 10^{-6}\,(t - t_1)$

b) ges: $v(t^* = \{20°C, 30°C, \dots, 70°C\})$

Ansatz aus a) mit angepasster Integralgrenze.

$v(t^*) = v_1 \cdot \exp\left(\beta_1 (t^* - t_1) + \frac{C}{2}(t^* - t_1)^2\right)$

Ergebnisse siehe Tabelle auf links

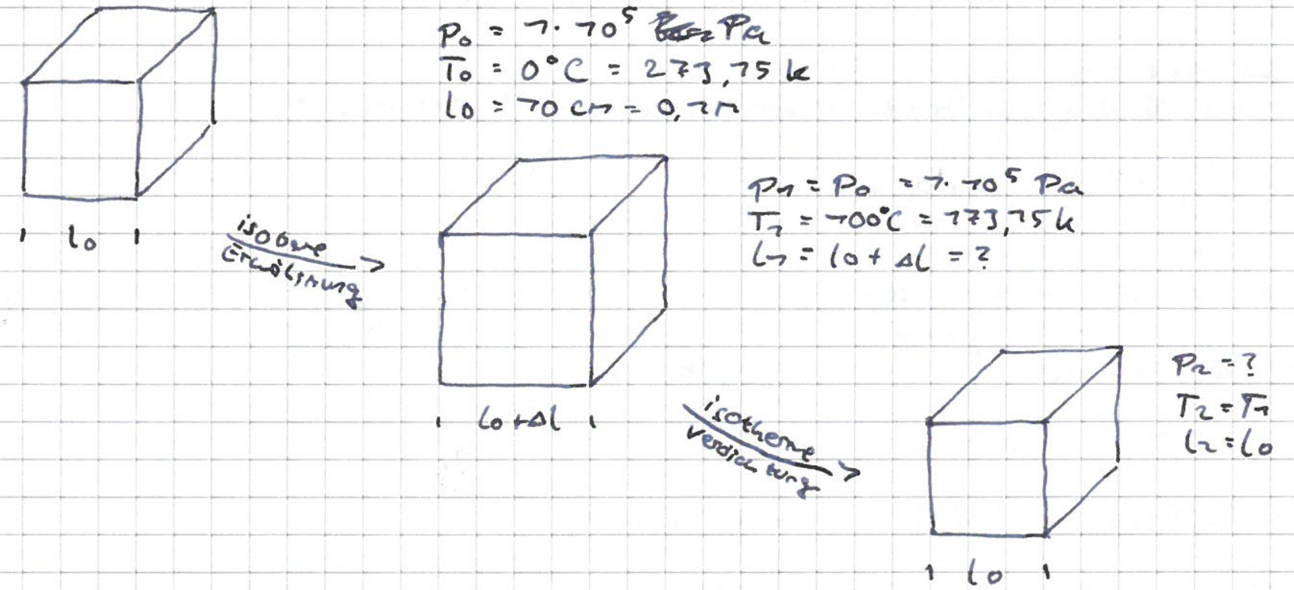
Thermo Vortragsübung 3.11.22

Vorüberlegungen

Zustandsgrößen (T, h, p)	Prozessgrößen (Q)
Wegunabhängig	Wegabhängig
Totale Differential dT -> gibt die absolute Änderung an.	Änderung δQ -> Gibt absolute Änderung an
Partielle Differentiale ∂T -> gibt die partielle Änderung nach einer abhängigen ZG an, die andere ZG wird konstant gehalten.	-> Eine partielle Änderung gibt es für Prozessgrößen nicht -> weil Wegabhängigkeit. -> δ soll als Schreibweise die Wegabhängigkeit verdeutlichen

Bsp: $\left(\frac{\partial T}{\partial v}\right)_p$; $T = f(v, p)$

11. Aufgabe: Thermische Zustandsgl. + Koeffizienten



geg: isotherm: Längenausdehnungskoeff.

$$\beta_l = 85 \cdot 10^{-6}\ \frac{1}{K} = \text{konst.}$$

isotherm. Kompressibilitätskoeff.

$$\chi = 3 \cdot 10^{-10}\ \frac{m^2}{N} = \text{konst.}$$

ges: $\Delta l_0 = l_1 - l_0$

$$\beta_l = \text{konst} \rightarrow \beta_l = \frac{1}{l}\left(\frac{\partial l}{\partial T}\right)_p = \frac{1}{l}\left(\frac{dl}{dT}\right)_p$$

$$\beta_l = \frac{1}{l}\frac{dl}{dT} \quad | \cdot dT$$

$$\beta_l\, dT = \frac{1}{l}\, dl$$

$$\beta_l \int_0^1 dT = \int_0^1 \frac{1}{l}\, dl$$

$$\beta_l (T_1 - T_0) = \ln\left(\frac{l_1}{l_0}\right) \quad \longleftrightarrow \quad l_1 = l_0 \cdot \exp(\beta_l (T_1 - T_0)) = 10{,}08536\ cm$$

$$\longrightarrow \quad \Delta l = l_1 - l_0 = 8{,}536 \cdot 10^{-4}\ m$$

b) ges: Isobarer Ausdehnungskoeff. $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ ②

↳ jetzt: Volumenänderung statt Längenänderung

$\beta = \frac{1}{V}\frac{dV}{dT}$, da $p = konst$; $\beta = f(V)$

$\beta = \frac{1}{V}\frac{dV}{dT} \quad | \cdot dT \quad | \cdot \int_0^1$

$\beta \int_0^1 dT = \int_0^1 \frac{1}{V} dV$

Potenzgesetze

$\beta(T_1 - T_0) = \ln\left(\frac{V_1}{V_0}\right) = \ln\left(\frac{L_1^3}{L_0^3}\right) = 3 \cdot \ln\left(\frac{L_1}{L_0}\right)$

$\beta = \frac{3}{T_1 - T_0}\ln\left(\frac{L_1}{L_0}\right) = 3 \cdot \beta_L = 2{,}550 \cdot 10^{-4}\ \frac{1}{K}$

c) ges: Thermische Zustandsgleichung ϕ

$\phi(T, p, V) = 0 \quad \rightarrow p(V,T) \quad ; V = V(p,T) \quad ; T = T(p,V)$

bekannt: $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ (isobarer Ausdehnungskoeff.)

$\varkappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$ (isothermer Kompressibilitätskoeff.)

⇒ Partielle Ableitungen von V sind bekannt.

→ Strategie: Bildung der totalen Differentiale von ~~V~~ $V = V(p,T)$

$dV = \underbrace{\left(\frac{dV}{dT}\right)_p}_{\beta \cdot V} dT + \underbrace{\left(\frac{\partial V}{\partial p}\right)_T}_{-\varkappa \cdot V} dp$

$dV = \beta \cdot V \cdot dT - \varkappa \cdot V \cdot dp \quad | \cdot \frac{1}{V}$

$\frac{1}{V} dV = \beta\, dT - \varkappa\, dp \quad \Big| \int_0^{V,T,p} \longrightarrow \beta, \varkappa = konst$

$\ln\left(\frac{V}{V_0}\right) = \beta(T - T_0) - \varkappa(p - p_0)$

$\beta(T - T_0) - \varkappa(p - p_0) - \ln\left(\frac{V}{V_0}\right) = 0 = \phi(T, p, V)$

d) ges: p_2 $\quad 1 \rightarrow 2$ isotherme Verdichtung ; $p_0 = p_1$, $V_2 = V_0$

→ Therm. Zustandsgleichung umformen nach p

$\beta(p_2 - p_1) - \varkappa(T_2 - T_1) - \ln\left(\frac{V_2}{V_1}\right) = 0$

$p_2 = \frac{1}{\varkappa}\left[\cancel{\beta(T_2 - T_1)}^{\,=0} - \ln\left(\frac{V_2}{V_1}\right)\right] + p_1 = 857{,}0\ bar = 8{,}570 \cdot 10^{7}\ Pa$

e) ges: γ isochorer Spannungskoeff. (konst., da β und $\varkappa$ auch konst)

$\gamma = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{p}\left(\frac{dp}{dT}\right)_V$

aus Aufgabe c: $\underbrace{dV}_{isochor}^{=0} = \beta V dT - \varkappa V dp \quad | \cdot \frac{1}{V}$

$0 = \beta\, dT - \varkappa\, dp$

$\underbrace{\left(\frac{dp}{dT}\right)_V}_{p \cdot \gamma} = \frac{\beta}{\varkappa} \quad \rightarrow \gamma = \frac{\beta}{p \cdot \varkappa}$ $\quad$ Zusammenhang: $\beta = p \cdot \gamma \cdot \varkappa$

$$U_{See} = 4{,}18\,\frac{kJ}{kgK} \cdot 278{,}15\,K = 1162{,}67\,\frac{kJ}{kg} \cdot m = U_1$$

$$U_D = 4{,}18\,\frac{kJ}{kgK} \cdot 308{,}15\,K = 1288{,}07\,\frac{kJ}{kg} \cdot m = U_2$$

$$0 = \dot{m}\left(h_1 + \frac{c_1^2}{2} + g z_1\right) - \dot{m}\left(h_2 + \frac{c_2^2}{2} + g z_2\right) + \dot{Q}_{12} + \dot{W}_{t,12}$$

$$0 = \dot{m}(h_1 - h_2) + \dot{m}\left(\frac{c_1^2 - c_2^2}{2}\right) + \dot{m}(g z_1 - g z_2) + \dot{Q}_{12} + \dot{W}_{t,12}$$

$$0 = \dot{m}\left(h_1 - h_2 + \frac{c_1^2 - c_2^2}{2} + g z_1 - g z_2\right) + \dot{Q}_{12} + \dot{W}_{t,12}$$

$$0 = \dot{m}\left((U_1 + \underbrace{p_1 v_1}_{konst}) - (U_2 + \underbrace{p_2 v_2}_{konst}) + \frac{c_1^2 - c_2^2}{2} + g z_1 - g z_2\right) + \dot{Q}_{12} + \dot{W}_{t,12}$$

$$0 = \dot{m}\left(U_1 - U_2 + \frac{c_1^2 - c_2^2}{2} + \cancel{g z_1} - g z_2\right) + \dot{Q}_{12} + \dot{W}_{t,12}$$

↳ Arbeit Pumpe

$$\dot{W}_{t,12} = -\dot{m}\left(U_1 - U_2 + \frac{c_1^2 - c_2^2}{2} \cancel{z_1} - g z_2\right) + \underbrace{\dot{Q}_{12}}_{\text{Verlust + Wärme Heizer}}$$

Thermo Vortragsübung 22.11.22

$\boxed{12}$ Ideales Gas

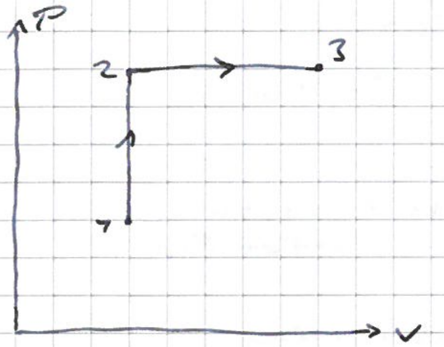
$1 \to 2$ isochor $\qquad p_2 > p_1$

$2 \to 3$ isobar $\qquad V_3 > V_2$

$R = 287{,}1 \frac{J}{kgK}$

$pV = mRT$ keine Arbeit außer Volumenänderungsarbeit.

a)



b) $p_1 = 5\ bar \qquad T_1 = 350\,K$

$v_1 = v_2$

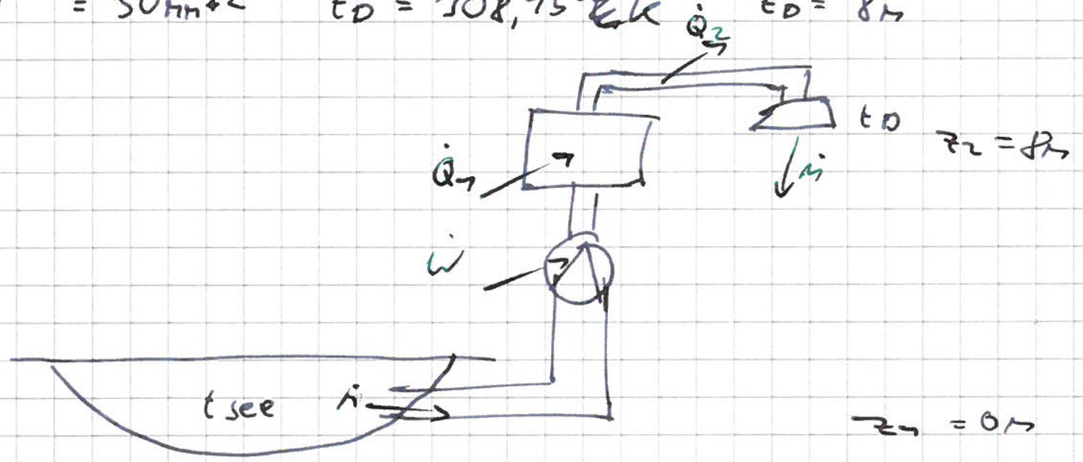
$V = \frac{mRT}{p} \quad | : m$

$v = \frac{RT}{p} = 0{,}2009 \frac{m^3}{kg}$

$\boxed{19}$ $t_{see} = 278{,}15°K$

b) $q_{verl} = 2 \frac{J}{g}$

$A = 50\,mm^2 \qquad t_D = 308{,}15°K \qquad z_D = 8\,m$



~~[illegible]~~ Volumenstrom: $10 \frac{L}{min} = \cancel{0{,}006 \frac{m^3}{s}}\ \frac{1}{6000} \frac{m^3}{s}$

Geschwindigkeit $= \cancel{0{,}0006}\ \frac{1}{6000} \frac{m^3}{s} \cdot 50\,m^2 = \cancel{[illegible]} = 3{,}33 \frac{m}{s}$

c) $12 \frac{L}{min} = \cancel{\frac{12}{7200} \frac{m^3}{s}}\ \frac{12}{6000} \frac{m^3}{s}$

$U = 220\,V$, $16\,A$

23

b) $pv = RT + ap - \frac{bp}{T}$

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \frac{1}{v}\left(R + bp \cdot T^{-2}\right)$$

c)

$$\gamma = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_v = \frac{1}{p}\left(R + bpT^{-2}\right)$$

d)

$$\chi = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = -\frac{1}{v}\left(a - \frac{b}{T} - v\right)$$

Thermo VÜ 17.11.22

22

Zustand 1 : $t_1 = 0°C$ $\quad S_1 = 0 \frac{J}{K}$ $\quad m = 5kg$

$273,15°K$

$\downarrow$ isotherm rev

Zustand 2 : $t_2 = 0°C$

$\downarrow$ rev

Zustand 3 : $t_3 = 10°C = 283,15°K$

Für 1kg Eis : $\Delta Q = 333,3\ J$

Erwärmung von 1kg Wasser um $\Delta T = 1K$: $\Delta Q = 4180\ J$

$c_{H2O} = 4180 \frac{J}{kgK}$

a) $Q_{12} = T(S_2 - S_1) \longrightarrow S_2 = \frac{Q_{12}}{T} + S_1$

$5 \cdot 333,3 = 273,15(S_2 - 0)$

$S_2 = 6,101 \frac{kJ}{K}$

$\rightarrow S_2 - S_1 = 6,101 \frac{kJ}{K}$

b) ~~$S_3 = \frac{Q}{T}$~~ Isochore Zustandsänderung

$S_3 - S_2 = m\, c_v \ln\left(\frac{T_3}{T_2}\right)$

$= 5 \cdot 4180 \cdot \ln\left(\frac{283,15}{273,15}\right) =$

$= 751,74 \frac{J}{K}$

24

$U(v,T) = A \cdot v^a \cdot T^b + c_{v0} \cdot T$

a) $c_v(T,v) = A \cdot v^a \cdot b \cdot T^{b-1} + c_{v0}$

$c_T(T,v) = A \cdot a \cdot v^{a-1} \cdot T^b$

$[u] = \frac{J}{kg} = \frac{\frac{kg \cdot m^2}{s}}{kg} = \frac{kg^2 \cdot m^2}{s^2}$

b) $\frac{kg^2 \cdot m^2}{s^2} = \left(\frac{kg^3}{K m s^2}\right)\left(\frac{m^3}{kg}\right)(K) + \frac{kg^2 \cdot m^2}{s^2 \cdot K}$

$[A] = \frac{kg^3}{K \cdot m \cdot s^2}$ $\qquad [c_{v0}] = \frac{kg^2 \cdot m^2}{s^2 \cdot K}$

23 $\quad pv = RT + \left(a - \frac{b}{T}\right)p$

a) $\frac{N}{m^2}\frac{m^3}{kg} = \frac{J\,\cancel{K}}{kg\,\cancel{K}} + \left(a - \frac{b}{K}\right) \cdot \frac{N}{m^2}$ $\qquad \frac{\frac{kg\,m}{s^2}}{m^2} = \frac{kg}{s^2 m}\frac{m^3}{kg} = \frac{m^2}{s^2}$

~~$[b] = \frac{J}{kg}$~~

$a \frac{N}{m^2} - b \cdot \frac{N}{m^2 K}$ $\qquad \frac{a \cdot kg}{m^3 \cdot s^2}$ $\qquad [a] = \frac{m^3}{kg}$

$\frac{b \cdot kg}{m \cdot K \cdot s^2}$ $\qquad [b] = \frac{m^3 \cdot K}{kg}$

f) ges: $W_{V,12}$

$$W_{V,12} = -\int_1^2 p\,dv = -p(v_2 - v_1) = -p\,(v_2 - v') = -135{,}4\ \frac{kJ}{kg}$$

g) und h)

ges: Freiheitsgrade

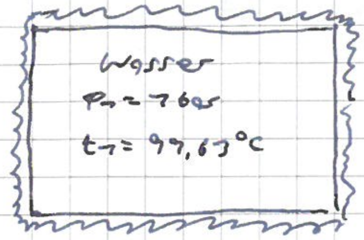
Gibbsche Phasenregel : $F = k + 2 - P$

Hier: $P =$ $\quad$ $k = 1$ (nur Wasser)

- Vor und nach der Erwärmung: $P = 1 \rightarrow F = 2$ $\rightsquigarrow p(v, T)$
- Während der Erwärmung: $P = 2 \rightarrow F = 1$ $\rightsquigarrow p(v)$
- 3 Phasen: $P = 3 \rightarrow F = 0$ $\rightsquigarrow$ Tripelpunkt

Thermo Vortragsübung 1.12.21

26 | - Nassdampf

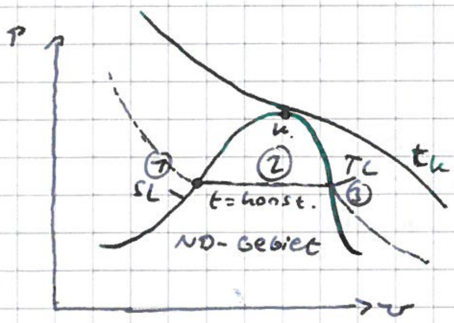


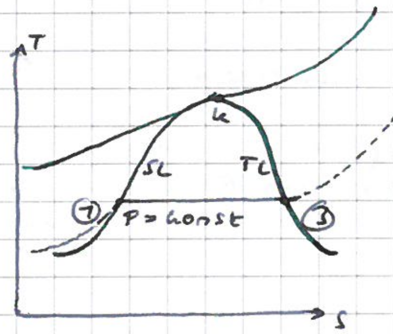
isobar Wärme zufuhr

Zustand 1: auf Siedelinie ↳ flüssig

Zustand 3: vollständig verdampft

a)





b) ges: Spez. Wärme $q_{13} = q_v$ Mithilfe der spez. Entropie s.

2. HS: $ds = \frac{\delta q_{rev}}{T} + ds_{prod}$

hier: isobar-isotherme Zustandsänderung
↳ Gesamte Wärme bewirkt Volumenvergrößerung
↳ reversibel, d.h. $\delta q_{rev} = \delta q_v$; $ds_{prod} = 0$

$\rightarrow ds = \frac{\delta q_{rev}}{T} = \frac{\delta q_v}{T} \quad \Big| \cdot \int_1^3$

$s_3 - s_1 = \underbrace{s''}_{\text{3 auf TL}} - \underbrace{s'}_{\text{1 auf SL}} = \frac{q_v}{T_1} \quad \longleftrightarrow q_v = T_1(s'' - s') = 2256\,\frac{kJ}{kg}$

c) ges: Dampfgehalt x_2 bei geg. v_2

Dampfgehalt: $x = \frac{m''}{m'' + m'} = \frac{\text{Dampfmasse}}{\text{Gesamtmasse}}$

$v = v' \underbrace{\frac{m'}{m'' + m'}}_{(1-x)} + v'' \underbrace{\frac{m''}{m'' + m'}}_{x} = v'(1-x) + v''(x)$

$= v' + x(v'' - v')$ analog für alle anderen ZG im Nassdampfgebiet.

$v_2 = v' + x_2(v'' - v')$

$x_2 = \frac{v_2 - v'}{v'' - v'} = 0{,}800$

d) ges: h_2, q_{12}

$h_2 = h' + x_2(h'' - h') = 2222\,\frac{kJ}{kg}$

q_{12}: Geschlossenes System, isobare ZÄ, nur Volumenänderungsarbeit

$q_{12} = h_2 - h_1 = h_2 - h' = 1805\,\frac{kJ}{kg}$

e) ges: $q_{13} = q_v$

$q_v = h'' - h' = 2256\,\frac{kJ}{kg} = r$ $\qquad r = h'' - h'$

↳ spez. Verdampfungsenthalpie

Thermische ZGL

$$\left.\begin{array}{l} f(T,v) \longrightarrow -p = \left(\frac{\partial f}{\partial v}\right)_T \\ -s(T,v) = \left(\frac{\partial f}{\partial T}\right)_v \end{array}\right\} s \text{ eleminieren } \rightarrow F(T,v,p)=0$$

kanonische Form wird nach dem spez. Volumen differenziert

$$\left(\frac{\partial f}{\partial v}\right)_T = \left(\frac{\partial}{\partial v}\right)_T \left\{ \underbrace{\cancel{(c_v - s_0)(T-T_0)}}_{=0} - T\left(\underbrace{\cancel{c_v \cdot \ln\left(\frac{T}{T_0}\right)}}_{=0} + R\cdot \ln\left(\frac{v}{v_0}\right)\right)\underbrace{\cancel{\cdot f_0}}_{=0} \right\}$$

$$= \left(\frac{\partial}{\partial v}\right)_T \left\{ - T \cdot R \cdot \ln\left(\frac{v}{v_0}\right) \right\}$$

$$= -TR\frac{1}{v} \stackrel{!}{=} -p$$

$$\Leftrightarrow \quad \underline{\underline{pv = RT}} \quad \text{(thermische ZGL)}$$

kalorische ZGL

$$\left.\begin{array}{l} -s(T,v) = \left(\frac{\partial f}{\partial T}\right)_v \\ f(T,v) \end{array}\right\} s \text{ eleminieren } \rightarrow u(T,v)$$

kanonische Form wird nach der Temp. differenziert

$$\left(\frac{\partial f}{\partial T}\right)_v = \left(\frac{\partial}{\partial T}\right)_v \left\{ (c_v - s_0)(T-T_0) - T\left(c_v \cdot \ln\left(\frac{T}{T_0}\right) + R\cdot\ln\left(\frac{v}{v_0}\right)\right)\cdot f_0 \right\}$$

$$= \cancel{c_v} - s_0 - \left(c_v \cdot \ln\left(\frac{T}{T_0}\right) + R\cdot\ln\left(\frac{v}{v_0}\right) + \cancel{T\cdot\left(c_v \cdot \frac{1}{T}\right)}\right) \qquad |\cdot(-1)$$

$$-\left(\frac{\partial f}{\partial T}\right)_v = +s_0 + c_v \ln\left(\frac{T}{T_0}\right) + R\cdot\ln\left(\frac{v}{v_0}\right) = s \qquad (5)$$

Totales differential von $s(T,v)$ bilden

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \qquad (6)$$

(5) und (6) zusammenlegen:

$$ds = c_v \frac{1}{T} dT + R \cdot \frac{1}{v} dv$$

$$T ds = c_v dT + \underbrace{\frac{RT}{v}}_{p} dv = c_v dT + p dv$$

Gibbsche Fundamentalgleichung

$$du = T ds - p dv$$

$$du = c_v dT \qquad \rightarrow \text{kalorische ZGL}$$

$u(T)$, da ideales Gas.

Thermo Vortragsübung 7.12.22

(25) geg: ideales Gas , c_v = konst.

ges: kanonische Zustandsgleichung der spez. freien Energie $f(T, v)$

Wiederholung Vorlesung

Eine kanonische Zustandsgleichung hat den gleichen Informationsgehalt wie die thermische $(F(p, v, T) = 0)$ und die kalorische $(u = u(T, v))$ ZGL. zusammen.
-> Aus einer kanonischen ZGL lassen sich die thermischen und die kalorische ZGL ableiten ("Potentialeigenschaft")
-> Vollständige Beschreibung eines idealen Gases.

thermische ZGL $pv = RT$ (1)

kalorische ZGL $u_1 - u_2 = c_v(T - T_0)$ (2)

} kanonische ZGL $f(T, v)$

Ansatz zur Bestimmung von $f(T, v)$:

$f = u - Ts$ (Def. der spez. freien Energie)

$$f - f_0 = \underbrace{u - u_0}_{c_v(T-T_0)} - Ts + T_0 s_0 \quad (3)$$

-> s eleminieren!

Gilt für reversible Prozesse

Ansatz zur Eleminierung von s durch T und v

Thermische und kalorische ZGL in die Gibbsche Fundamentalgleichung einsetzen:

↳ $du = Tds - pdv$ (1. HS in differentieller Form für die innere Energie)

$$\to ds = \underbrace{\frac{1}{T}du}_{\substack{\text{kalorische} \\ \text{ZGL}}} + \underbrace{\frac{p}{T}}_{\substack{\text{Thermische} \\ \text{ZGL}}} dv$$

$$\to ds = \frac{c_v}{T} dT + \frac{R}{v} dv$$

Integriert:

$$\to s - s_0 = c_v \cdot \ln\left(\frac{T}{T_0}\right) + R \cdot \ln\left(\frac{v}{v_0}\right) \quad (4)$$

$$f - f_0 = c_v(T - T_0) - Ts + T_0 s_0 \quad (3)$$

(3) in (4) einsetzen

$$f - f_0 = c_v(T - T_0) - T\left(c_v \cdot \ln\left(\frac{T}{T_0}\right) + R \cdot \ln\left(\frac{v}{v_0}\right) + s_0\right) + T_0 s_0$$

$$= (c_v - s_0)(T - T_0) - T\left(c_v \ln\left(\frac{T}{T_0}\right) + R \ln\left(\frac{v}{v_0}\right)\right) = f(T, v)$$

Potentialeigenschaft nachweisen "quasi rückwärts"

$du = Tds - pdv$

$f = u - Ts$

$df = du - Tds - sdT$

$df = \underline{-s}dT \underline{\underline{-p}}dv$

Ansatz: Koeffizientenvergleich zwischen
- differentielles Form des 1. HS der spez. freie Energie:
- Totales differential df $f(T, v)$

$$df = \underline{\left(\frac{\partial f}{\partial T}\right)_v} dT + \underline{\underline{\left(\frac{\partial f}{\partial v}\right)_T}} dT$$

$$\to \left(\frac{\partial f}{\partial v}\right)_T = p \quad ; \quad \left(\frac{\partial f}{\partial T}\right)_v = -s$$

4

1. HS für offene, stationäre Systeme :

$$0 = \dot{m}\left(h_2 + \frac{c_2^2}{2}\right) - \dot{m}\left(h_3 + \frac{c_3^2}{2}\right)$$

$$= h_2 - h_3 + \frac{1}{2}\left(c_2^2 - c_3^2\right) = c_p\left(T_2 - T_3\right) + \frac{1}{2}\left(c_2^2 - c_3^2\right) = 0 \qquad \text{III}$$

I $T_3 = \frac{p_3}{R \rho_3}$

II $\rho_3 = \frac{\dot{m}}{A_3 c_3}$

III $0 = c_p(T_2 - T_3) + \frac{1}{2}\left(c_2^2 - c_3^2\right)$

$\cancel{\rho_{3,1} = -0{,}011371 \frac{kg}{m}}$; $\rho_{3,2} = 1{,}672 \frac{kg}{m^2}$;

$\cancel{T_{31} = -12960\ K}$ $\quad T_{32} = 598{,}2\ K$

$\cancel{c_{31} = -12160 \frac{m}{s}}$ $\quad c_{32} = 99{,}71 \frac{m}{s}$

↳ Unphysikalisch!

$\rightarrow T_3 = 598{,}2\ K$

e) ges: c_3

aus Aufgabe d) bekannt: $c_3 = 99{,}71 \frac{m}{s}$

Thermo VÜ 8.12.22

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a) ges: spez. Gaskonstante R, ρ_1, ρ_2

- Für ein ideales Gas gilt:

$$\left.\begin{array}{l} c_p - c_v = R \\ \kappa = \frac{c_p}{c_v} \end{array}\right\} c_p = \frac{\kappa R}{\kappa - 1} \longleftrightarrow R = c_p \cdot \frac{\kappa-1}{\kappa} = 287{,}1 \frac{J}{kgK}$$

$$pv = RT \quad \text{mit } \rho = \frac{1}{v} \quad \Rightarrow \rho = \frac{p}{RT}$$

$$\rho_1 = \frac{p_1}{R \cdot T_1} = 3{,}870 \frac{kg}{m^3} \quad ; \quad \rho_2 = \frac{p_2}{R \cdot T_2} = ? \quad \rightarrow \text{Massenerhaltung}$$

$$\dot m = \text{const} = \rho \cdot c \cdot A \quad \Rightarrow \rho_2 = \frac{\dot m}{A_2 \cdot c_2} = 2{,}081 \frac{kg}{m^3}$$

b) ges: c_1 ; $\dot P = \dot W$

Aus Massenerhaltung: $c_1 = \frac{\dot m}{A_1 \cdot \rho_1} = 129{,}2 \frac{m}{s}$

stationärer Fließprozess: 1. HS für stationäres, offenes System: $\frac{d}{dt} \stackrel{!}{=} 0$
↳ ganz viel kürzt sich weg, übrig bleibt:

$$0 = \dot m \left(h_1 + \frac{c_1^2}{2}\right) - \dot m \left(h_2 + \frac{c_2^2}{2}\right) - P$$

$$\rightarrow P = \dot m \left(h_1 - h_2 + \frac{1}{2}(c_1^2 - c_2^2)\right)$$

$$dh = c_p(T)\, dT \longrightarrow dh = c_p\, dT$$
↳ $c_p = \text{const.}$

$$\rightarrow h - h_0 = c_p(T - T_0) \longleftrightarrow h_1 - h_2 = c_p(T_1 - T_2)$$

$$P = \dot m \left(c_p(T_1 - T_2) + \frac{1}{2}(c_1^2 - c_2^2)\right) = 15{,}33\, MW$$

c) ges: Widerstandsbeiwert ζ

$$p_2 - p_3 = \zeta \cdot \frac{\rho_2}{2} c_2^2$$

$$\rightarrow \zeta = \frac{2(p_2 - p_3)}{\rho_2 \cdot c_2^2} \qquad \text{mit } p_2 = \rho_2 R T_2 \quad \text{und} \quad p_3 = 0{,}8 \cdot p_2$$

$$= \frac{2(RT_2\rho_2 - 0{,}8 \cdot RT_2\rho_2)}{\rho_2 \cdot c_2^2} = \frac{2RT_2(1-0{,}8)}{c_2^2} = 10{,}77$$

Kontrolle über Einheiten:

$$[\zeta] = \frac{\frac{J}{kgK} \cdot K}{\frac{m^2}{s^2}} = \frac{\frac{kg \cdot m^2}{s^2} \cdot \frac{1}{kg}}{\frac{m^2}{s^2}} = \frac{1}{1} \qquad \text{keine Einheit } \checkmark$$

d) ges: p_3, T_3

$$p_3 = 0{,}8 \cdot p_2 = 0{,}8 \cdot \rho_2 \cdot RT_2 = 2{,}871 \cdot 10^5\, Pa = 2{,}871\, bar$$

$$T_3 = \frac{p_3}{R\,\underbrace{\rho_3}_{\text{unbekannt}}} \quad (I) \qquad ; \qquad \rho_3 = \frac{\dot m}{A_3 \underbrace{c_3}_{\text{unbekannt}}} \quad (II)$$

→ T_3 nicht allein aus thermischer ZG bestimmbar

$h_1 = h_1' + x_1 (h_1'' - h_1') = 1412 \frac{kJ}{kg}$

$h_2 = \underbrace{h_2'}_{} + x_2 (\underbrace{h_2''}_{} - \underbrace{h_2'}_{}) =$

unbekannt $\rightarrow$ Interpolieren bei $p = 2{,}5$ bar zwischen $p = 1{,}9854$ bar und $p = 2{,}701$ bar

$h_2' = 534{,}3 \frac{kJ}{kg}$; $h_2'' = 2716 \frac{kJ}{kg}$

$h_2 = 785{,}2 \frac{kJ}{kg}$

$Q_{12} = -7477 \, kJ$

f) ges: Volumenänderungsarbeit $W_{V,23}$; Wärme Q_{23} (isobare Expansion)

$W_{V,23} = -\int_2^3 p \, dV = -p \int_2^3 dV = -m \, p_2 (v_3 - v_2)$

Zustand 3 liegt auf der Taulinie : $v_3 = v_2'' = 0{,}7208 \frac{m^3}{kg}$

$-W_{V,23} = -m \cdot p_2 (v_2'' - v_2) = -379{,}5 \, kJ$

$-Q_{23} = m \underbrace{(U_3 + p_3 v_3)}_{h_3} - m \underbrace{(U_2 + p_2 v_2)}_{h_2}$

(1. HS für geschlossenes System)

$dU = \delta Q + \delta W_V$

$U_3 - U_2 = Q_{23} - W_{V,23} = Q_{23} - m p_2 (v_2'' - v_2)$

$Q_{23} = m (h_3 - h_2) = m (h_2'' - h_2)$

$= 4545 \, kJ$

Alternative zu f:

2. HS für geschlossene Systeme: $dS = \delta Q_{rev} : T$

$\Leftrightarrow \delta q_{rev} = T ds$

$| T = konst ; dp = konst$; Kopplung p und T im ND.

$Q_{23} = T_2 m (s_3 - s_2) = T_2 m (s_2'' - s_2)$

T_2, s_2'', s_2 mit Lin. Interpolation zw. $p = 1{,}984$ bar und $p = 2{,}707$ bar

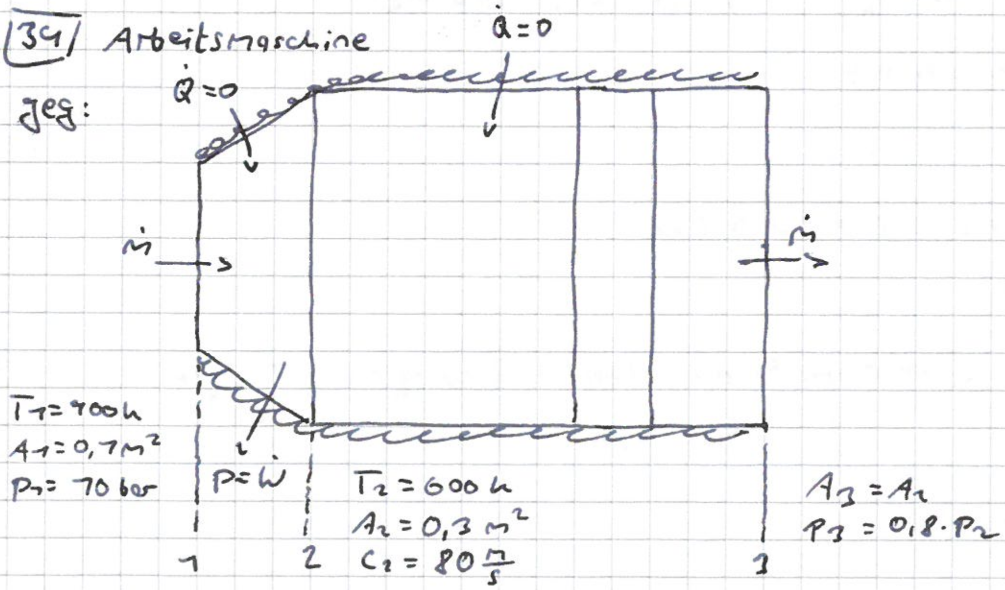
$T_2 = 400{,}3 \, K$ $\quad s_2' = 1{,}604 \frac{kJ}{kgK}$ $\quad s_2'' = 7{,}055 \frac{kJ}{kgK}$

$s_2 = s_2' + x_2 (s_2'' - s_2') = 2{,}232 \frac{kJ}{kgK}$

$Q_{23} = 4545 \, kJ$

34 Arbeitsmaschine

geg:



$\dot m = 50 \frac{kg}{s}$

$c_p = 1005 \frac{J}{kgK} = konst$

$\kappa = 1{,}4$

Ideales Gas

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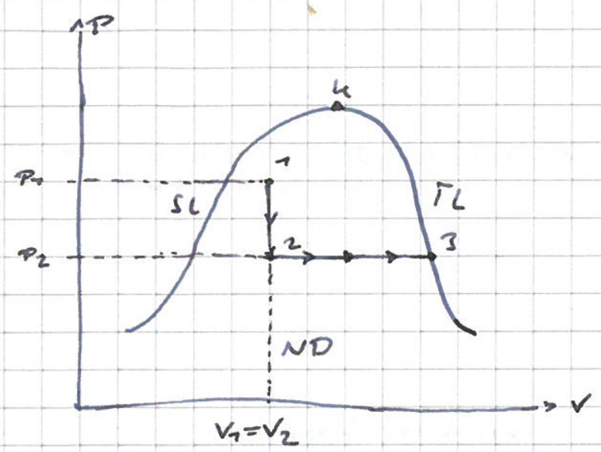
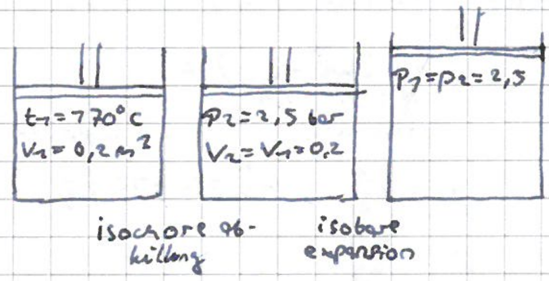
a)





geg: $m = 2{,}35$ kg

1,2 im Nassdampfgebiet

3 auf Taulinie

Im Nassdampfgebiet: Druck und Temp. sind aneinander gebunden: $2 \to 3$: T const.

b) ges: p_1, p_3 kein Ideales Gas

p_1: Bestimmung aus Wasserdampftafel für Wasser bei $t_1 = 170\,°C$

$p_1(t_1) = 7{,}920$ bar

p_3: Gleich wie $p_2 = 2{,}5$ bar

c) ges: v_1, x_1

$v_1 = \frac{V_1}{m} = 8{,}511 \cdot 10^{-2} \frac{m^3}{kg}$

allg: $v_1 = v' + x_1 (v_1'' - v_1')$

Aus Dampftafel: $v_1'(t_1 = 170°C) = 0{,}001145$; $v_1''(t_1 = 170°C) = 0{,}2426 \frac{m^3}{kg}$

$x_1 = \frac{v_1 - v_1'}{v_1'' - v_1'} = 0{,}3478$

d) ges: v_2', v_2'', x_2

$p_2 = 2{,}5$ bar, im ND Kopplung von p und T

$\to$ Lineare Interpolation zwischen $p = 1{,}9854$ bar und $p = 2{,}701$ bar

allg: $y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$

$p_2 = 2{,}5$ bar ; $v_2' = 1{,}067 \cdot 10^{-3} \frac{m^3}{kg}$; $v_2'' = 0{,}7038 \frac{m^3}{kg}$

Dampfgehalt wie bei c) : $x_2 = 0{,}1152$

e) ges: Q_{12}

1. HS für geschlossene Systeme: $dU = \delta Q + \delta W$

$1 - 2$ isochor $\delta W_v \stackrel{!}{=} 0$

$dU = \delta Q \to U_2 - U_1 = Q_{12}$ zugeführte Wärme entspricht Änderung innere Energie

mit $H = U + pV \leftrightarrow U = H - pV$

$Q_{12} = \left[\underbrace{(h_2 - h_1)}_{\text{unbekannt}} - v_1 (p_2 - p_1) \right] m$

Zustand	p (bar)	v $\left(\frac{m^3}{kg}\right)$	T (K)
1	1	0,83	289,14
2	10	~~0,4498~~ 0,4499	1567
3	~~2,999~~	1,5	1567
4	2,000	~~1,5~~ 1,5	1045
5	1,000	2,462	857,5

Zustand 1: Ideale Gasgleichung: $pv = RT$

$R = c_p - c_v$ und $k = \frac{c_p}{c_v}$ $\rightarrow$ $R = c_p - \frac{c_p}{k} = 287,1 \frac{J}{kg\,K}$

$T_1 = \frac{p_1 v_1}{R} = 289,1\,K$

Zustand 2: $1 \rightarrow 2$: polytrope Verdichtung $pv^n = konst$

$p_1 v_1^n = p_2 v_2^n \leftrightarrow v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{1/n} = 0,4498 \frac{m^3}{kg}$

$T_2 = \frac{p_2 v_2}{R} = 1567\,K$

Zustand 3: $2 \rightarrow 3$: isotherme Entspannung $T_3 = T_2 = 1567\,K$

$p_3 = \frac{RT_3}{v_3} = 2,999\,bar$

Zustand 4: $3 \rightarrow 4$: isochore Abkühlung $v_4 = v_3 = 1,5 \frac{m^3}{kg}$

$p_3/T_3 = R/v_3$; $\frac{p_4}{T_4} = R/v_4 \xrightarrow{v_3 = v_4} p_3/T_3 = \underbrace{\frac{p_4}{T_4}}_{unbekannt} = konst$ (1)

Zustand 5: $4 \rightarrow 5$: adiabate Abkühlung $pv^k = konst$

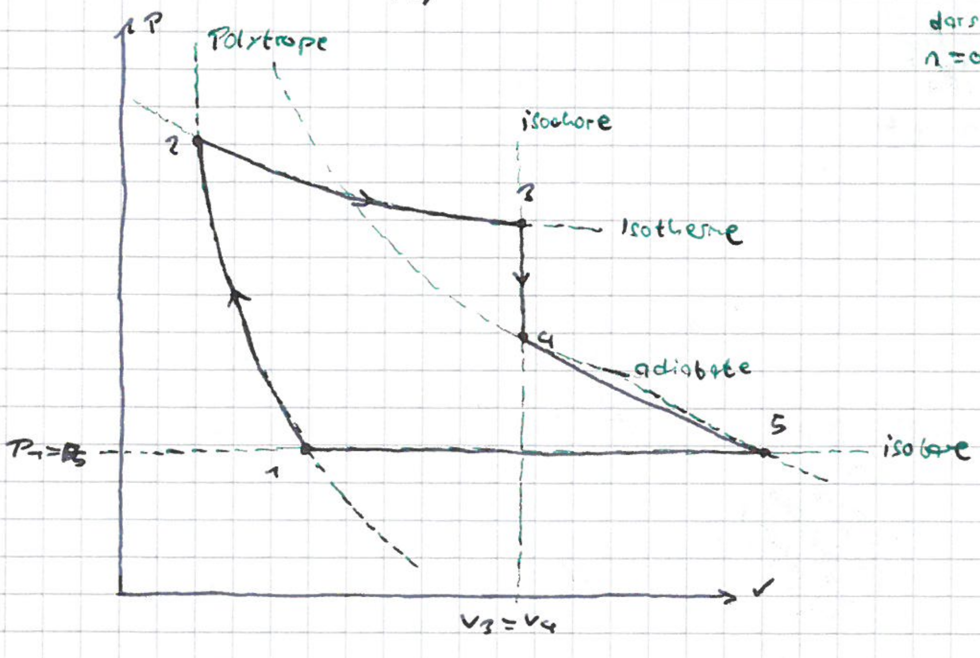
$\underbrace{p_4 \cdot v_4^k}_{unbekannt} = \underbrace{p_5 \cdot v_5^k}$ (2)

$5 \rightarrow 1$: isobare ZÄ $p_5 = p_1 = 1\,bar$

$\frac{v_5}{T_5} = \frac{v_1}{T_1} \rightarrow v_5 = v_1 \cdot \frac{T_5}{T_1} = 2,462 \frac{m^3}{kg}$

In (2): $p_4 = p_5 \cdot \left(\frac{v_5}{v_4}\right)^k = 2,000\,bar$

In (1): $T_4 = T_3 \cdot \left(\frac{T_3}{T_4}\right)^k = 1045\,K$



Alle ZÄ lassen sich durch $pv^n = konst$ darstellen. $n = 1$: isotherm, $n = k$: adiabat, $n = 0$: isobar, $n = \infty$: isochor

~~Academic writing WR [illegible]~~ 3)

1: This precipitation is called snow and is in the form of...

2: The meeting was postponed, because it was inconviniently sceduled at 9 a.m. ...

3: A variety of measures were used to quantify happiness ...

c) ges η_{th}

$\eta_{th} = \frac{|w_{ges}|}{q_{zu}}$ wie bei f)

$q_{zu} \Rightarrow$ nur zugeführte Wärme! ($q_{ij} > 0$)

<u>1 -> 2</u>: Polytrope ZÄ

$w_{v,12} = \frac{R_i T_1}{n-1}\left(1-\left(\frac{v_1}{v_2}\right)^{n-1}\right) = 132{,}9 \frac{kJ}{kg}$

$q_{12} = c_v \frac{n-k}{n-1}(T_2 - T_1)$ mit $c_v = \frac{c_p}{k} = 772{,}1 \frac{J}{kgK}$

$q_{12} = 784{,}3 \frac{kJ}{kg} > 0$

<u>2 -> 3</u>: isotherme ZÄ

$w_{v,23} = -RT_2 \ln\left(\frac{v_3}{v_2}\right) = -541{,}8 \frac{kJ}{kg}$

$du = \delta q + \delta w$ isotherm: $du = 0 \rightarrow q_{23} = -w_{v,23} = 541{,}8 \frac{kJ}{kg}$

<u>3 -> 4</u>: isochore ZÄ

$w_{v,34} = 0$

$q_{34} = u_4 - u_3 = c_v(T_4 - T_3) = -374{,}2 \frac{kJ}{kg}$

<u>4 -> 5</u>: adiabate ZÄ

$du = \overset{=0}{\cancel{\delta q}} + \delta w \rightarrow q_{45} = 0$

$w_{v,45} = -\frac{p_4 v_4}{k-1}\left(1-\left(\frac{v_4}{v_5}\right)^{k-1}\right) = -135{,}0 \frac{kJ}{kg}$

<u>5 -> 1</u>: isotherme ZÄ

$w_{v,51} = -p(v_1 - v_5) = 163{,}2 \frac{kJ}{kg}$

$q_{51} = c_p(T_1 - T_5) = -571{,}3 \frac{kJ}{kg} < 0$

$w_{ges} = -380{,}6 \frac{kJ}{kg}$

$q_{zu} = q_{12} + q_{23} = 1326 \frac{kJ}{kg}$

$\downarrow$

$\eta_{th} = \frac{|w_{ges}|}{q_{zu}} = 0{,}287 \hat{=} 28{,}70\,\%$

(1)

$V_A = 150$ liter

$V_B = 35$ liter

$t_{A,1} = t_{B,1} = 41°C$

$p_{B,1} = 3,5$ bar

$p_{B,2} = 5,5$ bar

$\frac{p_{CO_2,3}}{p_{H_2,3}} = \frac{3}{1}$

a) $R_{CO_2} = \frac{R_m}{M_{CO_2}} = 188,9 \frac{J}{kgK}$

$R_{H_2} = \frac{R_m}{M_{H_2}} = 4124 \frac{J}{kgK}$

$p_{1,B} V_{1,B} = m_{CO_2} R_{CO_2} T_{1,B}$

$m_{CO_2} = 0,2064$ kg

$n_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = 4,69$ mol

b)

$\psi_{H_2} = 1 - \psi_{CO_2}$

$\frac{p_{CO_2,1}}{p_{H_2,3}} = \frac{\psi_{CO_2}}{\psi_{H_2}} = \frac{3}{1}$

$\frac{\psi_{CO_2}}{1-\psi_{CO_2}} = \frac{3}{1}$

$n_{ges} = \frac{n_{CO_2}}{\psi_{CO_2}} = 6,253$ mol

$\psi_{CO_2} = 0,75$

$\psi_{H_2} = 0,25$

$n_{H_2} = \psi_{H_2} \cdot n_{ges} = 1,563$ mol

c) $p_{A,1} V_{A,1} = m_{H_2} R_{H_2} T_{A,1}$

$m_{H_2} = M_{H_2} \cdot n_{H_2} = 3,151 \cdot 10^{-3}$ kg

$V_{A,1} = V_A - V_B = 115$ liter

$p_{A,1} = 0,3550$ bar

d)

$n_G = 6{,}253$ mol

$M_G = \frac{m_G}{n_G} = 0.03357 \frac{kg}{mol}$ $\qquad m_G = 0{,}2096$ kg

$R_G = \frac{1}{m_G}\left(m_{CO_2} R_{CO_2} + m_{H_2} R_{H_2}\right) = 248{,}0 \frac{J}{kgK}$

$c_{CO_2} = \frac{m_{CO_2}}{m_G} = 0{,}9847$

e)

$Q_{12} = c_v (T_2 - T_1) \cdot m_{CO_2}$

$c_v = -R + c_p = 657{,}7 \frac{J}{kgK}$ $\qquad \frac{p_1}{T_1} = \frac{p_2}{T_2} \rightarrow T_{2B} = 493{,}7$ K

$Q_{12} = 2{,}506 \cdot 10^4$ J

$S_2 - S_1 = c_v \ln\left(\frac{T_2}{T_1}\right)$

$= 67{,}39 \frac{J}{K}$

f) T_3

$p_3 V_3 = m_3 R_{ges} T_3$

~~p_3 … bar~~

$V_3 = 150$ liter

$m_3 = m_G = 0{,}2096$ kg

$R_{ges} = R_G = 248{,}0 \frac{J}{kgK}$

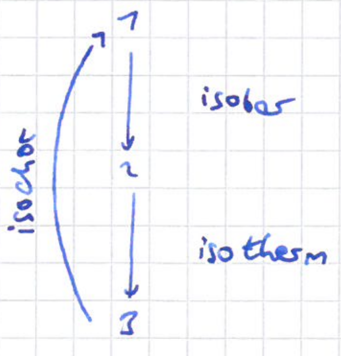
~~$T_3 = 460{,}8$ K~~ $\qquad$? $T_3 = 460{,}1$ K

g) $p_3 V_3 = m_3 R_{ges} T_3$

$p_3 = 1{,}595$ bar

$c_{v,g} = c_{CO_2} c_{v,CO_2} + c_{H_2} c_{v,H_2}$

(2) Van-der-Waals-Gas



a) $a = \frac{a_m}{M^2} = \cancel{4/020}\ 53{,}03\ \frac{m^5}{kg\,s^2}$

$b = \frac{V_k}{3} = 8{,}590 \cdot 10^{-4}\ \frac{m^3}{kg}$

$(\bar{p}_2 + \frac{3}{\bar{v}_2^2})(3\bar{v} - 1) = 8\bar{T}$

$\bar{p}_2 = 2{,}738$

$R = \frac{R_m}{M} = 472{,}0\ \frac{J}{kgK}$

I $a = 3 p_k v_k^2$

II $\frac{3}{8} = \frac{p_k v_k}{R T_k}$

Lineares Gleichungssystem lösen

$V_k = 2{,}577 \cdot 10^{-3}\ \frac{m^3}{kg}$

$p_k = 26{,}62$ bar

$b = 8{,}590 \cdot 10^{-4}\ \frac{m^3}{kg}$

b) $s_1 - s_3 = 490\ \frac{J}{kgK}$

ges: $\cancel{\bar{p}_2}, \cancel{\bar{T}_3}, \cancel{\bar{T}_1}, \cancel{\bar{p}_1}, \cancel{\bar{v}_1}, \cancel{\bar{v}_3}, \cancel{\bar{p}_3}$

$\bar{p}_2$ in a) berechnet

$\bar{p}_2 = 2{,}738$

$\bar{p}_1 = 2{,}738$ (isobar)

$\bar{T}_3 = 1{,}5$ (isotherm)

$\bar{v}_3 = \bar{v}_1$ (isochor)

$s_1 - s_3 = c_v \ln\left(\frac{\bar{T}_1}{\bar{T}_3}\right)$

$\bar{T}_1 = 2{,}996$

$(\bar{p}_1 + \frac{3}{\bar{v}_1^2})(3\bar{v}_1 - 1) = 8\bar{T}_1$

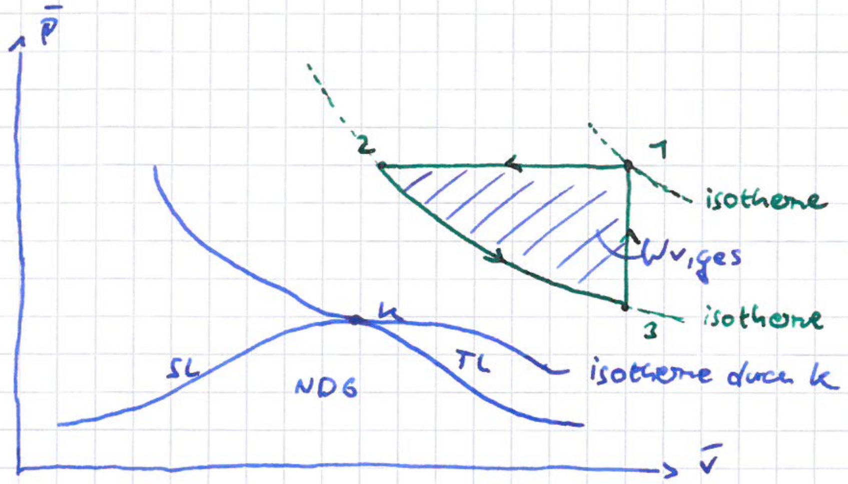
$\bar{v}_1 = 2{,}979$

$\bar{v}_3 = 2{,}979$

$(\bar{p}_3 + \frac{3}{\bar{v}_3^2})(3\bar{v}_3 - 1) = 8\bar{T}_3$

$\bar{p}_3 = 1{,}195$

c)



d)

$$W_{v,12} = -p_1 (v_2 - v_1)$$

$$W_{v,23} = -RT_2 \ln\left(\frac{v_3 - b}{v_2 - b}\right) + \frac{a}{v_2} - \frac{a}{v_3}$$

$$W_{v,31} = 0$$

$$p_1 = \bar{p}_1 \cdot p_k = 12{,}87 \text{ bar}$$

$$p_2 = p_1 = 12{,}87 \text{ bar}$$

$$p_3 = \bar{p}_3 \cdot p_k = 11{,}87 \text{ bar}$$

$$v_1 = \bar{v}_1 \cdot v_k = 1{,}522 \cdot 10^{-3} \ \frac{m^3}{kg}$$

$$v_2 = \bar{v}_2 \cdot v_k = 2{,}835 \cdot 10^{-3} \ \frac{m^3}{kg}$$

$$v_3 = \bar{v}_3 \cdot v_k = 1{,}522 \cdot 10^{-3} \ \frac{m^3}{kg}$$

$$T_1 = \bar{T}_1 \cdot T_k = 133{,}0 \text{ K}$$

$$T_2 = \bar{T}_2 \cdot T_k = 66{,}60 \text{ K}$$

$$T_3 = T_2 = 66{,}60 \text{ K}$$

$$W_{v,12} = 34160 \text{ J/kg}$$

$$W_{v,23} = -21700 \text{ J/kg}$$

$$W_{v,31} = 0 \text{ J/kg}$$

$$W_{v,ges} = W_{v,12} + W_{v,23} + W_{v,31} = 12460 \text{ J/kg}$$

e)

$$\beta_1 = \frac{(v_1 - b) R v_1^2}{RT_1 v_1^3 - 2a(v_1 - b)^2} = 8{,}345 \cdot 10^{-3} \ K^{-1}$$

$$\gamma_1 = \frac{R v_1^2}{RT_1 v_1^2 - a(v_1 - b)} = 8{,}486 \cdot 10^{-3} \ K^{-1}$$

$$\varkappa_1 = \frac{(v_1 - b)^2 v_1^2}{RT_1 v_1^3 - 2a(v_1 - b)^2} = 1{,}350 \cdot 10^{-7} \ K^{-1}$$

$$c_{p,1} = \frac{T_1 v_1 \beta_1^2}{\varkappa_1} + c_v = 1224 \ \frac{J}{kgK}$$

3 $T_0 = 365{,}8\,K$

$\dot{V}_1 = 5{,}32\,\frac{m^3}{s}$

$c_1 = 98\,\frac{m}{s}$

$Ma_2 = 2{,}1$

$\rho_1 = 3{,}5\,\frac{kg}{m^3}$

$R = 211\,\frac{J}{kgK}$

$c_p = 1175\,\frac{J}{kgK}$ rev. ad.

a) $c_p = \frac{k}{k-1} R$

$k = 1{,}36$

$T_0 = T_1 + \frac{c_1^2}{2c_p}$

$T_1 = 361{,}7\,K$

$c_s = \sqrt{kRT_1} = 391{,}1\,\frac{m}{s}$

$Ma = \frac{c_1}{c_s} = 0{,}2506$

b) Ja, weil $Ma_2 > 1$ und $Ma_1 < 1$

$\frac{T^*}{T_0} = \frac{2}{k+1}$

$T^* = 310{,}0\,K$

c) $\frac{\rho_2}{\rho_1} = \frac{2k\,Ma_2^2 - k + 1}{k+1}$ (falsche Formel, aber richtig eingegeben)

$\rho_2 = 1{,}206\,\frac{kg}{m^3}$

$$\frac{\rho_0}{\rho_2} = \left(1 + \frac{k-1}{2} Ma_2^2\right)^{\frac{1}{k-1}}$$

$$\rho_0 = 6{,}773 \ \frac{kg}{m^3}$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{k-1}{2} Ma_1^2\right)^{\frac{1}{k-1}}$$

$$\rho_1 = 5{,}925 \ \frac{kg}{m^3}$$

$$\dot{m} = \rho_1 c_1 A_1 = \rho_1 \cdot \dot{V}_1 = 37{,}52 \ \frac{kg}{s}$$

$$\frac{H_1}{H^*} = \frac{A_1}{A^*} = \frac{1}{Ma_1}\left(\frac{2}{k+1}\left(1 + \frac{k-1}{2} Ma_1^2\right)\right)^{\frac{k+1}{2(k-1)}}$$

$$\frac{H_1}{H^*} = 2{,}407$$

$$\frac{H_2}{H^*} = \frac{A_2}{A^*} = \quad " \quad " \quad " \quad "$$

$$= 1{,}879$$

$$\frac{H_2}{H_1} = \frac{H_2}{H^*} \cdot \frac{H^*}{H_1} = 0{,}7806$$

$$H_1 = ???$$

e) $$\frac{p_0}{p_3} = \left(1 + \frac{k-1}{2} Ma_3^2\right)^{\frac{k}{k-1}}$$

$$Ma_3 = \sqrt{\frac{(k-1)(Ma_2^2-1)+k+1}{2k(Ma_2^2-1)+k+1}}$$

$$= 0{,}5553$$

~~$p_{0,3}$~~

$$\frac{p_3}{p_2} = \frac{\frac{k+1}{k-1}\frac{\rho_3}{\rho_2} - 1}{\frac{k+1}{k-1} - \frac{\rho_2}{\rho_1}}$$

$$\frac{p_1}{p_2} = \frac{2k Ma_2^2 - k + 1}{k+1} = 4{,}930$$

$$\frac{T_3}{T_2} = \frac{(2k Ma_2^2 - k + 1)(2 + (k-1) Ma_2^2)}{(k+1)^2 Ma_2^2} = 1{,}699$$

$$s_{03} - s_{02} = c_p \ln\left(\frac{T_3}{T_2}\right) - R \ln\left(\frac{p_3}{p_2}\right) = 126{,}6 \ \frac{J}{kgK}$$

$$\dot{m} = \frac{\Delta \dot{S}_{23}}{s_3 - s_2}$$

$$\Delta \dot{S}_{23} = 3990 \ \frac{J}{Ks}$$

f) totales Druck / Totaldruck

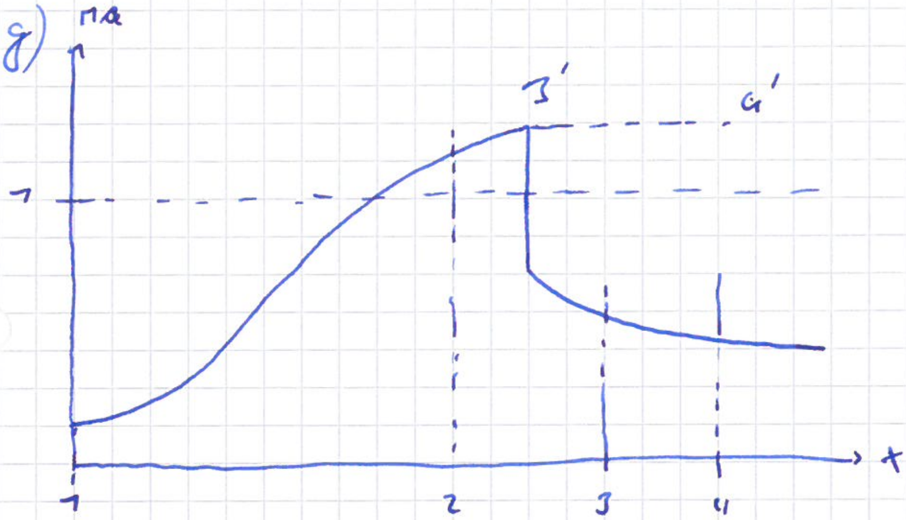
$p_{0,4} = p_{0,3} \Rightarrow \rho_{0,2} R T_{0,2} = 6{,}958 \text{ bar} = p_{,02}$

$s_3 - s_2 = -R \ln\left(\frac{p_{0,3}}{p_{0,2}}\right)$

$p_{0,3} = 4{,}626 \text{ bar}$

$p_{0,4} = p_{0,3} = 4{,}626 \text{ bar}$

g)



(4) $\Pi_G = 15$, $p_{1,G} = 1 bar$, $T_{1,G} = 500 K$, $T_{3,G} = 1850 K$

$c_{p,G} = 1008 \frac{J}{kgK}$ und $\kappa_G = 1,4$

a) $p_{2,G} = \Pi_G \, p_{1,G} = 15 bar$

Joule:
1
↓ Isentrop
2
↓ Isobar
3 Isentrop
↓
4
↓ Isobar
1

$T_2 = T_{1,G} \left(\frac{p_{2,G}}{p_{1,G}}\right)^{\frac{\kappa_G-1}{\kappa_G}} = 1084 K$

$p_3 = p_2 = 15 bar$

$T_3 = 1850 K$

$T_4 = T_{3,G} \left(\frac{p_4}{p_3}\right)^{\frac{\kappa_G-1}{\kappa_G}} = 853,4 K$

$p_4 = p_1 = 1 bar$

b)
1 Gesättigtes Wasser
↓ rev. ad. Verdichtung
2 $T_{2,D} = 448 K$
↓ Isobare Erwärmung in überhitzten Zustand $\Delta T_{G-D} = 47,03 K$
3
↓ rev. adiabat entspannt
4 $p_{4,D} = 8 bar$ Gesättigter Zustand

$T_{3,D} = T_{4,G} - \Delta T_{G-D} = 806,4 K$

$T_{2,D} = 448 K$

$T_{4,D} = 443,6 K$

$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\kappa}{\kappa-1}}$

$p_{3,D} = 100 bar$

$p_{2,D} = 100 bar$

$p_1 = p_4 = 8 bar$ $T_1 = T_4 = 443,6 K$

c) $q_{41} = c_p(T_{4a} - \cancel{T_{1}})$ (weil isobar)

$= -356{,}2 \frac{kJ}{kg}$

$q_{verdampfung} = \cancel{c_p t} \; h'' - h' = 1379 \frac{kJ}{kg}$

$T_1'' = 110{,}96\,°C$

$q_{\text{Überhitzung}} = 1037 \frac{kJ}{kg}$

e) $\eta_{th} = 1 - \frac{T_{1,c}}{T_{2,c}} = 0{,}5387$

$w_{t,c} = -q_{ges} = -q_{23} + |q_{41}| = -476{,}0 \frac{kJ}{kg}$

(1) $g(T,p) = \frac{AT^2}{p} + pBT$

a) $[A\cdot]\frac{K^2}{\frac{kg}{s^2\cdot m}} \stackrel{!}{=} [B]\cdot\frac{kg}{s^2\cdot m}\cdot K \stackrel{!}{=} \frac{J}{kg} \stackrel{!}{=} \frac{m^2}{s^2} = [A]\cdot\frac{K^2\cdot s^2 m}{kg}$

$[B] = \frac{m^3}{kg\cdot K}$

$[A] = \frac{m\cdot kg}{K^2\cdot s^4}$

Es handelt sich um die kanonische Zustandsgleichung $g(T,p)$ für die spezifische freie Enthalpie

b) $F(p,v,T) = 0$

Totales Diff.:

$dg = \left(\frac{\partial g}{\partial T}\right)_p dT + \left(\frac{\partial g}{\partial p}\right)_T dp$

Gibbs.:

$v = \left(\frac{\partial g}{\partial p}\right)_T$

$dg = -s\,dT + v\,dp$

$v = BT - \frac{AT^2}{p^2}$; $-s = \frac{2AT}{p} + pBT$

$0 = BT - v - \frac{AT^2}{p^2} = F(p,v,T)$

c) $u(s,p)$

$dh = Tds + Vdp$; $h = u + pv$

$du = Tds - pdV$; $g = h - Ts$;

$u = Ts - pv$

I $g = u + pV - Ts$

II $g = \frac{AT^2}{p} + pBT$

$u(s,p) = sT + \frac{T}{p}(AT^2 + Bp^2T - p^2v)$; T und v einsetzen

$u(s,p) = \frac{B\cdot p^2\cdot(s+Bp)}{2\cdot A}$

d) $\beta(T,v)$ $\varkappa(p,v)$

$p(v-b) = C_0 T^a$

$v(T,p) = \frac{1}{p}(C_0 \cdot T^a) + b$ $\quad$; $C_0 = \frac{p(v-b)}{T^a}$

$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$ $\quad [\frac{1}{K}]$

$= \frac{1}{v} \cdot \frac{1}{p}(a \cdot C_0 \cdot T^{a-1})$

$\varkappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$ $\quad [\frac{ms^2}{kg}]$

$= -\frac{1}{v} \cdot \frac{1}{p^2}(-C_0 T^a)$

e) $\beta = p \gamma \varkappa$

$\gamma = \frac{\beta}{p\varkappa} = \frac{a}{T}$ $\quad ↯$

f) $↯$

g) $c_p - c_v = \frac{T v \beta^2}{\varkappa} = a^2 C_0 T^{a-1}$ $\quad ↯$ $\quad$ Folgefehler?

$a = 1$

$C_0 = R$

$b = 0$

② Van-der-Waals-Gas

$1 \xrightarrow{\text{isobar}} 2 \xrightarrow{\text{rev. adiabat}} 3 \xrightarrow{\text{isochor}} 1$

a) a, b

$\frac{3}{8} = \frac{p_k v_k}{R T_k}$ $\longrightarrow$ $v_k = 6{,}153 \cdot 10^{-3}\ m^3/kg$ $\quad$ mit $R = \frac{R_M}{M} = 188{,}5\ \frac{J}{kgK}$

$a = 3 p_k v_k^2$ $\longrightarrow$ $a = 482{,}8\ \frac{m^5}{kg\,s^2}$

$b = v_k/3$ $\longrightarrow$ $b = 2{,}051 \cdot 10^{-3}\ \frac{m^3}{kg}$

b)

$$\bar{T}_1 = \frac{T_1}{T_k} = 0{,}9236$$

$$\left(p_1 + \frac{a}{v_1^2}\right)(v-b) = RT_1 \quad ; \quad p_1 = \bar{p}_1 \cdot p_k = 25{,}08 \text{ bar}$$

$$T_1 = 341{,}7 \text{ K} \qquad v_1 = \bar{v}_1 \cdot v_k = 1{,}846 \cdot 10^{-2} \ \frac{m^3}{kg}$$

$$\beta_1 = \frac{(v_1-b)Rv_1^2}{RT_1v_1^3 - 2a(v_1-b)^2}$$

$$= 1{,}260 \cdot 10^{-3} \ K^{-1}$$

$$\kappa_1 = \frac{(v_1-b)^2 v_1^2}{RT_1v_1^3 - 2a(v_1-b)^2}$$

$$= 6{,}320 \cdot 10^{-7} \ K^{-1}$$

$$\gamma_1 = \frac{Rv_1^2}{RT_1v_1^2 - a(v_1-b)}$$

$$= 4{,}579 \cdot 10^{-3} \ K^{-1}$$

$$c_p = c_v + \frac{T_1 v_1 \beta_1^2}{\kappa_1}$$

$$= 2450 \ \frac{kJ}{kgK}$$

c) T_2, v_2, q_{12}

$$p_2 = p_1 = 25{,}08 \text{ bar}$$

$$w_{v,12} = -p_1(v_2 - v_1) = -1{,}5 \cdot 10^3 \ \frac{J}{kg}$$

$$v_2 = 2{,}145 \cdot 10^{-2} \ \frac{m^3}{kg}$$

$$T_2 = T_1 \frac{v_2-b}{v_1-b} + \frac{a}{R}(v_2-b)\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right)$$

$$= 366{,}1 \text{ K}$$

$$q_{12} = \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) + p_1(v_2 - v_1)$$

$$= 5{,}809 \cdot 10^4 \ \frac{J}{kg}$$

d) $T_3, p_3, v_3, w_{v,23}$

$$v_3 = v_1 = 1{,}846 \cdot 10^{-2} \ \frac{m^3}{kg}$$

$$\left(p_2 + \frac{a}{v_2^2}\right)(v_2-b)^{\frac{c_v+R}{c_v}} = \left(p_3 + \frac{a}{v_3^2}\right)(v_3-b)^{\frac{c_v+R}{c_v}}$$

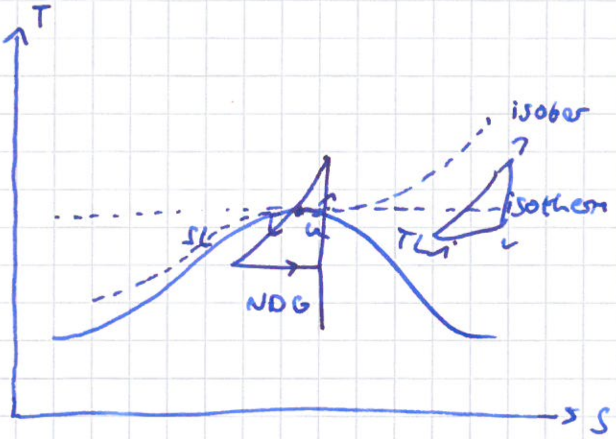
$$p_3 = 28{,}58 \text{ bar}$$

$$T_3 = T_2\left(\frac{v_2-b}{v_3-b}\right)^{\frac{R}{c_v}}$$

$$T_3 = 372{,}2 \text{ K}$$

$$w_{v,23} = \frac{a}{v_2} - \frac{a}{v_3} + c_v(T_3 - T_2) = 8{,}097 \cdot 10^3 \ \frac{J}{kg}$$

e)



Kälteprozess, weil linkslaufend

(3) Ideales Gas: $\kappa = 1{,}4 \quad R = 287 \frac{J}{kgK}$

$p_0 = 2{,}5 \text{ bar} \quad T_0 = 380 K$

$A^* = 1040 mm^2$

$w_1 = 20{,}04 \frac{m}{s}$

$Ma_2 = 2$

a) Düse wird kritisch durchströmt, Austritt bei $Ma_2 = 2 > 1$

$p^* = p_0 \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} = 1{,}327 \text{ bar}$

$Ma^* = 1$

b) $T^* = T_0 \cdot \left(\frac{2}{\kappa+1}\right) = 316{,}7 K$

$\dot m = \rho_1 w_1 A_1$

$= \rho^* c^* A^*$ $\qquad \rho^* = \frac{p^*}{RT^*} = 1{,}453 \frac{kg}{m^3}$

$c_s^* = \sqrt{\kappa R T^*} = 356{,}7 \frac{m}{s} = v$ (weil $Ma^* = 1$)

$\dot m = 0{,}5290 \frac{kg}{s}$

c) T_1, Ma_1

$T_0 = T_1 + \frac{c_1^2}{2c_p}$; $c_p = \frac{\kappa}{\kappa-1} R = 1005 \frac{J}{kgK}$ (c_1 ← w_1)

$T_1 = 379{,}8 K$

$\left(\frac{T_0}{T_1}\right)^{\frac{1}{\kappa-1}} = \left(1 + \frac{\kappa-1}{2} Ma^2\right)^{\frac{1}{\kappa-1}}$

$Ma = 0{,}05131$

d) A_2, p_2

$\frac{A_2}{A^*} = \frac{1}{Ma_2}\left(\frac{2}{\kappa+1}\left(1 + \frac{\kappa-1}{2} Ma_2^2\right)\right)^{\frac{\kappa+1}{2(\kappa-1)}}$

$A_2 = 1{,}755 \cdot 10^{-3} m^2$

$\frac{p_0}{p_2} = \left(\frac{T_0}{T_2}\right)^{\frac{\kappa}{\kappa-1}}$ $\qquad p_2 = 0{,}3195 \text{ bar}$

e)

$$Ma_3 = \sqrt{\frac{(k-1)(Ma_2^2-1)+k+1}{2k(Ma_2^2-1)+k+1}} = 0{,}5774$$

$$\frac{p_3}{p_2} = \frac{2k Ma_2^2 - k + 1}{k+1}$$

$$p_3 = 1{,}438 \text{ bar}$$

$$p_{0,3} = p_3 \left(1 + \frac{k-1}{2} Ma_3^2\right)^{\frac{k}{k-1}} = 1{,}803 \text{ bar}$$

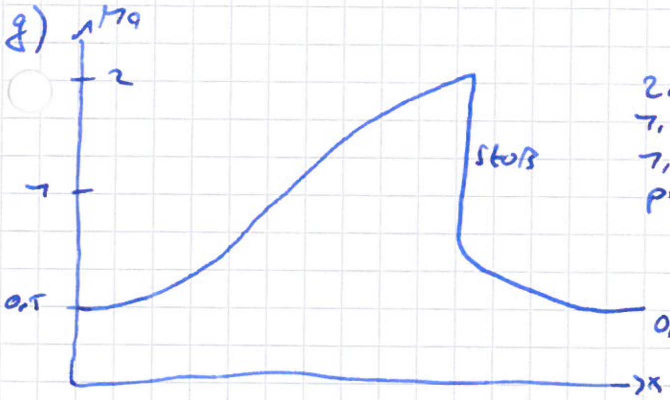
$$\Delta p_0 = p_{0,3} - p_{0,2} = -0{,}6970 \text{ bar}$$

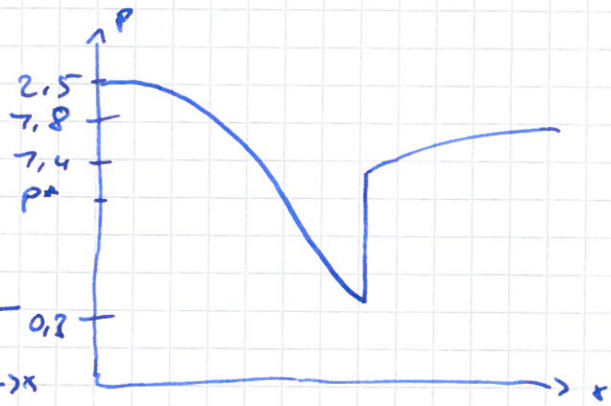
$$s_{3-2} = -R \ln\left(\frac{p_{0,3}}{p_0}\right) = 93{,}80 \; \frac{J}{kgK}$$

f) $T_{0,4} = T_{0,3} = T_0 = 380 \text{ K}$

$p_{0,4} = p_{0,3} = \; = 1{,}802$ ~~[illegible]~~ bar

g)





(4) 1 $T_1 = 256K$ $p_1 = 0,54 bar$ $c_1 = 780 \frac{m}{s}$

↓ rev. adiabat

2 $c_2 = 0 \frac{m}{s}$

$\eta_{sv} = 0,8$ $\pi_v = 12,4$

↓

3

↓ isobare Verbrennung

4 $T_4 = 1100K$

↓ adiabat expandiert $\eta_{sT} = 0,9$

5

↓ isobare Verbrennung

6 $T_6^+ = 1400K$

↓ rev. adiabate Entspannung

7 $A_7 = 0,25 m^2$ $A_7^* = 0,32 m^2$

↓

1

$\eta_{sv} = 0,8$ $\pi_v = 12,4$ (2→3) $\dot{m} = 74,5 kg/s$

a) ~~$\dot{m} = \rho_1 c_1 A_1$~~

$c_p = \frac{k}{k-1} R = 1005 \frac{J}{kgK}$

$T_2 = T_1 + \frac{c_1^2}{2c_p} = 272,7 K$

$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$

$p_2 = 0,6685 bar$

$\eta_{th} = 1 - \frac{T_1}{T_3}$

$T_3 = T_2 + \frac{T_2(\pi_v^{\frac{k-1}{k}} - T_2)}{\eta_v} = 630,3K$

$p_3 = \pi_v \cdot p_2 = 8,289 bar$

$p_4 = p_3$; $T_4 = 1400K$

$p_5 = p_4 \left(\frac{T_{5,rev}}{T_4}\right)^{\frac{k}{k-1}} = 2,024 bar$

$w_{t,23} = w_{t,45} \rightarrow T_3 - T_2 = T_4 - T_5$

$T_5 = 847,8K$

~~$T_{5,rev}$~~ $\eta_{th} = \frac{T_4 - T_5}{T_4 - T_{5,rev}} \rightarrow T_{5,rev} = 802,0K$

$T_6 = T_5 = 847,8K$ $p_6 = p_5 = 2,024 bar$

$T_6^+ = 1400 K$ $p_6^* = /$ $p_1 = p_7 = 0,54 bar$ $T_7 = 577,7K$

b) $T_6^* = 1400\,k$

$p_7^* = p_1 = 0{,}54$

$p_5 = p_6^* = 2824\,k$

$T_7^* = \left(\frac{p_7^*}{p_6^*}\right)^{\frac{\kappa-1}{\kappa}} T_6^* = 459{,}9\,k$

c)

d) $\dot{m} = \rho_7 A_7 c_7 \Rightarrow \rho = \frac{p}{RT} \Rightarrow \dot{m} = 79{,}5\,\frac{m}{s}$

$\rho_7 = \frac{p_7}{R T_7} = 3{,}26 \cdot 10^{-6}$

$c_7 = 974{,}1\,\frac{m}{s}$

$c_7^* = 1188\,\frac{m}{s}$ (genau gleicher Rechenweg

$F_{ohne} = \dot{m} \cdot (c_7 - c_1) = 5{,}469 \cdot 10^4\,N$

$F_{mit} = \dot{m} \cdot (c_7^* - c_1) = 7{,}570 \cdot 10^4\,N$

e) $\eta_{th\,ohne} = 1 - \frac{|q_{ab}|}{q_{zu}} = 1 - \frac{|T_7 - T_1|}{T_4 - T_3} = 43{,}64\,\%$

$\eta_{th\,mit} = 1 - \frac{|q_{ab}^*|}{q_{zu}}$

$= 1 - \frac{|T_7^* - T_1|}{(T_4 - T_3) + (T_6^* - T_5)} = 37{,}59\,\%$

Wirkungsgrad mit Nachbrenner ist kleiner "das mehr Arbeit ins System fließt" ??

f) $\eta_{m,v} = \frac{W_{t,rev}}{W_t}$ komische Herleitung...

$V = 15\,m^3$ ~~[illegible]~~

$t_1 = 22°C$ $\varphi_1 = 30\%$ $m_D = 0{,}255\,kg$ $t_D = 40°C$

$t_U = 4°C$ $\varphi_U = 80\%$

a) $x_1 = 5\,\frac{g}{kg} = 0{,}005\,\frac{kg}{kg}$

b) ~~[illegible]~~

$\rho_1 = \frac{(1+x_1)p_1}{(R_L + x R_D)T_1} = 1{,}177\,\frac{kg}{m^3}$

$m_1 = \rho \cdot V = 17{,}65\,kg$

$m_{W1} = m_1 \cdot 0{,}005 = 8{,}827\,g$

$m_{L1} = m_1 - m_{W1} = 17{,}57\,kg$

c) $\frac{\Delta h}{\Delta x}$ ~~[illegible]~~ $\frac{kJ}{kg}$

$= h_D = c_{pD} \cdot t_D + r_D = 2577\,\frac{kJ}{kg}$

$m_{L1} x_1 + m_D = m_{L1} x_2$

$x_2 = 19{,}51\,\frac{g}{kg}$

$t_2 = 24°C$ abgelesen

d) $9\,m^3$ mit feuchter Umgebungsluft ausgetauscht

$x_U = 4{,}1\,\frac{g}{kg}$

$m_{L2} x_2 + m_{D2} = m_{L2} x_{mix}$

$m_2 = \rho_2 \cdot V$

$\rho_2 = \frac{(1+x_2)p_2}{(R_L + x_2 R_D)T_2} = 1{,}159\,\frac{kg}{m^3}$

$m_2 = 6{,}955\,kg$

$\rho_U = \frac{(1+x_U)p_U}{(R_L + x_U R_D)T_U} = 1{,}254\,\frac{kg}{m^3}$

$m_U = 11{,}29\,kg$

$m_{L2} = m_2 - m_2 \cdot x_2 = 6{,}819\,kg$

$m_{LU} = m_U - m_U \cdot x_U = 11{,}24\,kg$

e) nach dem Mischen

$m_{L2} + m_{LU} = m_{L3}$ $\qquad m_{L3} = 78{,}77\ kg$

$m_{L2}\,x_2 + m_{LU}\,x_U = m_{L3}\,x_3$

$x_3 = 9{,}897\ \frac{g}{kg}$

Einzeichnen über Mischungsgerade zwischen U und 2

f) $0 = \dot{m}\left(h_2 + \frac{c_2^2}{2} + g z_2\right) - \dot{m}\left(h_3 + \frac{c_3^2}{2} + g z_3\right) + \dot{Q}_{23} + \dot{W}_{t,23}$

$0 = m_2 (h_2) - m_3 h_3$

$m_{L2} h_2 + m_{LU} h_U = m_{L3} h_3$

$x_{D2} = 79\ \frac{g}{kg} \qquad x_{E2} = 0{,}57\ \frac{g}{kg}$

$h_2 = c_{pL} t_2 + x_{D2}(c_{pD} t_2 + r_D) + x_{E2} c_W t_2 \qquad = 72{,}52\ \frac{kJ}{kg}$

$h_U = c_{pL} t_U + x_{UD}(c_{pD} t_U + r_D) \qquad = 74{,}37\ \frac{kJ}{kg}$

$h_3 = 36{,}79\ \frac{kJ}{kg}$

$x_{D,3} = 9{,}2\ \frac{g}{kg} \qquad x_{E,3} = 0{,}66\ \frac{g}{kg}$

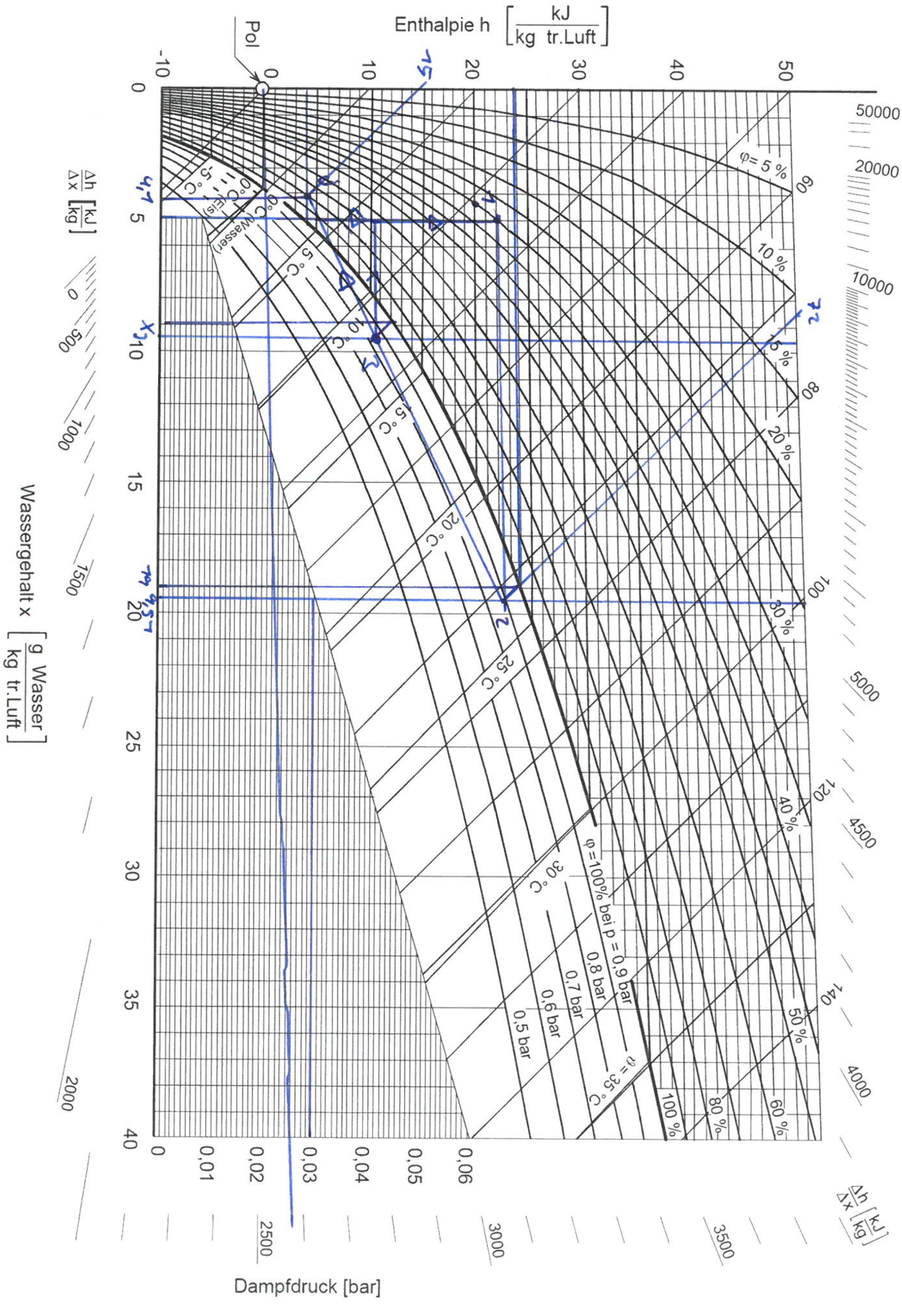
$h_3 = c_{pL} t_3 + x_{D,3}(c_{pD} t_3 + r_D)$

$t_3 = 72{,}89\,°C$

g) Erneutes Lüften

$h_4 = 77{,}9\ \frac{kJ}{kg}$

$q_{41} = h_1 - h_4 = 76{,}96\ \frac{kJ}{kg}$



Enthalpie h [kJ / kg tr.Luft]
Pol
Wassergehalt x [g Wasser / kg tr.Luft]
Δh/Δx [kJ/kg]
Dampfdruck [bar]
φ = 5 %
10 %
15 %
20 %
30 %
40 %
50 %
60 %
80 %
100 %
φ = 100% bei p = 0,9 bar
0,8 bar
0,7 bar
0,6 bar
0,5 bar
φ = 35 °C
0 °C (Eis)
0 °C (Wasser)
-5 °C
5 °C
10 °C
15 °C
20 °C
25 °C
30 °C

1) $F = n\left\{(\omega-\phi)T - \phi T \ln\left(\frac{V}{V_0}\right) - \omega T \ln\left(\frac{T}{T_0}\right) + \phi T \ln\left(\frac{T}{T_0}\right)\right\}$

a) $\phi T n \hat{=} \omega T \hat{=} (\omega-\phi) T n \hat{=} F\ [J] \hat{=} \left[\frac{kg\,m^2}{s^2}\right]$

$\phi = \left[\frac{kg\,m^2}{mol\,s^2\,K}\right]$

$\omega = \left[\frac{kg\,m^2}{mol\,s^2\,K}\right]$

kanonische Zustandsgleichung für Freihe ~~Enthalpie~~ Energie $F(T,V)$

b) $dF = \underbrace{\left(\frac{\partial F}{\partial T}\right)_V}\, dT + \underbrace{\left(\frac{\partial F}{\partial V}\right)_T}_{-p} dV$

$dF = -S\,dT - p\,dV$

$-p = \frac{n\phi T}{V}$

$p(T,V,n) = +\frac{n\phi T}{V}$ Therm. ZGL ideales Gas

$p = \frac{nR_m T}{V}$

$\phi \hat{=} +R_m$ Universelle Gaskonstante

c) $-S = \cancel{\ldots} - n\left(\ln\left(\frac{T}{T_0}\right)\cdot(\omega-\phi) - \phi\cdot\ln\left(\frac{V}{V_0}\right)\right)$

$S(n,T,V) = n\left(\ln\left(\frac{T}{T_0}\right)(\omega-\phi) - \phi\ln\left(\frac{V}{V_0}\right)\right)$

$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

$V = \frac{nR_m T}{p}$

$\beta = \frac{nR_m}{Vp}$

d) $U = F + TS$

$= -nT\underbrace{\left(2\phi\ln\left(\frac{V}{V_0}\right) - \omega + \phi\right)}_{\text{Hier laut Lösung nur } (\omega-\phi)}$

$\hookrightarrow (\omega-\phi) = C_{V,m}$

e) $U = U(S,V,n)$

$S(n,T,V)$ nach T umschreiben:

$T(S,n,V) = \left(\frac{V}{V_0}\right)^{\frac{\cancel{\phi}-\omega_1\ R_m}{C_{V,m}}}\cdot \exp\left(\frac{S}{\underbrace{n(\omega-\phi)}_{n C_{V,m}}}\right)\cdot T_0$

$T(S,V,n) = n\left(\frac{V}{V_0}\right)^{\frac{R_m}{C_{V,m}}}\cdot \exp\left(\frac{S}{n\cdot C_{V,m}}\right)\cdot T_0$

f) übersprungen...

② ad. reversibel

$1 \xrightarrow{\text{isobar}} 2 \xrightarrow{\text{isochor}} 3 \rightarrow 1$

Van der Waals Gas

$p_k = 4{,}599 \cdot 10^{6}\ Pa$

$T_k = 190{,}6\ K$

$v_k = 6{,}135 \cdot 10^{-3}\ \frac{m^3}{kg}$

$R = 518{,}3\ \frac{J}{kgK}$

$c_v = 1657\ \frac{J}{kgK}$

$v_1 = 2{,}761 \cdot 10^{-2}\ \frac{m^3}{kg}$

$p_2 = 1{,}238 \cdot 10^{7}\ Pa = p_1$

a) $a = 3 p_k v_k^2$

$= 519{,}3\ \frac{m^5}{kg\, s^2}$

$b = \frac{v_k}{3}$

$= 2{,}045 \cdot 10^{-3}\ \frac{m^3}{kg}$

$T_1 = 644{,}2\ K$ (über therm. ZGL)

$\bar{v}_1 = \frac{v_1}{v_k} = 4{,}5$

$\bar{p}_1 = \frac{p_1}{p_k} = 2{,}962$

b) $w_{v,12} = 1{,}898 \cdot 10^{5}\ \frac{J}{kg}$

$w_{v,12} = -p_1 (v_2 - v_1)$

$v_2 = 1{,}228 \cdot 10^{-2}\ \frac{m^3}{kg}$

$\bar{v}_2 = \frac{v_2}{v_k} = 2{,}002$

$T_2 = 372{,}5\ K$ (über therm. ZGL)

$q_{12} = \frac{a}{v_1} - \frac{a}{v_2} + c_v (T_2 - T_1) + p_1 (v_2 - v_1)$

$= -7{,}629 \cdot 10^{5}\ \frac{J}{kg}$

$\Delta s_{12} = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2 - b}{v_1 - b}\right)$

$= -1{,}673 \cdot 10^{3}\ \frac{J}{kgK}$

c) $w_{v,31} = -3{,}303 \cdot 10^{5}\ \frac{J}{kg}$

$w_{v,31} = \frac{a}{v_3} - \frac{a}{v_1} + c_v (T_1 - T_3)$ mit $v_3 = v_2 = 1{,}228 \cdot 10^{-2}\ \frac{m^3}{kg}$

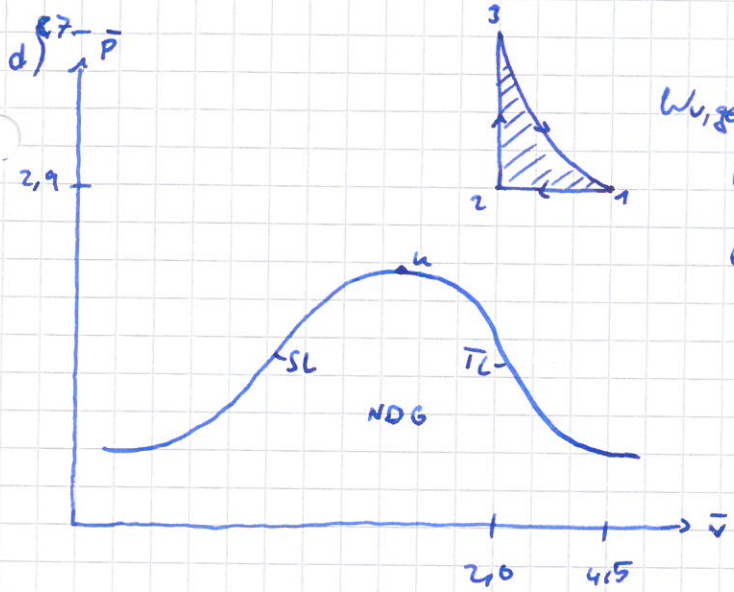
$T_3 = 857{,}7\ K$

$p_3 = 3{,}999 \cdot 10^{7}\ Pa$ (über therm. ZGL) $\bar{p}_3 = \frac{p_3}{p_k} = 8{,}695$

$w_{v,23} = 0$ $q_{23} = c_v (T_3 - T_2) = 8{,}034 \cdot 10^{5}\ \frac{J}{kg}$

$\Delta s_{23} = c_v \ln\left(\frac{T_3}{T_2}\right) = 1{,}673 \cdot 10^{3}\ \frac{J}{kgK}$

d)



$W_{v,ges}$ ist schraffierte Fläche

$W_{v,ges} = W_{v,12} + W_{v,23} + W_{v,31} = -1,\cancel{222}\,405 \cdot 10^5 \frac{J}{kg}$

Es wird Arbeit entnommen

e) $p_4 = 1$ bar $\qquad T_{zünd} = 868,2$ K

adiabate Drossel

$$\beta = \frac{(v-b)Rv^2}{RTv^3 - 2a(v-b)^2}$$

$$\beta = 1,12 \cdot 10^{-3} \frac{1}{K}$$

$$c_{p1} = \frac{R}{1 - \frac{(3\bar{v}-1)^2}{4\bar{T}\bar{v}^3}} + c_v = 2,284 \cdot 10^3 \frac{J}{kgK}$$

$$\delta_H = -\frac{v}{c_p}\left(\frac{RTv^3 - 2a(v-b)^2 - T(v-b)Rv^2}{RTv^3 - 2a(v-b)^2}\right)$$

$$= -2,129 \cdot 10^{-7} \frac{kms^2}{kg} \quad < 0 \quad \rightarrow \text{Temperaturerhöhung}$$

$$\delta_H \approx \frac{T_4 - T_3}{p_4 - p_3} \quad \rightarrow T_4 = 866,2 \text{ K}$$

Das Methan entzündet sich nicht

(3) Triebwerk Reiseflugzeug im Reiseflug

$p_\infty = 22630$ Pa

isentrope zu, ideales Gas

$R = 287 \frac{J}{kgK} \quad ; k = 1,4$

$p_{0,2} = 5,123 \cdot 10^5$ Pa

$T_{0,2} = 488,0$ K

$p_2 = 4,202 \cdot 10^5$ Pa

a) $\frac{p_{02}}{p_2} = \left(1 + \frac{\kappa-1}{2} Ma_2^2\right)^{\frac{\kappa}{\kappa-1}}$

$Ma_2 = 0{,}5297$

$Ma_2 = \sqrt{\frac{(\kappa-1)(Ma_1^2-1)+\kappa+1}{2\kappa(Ma_1^2-1)+\kappa+1}}$

$Ma_1 = 2{,}257$

$\frac{p_{01}}{p_1} = \left(1 + \frac{\kappa-1}{2} Ma_1^2\right)^{\frac{\kappa}{\kappa-1}}$

$p_{0,1} = 8{,}504 \cdot 10^5\ Pa$

$\frac{p_2}{p_1} = \frac{(\kappa+1)Ma_1^2}{2+(\kappa-1)Ma_1^2}$

$p_1 = 7{,}274 \cdot 10^4\ Pa$

$p_{0,1} - p_{0,2} = 3{,}381 \cdot 10^5\ Pa$

b) $\dot{S}_{12} = 4765\ \frac{J}{sK}$

$\rho_1 c_1 A_1 = \rho_2 c_2 A_2 = \dot{m}$ → mit $\dot{m}$ von später: $A_2 = 4{,}472 \cdot 10^{-2}\ m^2$

$\frac{T_{0,2}}{T_2} = 1 + \frac{\kappa-1}{2} Ma_2^2$

$T_2 = 467{,}1\ K$

$c_{2,s} = \sqrt{\kappa R T_2} = 430{,}4\ \frac{m}{s}$

$c_2 = Ma_{,2} \cdot c_{s,2} = 182{,}4\ \frac{m}{s}$

$c_{s,2} = \sqrt{\kappa \frac{p_2}{\rho_2}}$

$\rho_2 = 3{,}776\ \frac{kg}{m^3}$

$\dot{m} = \frac{\dot{S}_{12}}{\Delta s_{12}}$

~~$\Delta s_{12} = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$~~ mit $c_p = 1005\ \frac{J}{kgK}$

$\Delta s_{12} = -R \ln\left(\frac{p_{0,2}}{p_{0,1}}\right)$

$= 145{,}5\ \frac{J}{kgK}$

$\dot{m} = \frac{\dot{S}_{12}}{\Delta s_{12}} = 32{,}75\ \frac{kg}{s}$

c) $\dot{m} = 33{,}00\ \frac{kg}{s}$

$c_{s1} = \sqrt{\kappa R T_1} = \sqrt{\kappa \frac{p_1}{\rho_1}}$; $T_{0,1} = T_{0,2}$ → $\frac{T_{0,1}}{T_1} = 1 + \frac{\kappa-1}{2} Ma_1^2$ $T_1 = 247{,}7\ K$

$c_{s1} = $ ~~$401{,}4$~~

$c_1 = c_{s,1} \cdot Ma_1 = 1034\ \frac{m}{s}$

$\dot{m}_1 = A_1 \cdot \rho_1 \cdot c_1 \Rightarrow A_1 = 4{,}472 \cdot 10^{-2}\ m^2$

d) $p_{0,3} = 1,432 \cdot 10^6 \, Pa$

$\dot{m} = 33,00 \, \frac{kg}{s}$

$A^* = 1,6 \cdot 10^{-2} \, m^2$

$p^* = \rho^* R T^*$

$\frac{p^*}{p_{0,3}} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$

$p^* = 1,565 \cdot 10^5 \, Pa$

$T^* = \frac{\rho^* R}{p^*}$

$\dot{m} = A^* \cdot \rho^* \cdot c^*$; $c_s^* = \sqrt{\kappa \frac{p^*}{\rho^*}}$, $Ma^* = 1$

$\rho^* = 1,129 \cdot 10^6 \, \frac{kg}{m^3}$ ↯

$T^* = 656,3 \, K$

$\frac{T^*}{T_{0,3}} = \frac{2}{\kappa+1}$

$T_{0,3} = 787,6 \, K$

e) Düse in angepassten Zustand -> $p_4 = p_\infty$ $p_{0,4} = p_{0,3}$

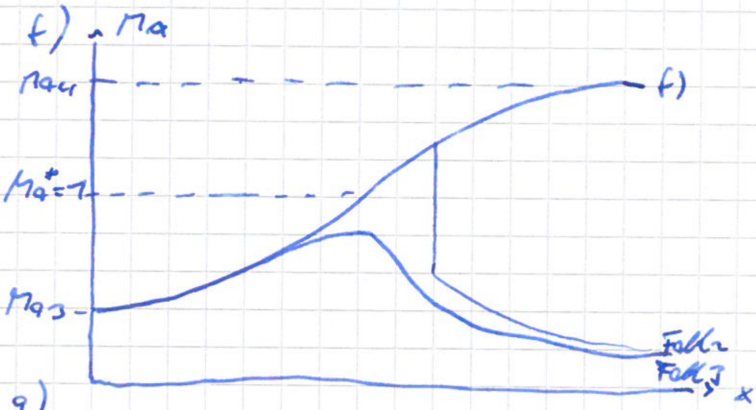
$\frac{p_{0,4}}{p_4} = \left(1 + \frac{\kappa+1}{2} Ma_4^2\right)^{\frac{\kappa}{\kappa-1}}$

$Ma_4 = 3,369$

$\frac{A_4}{A^*} = \frac{1}{Ma_4} \left(\frac{2}{\kappa+1}\left(1 + \frac{\kappa-1}{2} Ma_4^2\right)\right)^{\frac{\kappa+1}{2(\kappa-1)}}$

$A_4 = 9,610 \cdot 10^{-2} \, m^2$

f)



g)

Siehe oberes Diagramm

(4)

1 $p_1 = 1$ bar $T_1 = 280$ K

irreversibel adiabate Kompression $\eta_{is,V} = 85\%$ $\pi_V = 1{,}5$

2

Isobare Wärmezufuhr $\Delta T_{23} = -190$ K

3

Irreversibel adiabate Entspannung

4 $T_4 = 130$ K

Isobare Wärmezufuhr $\dot{Q}_L = 22$ W

5

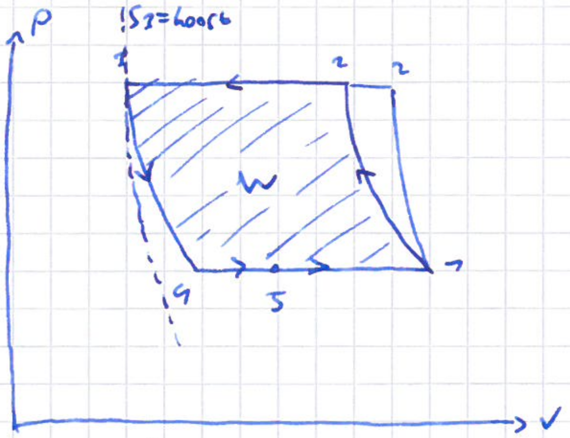
Isobare Wärmezufuhr

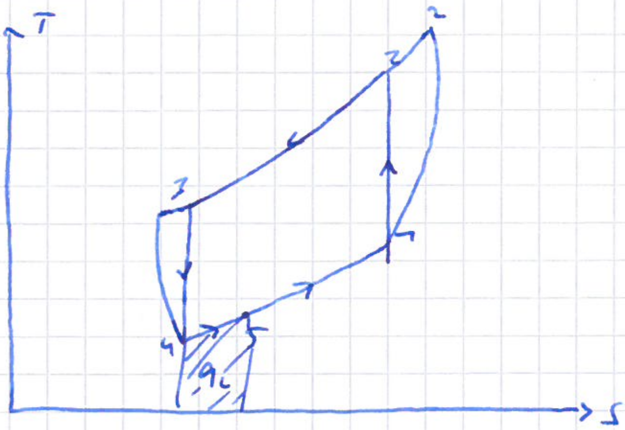
Isobare Wärmezufuhr im Wärmetauscher $\Delta T_{51} = -0{,}75 \cdot \Delta T_{23}$

Verluste als Q_V an Umgebung

$M_{Hm} = 7{,}2 \cdot 10^{-3} \frac{kg}{mol}$ $c_{p,Hm} = 2860$ J/kgK $R_m = 8{,}314 \frac{J}{mol K}$

a)





b) $\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1} \stackrel{!}{=} 0{,}85$

$\pi_V = \frac{p_2}{p_1}$

$p_2 = 1{,}5$ bar

$R = \frac{R_m}{M_m} = 1155$

~~$T_{2,rev} = T_1$~~

$c_{p,Hm} = \frac{k}{k-1} R$

$\frac{p_2}{p_1} = \left(\frac{T_{2,rev}}{T_1} \right)^{\frac{k}{k-1}}$

$k = 1{,}677$

$T_{2,rev} = 329{,}8$ K

$T_2 = 338{,}6$ K

c) $W_{t,T} = -Q_{t,T}$ (aus 1. Hauptsatz)

$= c_{p,Hm} \cdot (T_4 - T_3) = -5{,}32 \cdot 10^4$ J/kg

$T_3 = T_2 - 190$ K $= 148{,}6$ K

1

a) $\alpha T - \alpha T \ln\left(\frac{T}{T_0}\right) + \lambda T \ln\left(\frac{p}{p_0}\right) = g \quad \left[\frac{J}{kg}\right]$

$\alpha \cdot K \overset{!}{=} \left[\frac{J}{kg}\right] \rightarrow \left[\frac{J}{kgK}\right] = \alpha$

$\lambda \overset{!}{=} \left[\frac{J}{kgK}\right] = \left[\frac{kg\, m^2}{kgK s^2}\right] = \left[\frac{m^2}{K s^2}\right] = [\alpha, \lambda]$

$g = g(T, p) \rightarrow$ kanonische ZGL

b) $s = s(T, p)$

$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \underbrace{\left(\frac{\partial s}{\partial p}\right)_T}_{= v \cdot \beta} dp$

$dg = -s\, dT + v\, dp$

$dg = \underbrace{\left(\frac{\partial g}{\partial T}\right)_p}_{-s} dT + \underbrace{\left(\frac{\partial g}{\partial p}\right)_T}_{v} dp$

$-s = \cancel{\ln\left(\frac{p}{p_0}\right)\lambda + \alpha\left(-1 \ln\left(\frac{p}{p_0}\right)\right)}$

$s(p,T) = -\ln\left(\frac{p}{p_0}\right) \cdot \lambda + \alpha \cdot \ln\left(\frac{T}{T_0}\right)$

$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \quad \left[\frac{1}{K}\right]$

$v \cdot \beta = \left(\frac{\partial s}{\partial p}\right)_T = \frac{\lambda}{p}$

$\beta = \frac{\lambda}{vp}$

c) $p = p(v, T)$

$p = \frac{\lambda}{v\beta}$

$pv = RT \rightarrow p = \frac{RT}{v}$

$v = \frac{T \beta \lambda \cancel{\ldots}}{p}$

$p = \frac{T\lambda}{v}$

$\lambda \overset{!}{=} R$

d) $dh = T\, ds + v\, dp$

②

1

↓ isotherm verdichtet

2

↓ isochor Wärme entzogen

3

↓ isobar

1

a) $a = 3 p_k v_k^2$

$a = 14,41 \left[\frac{m^5}{kg\, s^2}\right]$

$b = \frac{v_k}{3} = 3,022 \cdot 10^{-4} \left[\frac{m^3}{kg}\right]$

$\frac{3}{8} = \frac{p_k v_k}{R T_k}$

$T_k = 289,8$ K

b) Therm. ZGL

$\bar{p}_1 = \frac{p_1}{p_k} \rightarrow p_1 = 13,49$ bar

$T_1 = \cancel{286,5 K} \; 354,7$ K

$q_{12} = R T_1 \ln\left(\frac{v_2 - b}{v_1 - b}\right)$

$v_1 = 1,636 \cdot 10^{-4}$ m^3

$T_2 = T_1$

p_2 mit therm. ZGL

$p_2 = 127,4$ bar

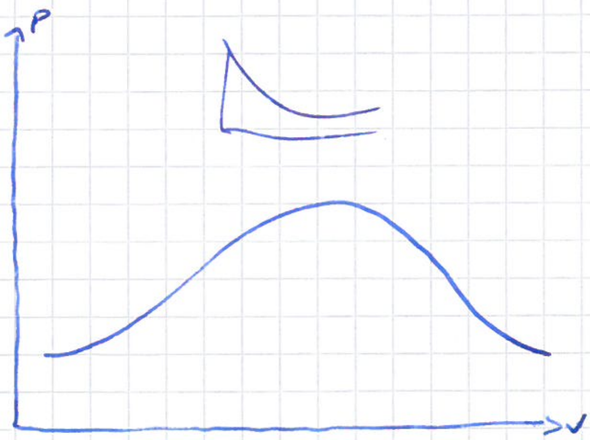
c) $v_3 = v_2$; $\bar{v}_2 = \bar{v}_3 = 0,8445$

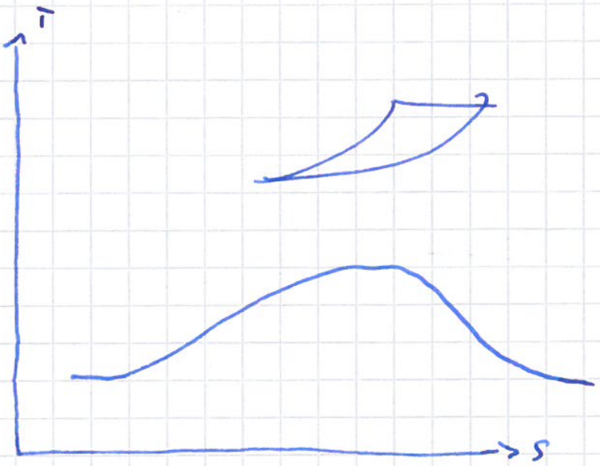
$p_3 = p_1$; $\bar{p}_3 = \bar{p}_1 = 1,6$

$p_3 = \frac{T_3}{T_2}\left(p_2 + \frac{a}{v_2^2}\right) - \frac{a}{v_2^2}$

$T_3 = 327,1$; $\bar{T}_3 = 1,112$

d)





(5)

$t_U = 3°C \quad \varphi_U = 70\%$

$t_H = 50°C$

$\dot{V}_U = 5 \frac{l}{s}$

$t_{DB} = 29°C$

a) $x = 3,2 \; g/kg_{tr. Luft}$

b) $\rho = \frac{(1+x)\,p}{(R_L + x R_D)T} = 1,259 \frac{kg}{m^3}$

$\dot{m}_U = \rho \cdot \dot{V} = 6,275 \frac{kg}{s} \cdot 10^{-3}$

$\dot{m}_{U,tr} = 6,275 \cdot 10^{-3} \frac{kg}{s}$

c) $\frac{\dot{m}_{LU}}{\dot{m}_{LH}} = \frac{l_U}{l_H} \qquad l_U = 4,3\,cm; \; l_H = 5,2\,cm$

$\dot{m}_H = 7,590 \cdot 10^{-3} \frac{kg}{s}$

$\dot{m}_{DB} = \dot{m}_H + \dot{m}_U = 1,387 \cdot 10^{-2} \frac{kg}{s}$

$h_H - h_U = \frac{\dot{Q}_H}{\dot{m}_H}$

$\dot{Q}_H = 3,567 \cdot 10^2 \frac{J}{s}$

d) Wasserbilanz : $\dot{m}_{DB}\, x_{DB} + \dot{m}_{H_2O} = \dot{m}_{DB'}\, x_{DB'}$

Energiebilanz : $\dot{m}_{DB}\, h_{DB} + \dot{m}_{H_2O}\, h_{H_2O} = \dot{m}_{DB'}\, h_{DB'}$

$h_{H_2O} = \frac{h_{DB'} - h_{DB}}{x_{DB'} - x_{DB}}$

$h_{DB} = c_{pL}\, t + x_D\,(c_{pD} t + r_D)$

$x_{DB} = x_U = x_H$

$h_{DB} = 37,35 \frac{kJ}{kg}$

④

1 ↓ isotherme Expansion
2 ↓ isochore Wärmeabfuhr
3 ↓ isotherme Kompression
4 ↓ isochore Wärmezufuhr
1

$R = 2077 \frac{J}{kgK}$

$c_p = 5200 \frac{J}{kgK}$

$\rho_1 = 4{,}874 \frac{kg}{m^3}$

$p_1 = 80$ bar

$m_1 = 0{,}7221$ g

$\Delta V = 160\,cm^3 = |V_2 - V_1|$

$T_4 = 0{,}5\,T_1$

$n = 1300\,min^{-1}$

a) $pv = RTm \quad ; \; v = \frac{m}{\rho}$

$T_1 = 800\,K$

$V_1 = 1{,}5 \cdot 10^{-4}\,m^3 = 150\,cm^3$

$T_2 = T_1 = 800\,K$

$V_2 = V_1 + 160\,cm^3 = 310\,cm^3$

$p_2 = \frac{V_1}{V_2} \cdot p_1$

$p_2 = 38{,}71$ bar

$T_3 = T_4 = 400\,K$

~~mit $p_3 V_3 = mRT_3$~~

$p_3 = \frac{p_2}{T_2} \cdot T_3$

$p_3 = 19{,}36$ bar

$V_3 = V_2 = 310\,cm^3$

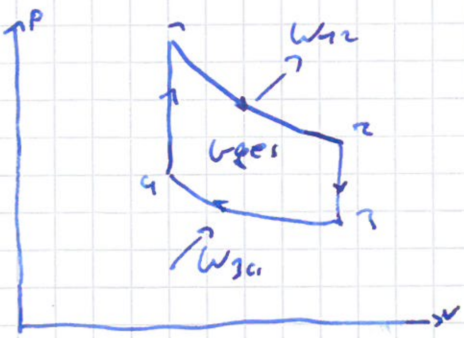
$T_4 = T_3 = 400\,K$

$V_4 = V_1 = 150\,cm^3$

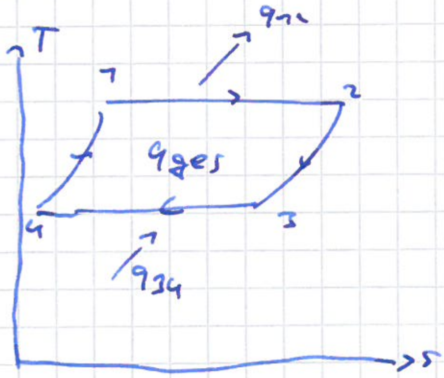
$p_4 = \frac{V_3}{V_4} \cdot p_3$

$p_4 = 40$ bar

b)



Stirling Prozess



c)

d) Siehe Diagramme

e) $Q_{1-2} = p_1 V_1 \cdot \ln\left(\frac{p_1}{p_2}\right) = 877{,}7\ J$

$Q_{2-3} = c_v \cdot m \cdot (T_3 - T_2) = -902\ J$

$Q_{3-4} = p_3 \cdot V_3 \cdot \ln\left(\frac{p_3}{p_4}\right) = -435{,}6\ J$

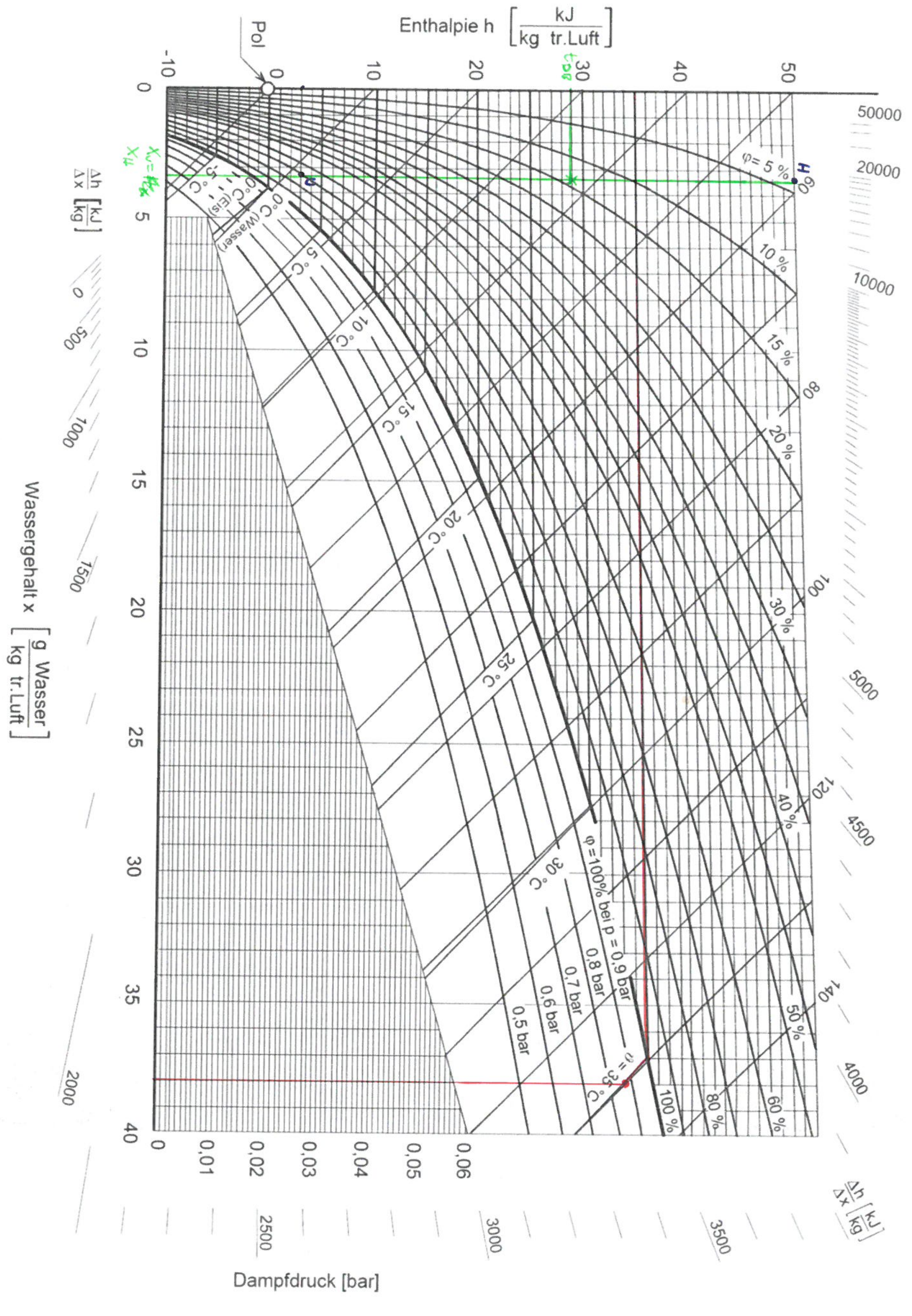
$Q_{4-1} = c_v \cdot m \cdot (T_1 - T_4) = 902\ J$

$W_{ges} = -(Q_{12} + Q_{23} + Q_{34} + Q_{41}) = -435{,}5\ J$

$c_v = c_p - R = 3723\ \frac{J}{kg\,K}$

$P_{el} = -\frac{W_{ges} \cdot n}{60} = 9{,}436\ kW$

f) $\eta_{th} = 1 - \left(\frac{T_4}{T_1}\right) = 50\%$



Enthalpie h [kJ / kg tr.Luft]
Pol
Wassergehalt x [g Wasser / kg tr.Luft]
Dampfdruck [bar]
Δh/Δx [kJ/kg]
φ = 5 %
10 %
15 %
20 %
30 %
40 %
50 %
60 %
80 %
100 %
φ = 100% bei p = 0,9 bar
0,8 bar
0,7 bar
0,6 bar
0,5 bar
0°C (Eis)
0°C (Wasser)
-5 °C
5 °C
10 °C
15 °C
20 °C
25 °C
30 °C
ϑ = 35 °C

① $h = T\left(\frac{AT}{p} + Bp + s\right)$

a) kanonische Form wäre $h = h(s,p)$, T müsste eleminiert werden

↳ keine kanonische Form

$\cancel{[Bp] \overset{!}{=} [s]}$

$[Bp] \overset{!}{=} [s]$

$$B \cdot \frac{kg}{s^2 m} = \frac{J}{kg\,K} = \frac{m^2}{s^2 K}$$

$$[B] = \frac{m^3}{kg\,K}$$

$$\left[\frac{AT}{p}\right] \overset{!}{=} [Bp] \overset{!}{=} [s]$$

$$A \cdot \frac{K}{\frac{N}{m^2}} \overset{!}{=} \frac{K}{\frac{\frac{kg\,m}{s^2}}{m^2}} = \frac{K \cdot m \cdot s^2}{kg}$$

$$[A] = \frac{kg \cdot m}{K^2 \cdot s^4}$$

b) Allgemeine kanonische Form: $g = g(p,T)$

$g = h - Ts$

$dG = -S\,dT + V\,dp \quad ; \quad dg = -s\,dT + v\,dp$

Totales Differential:

$$dg = \underbrace{\left(\frac{\partial g}{\partial p}\right)_T}_{v} dp + \underbrace{\left(\frac{\partial g}{\partial T}\right)_p}_{-s} dT$$

$$g = T\left(\frac{AT}{p} + Bp + s\right) - Ts$$

$$\left(\frac{\partial g}{\partial p}\right)_T = \frac{(p^2 \cdot B - AT)T}{p^2} = v$$

$$v(T,p) = \frac{(p^2 B - AT)T}{p^2}$$

Seite 2

c) $h = g + Ts$

$dh = T\,ds + v\,dp$

$h = h(s,p)$

$\cancel{dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \underbrace{\left(\frac{\partial h}{\partial p}\right)_s}\, dp}$

$$dg = \underbrace{\left(\frac{\partial g}{\partial p}\right)_T}_{v} dp + \underbrace{\left(\frac{\partial g}{\partial T}\right)_p}_{-s} dT$$

$s = s(h,p)$

$$\left(\frac{\partial g}{\partial T}\right)_p = \frac{1}{p}\left(2AT + Bp^2\right) = -s$$

$$T = -\frac{p}{2A}\left(s + Bp\right)$$

Einsetzen in erste gegebene Gleichung

$$h(s,p) = \left(-\frac{p}{2A}(s+Bp)\right)\left(\frac{AT}{p} + Bp + s\right)$$

d)

$$(c_p - c_v) = \left[\left(\frac{\partial u}{\partial v}\right)_T + p\right]\left(\frac{\partial v}{\partial T}\right)_p$$

$du = T\,ds - p\,dv$

$$\frac{du}{dv} = T\frac{ds}{dv} - p$$

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T - p \qquad \text{Maxwell}$$

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p$$

$$(c_p - c_v) = = \left[\cancel{(\ldots)}\left(T\left(\frac{\partial p}{\partial T}\right)_v\right)\right]\left(\frac{\partial v}{\partial T}\right)_p$$

e) $B = 0$

$$\delta_h = \left(\frac{\partial T}{\partial p}\right)_h = ???$$

Formel anders für reale Stoffe?

f) Baut auf e) auf

(2)

a) Helium, Van der Waals, Gas, c_v = konst.

1

↓ isochor

2

↓ isotherm

3

↓ isobar

→ 4

a) ges: v_k, T_k, p_k, a, p_1, $\tilde{v}_1$, $\tilde{p}_1$

$$b = \frac{v_k}{3} \quad \rightarrow \quad v_k = 1{,}869 \cdot 10^{-3} \ \frac{m^3}{kg}$$

$$\tilde{T} = \frac{T}{T_k} \quad \rightarrow \quad T_k = 190{,}6 \ K$$

$$\frac{3}{8} = \frac{p_k v_k}{R T_k} \quad \rightarrow \quad p_k = 4{,}708 \cdot 10^{6} \ Pa$$

$$a = 3 p_k v_k^2 \rightarrow a = 874{,}6 \ \frac{m^5}{kg\, s^2}$$

$$T_1 = 565{,}7 \ K$$

$$v_1 = 1{,}574 \cdot 10^{-2} \ \frac{m^3}{kg}$$

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \quad \rightarrow$$

$$p_1 = 1{,}882 \cdot 10^{7} \ Pa$$

$$\tilde{v}_1 = \frac{v_1}{v_k} = 2{,}000$$

$$\tilde{p}_1 = \frac{p_1}{p_k} = 3{,}997$$

b) ges: T_2, p_2, $\tilde{T}_2$, $\tilde{p}_2$

$$\Delta S_{12} = c_v \ln\left(\frac{T_2}{T_1}\right)$$

$$T_2 = 190{,}6 \ K$$

~~p_2 über Zustandsgleichung~~

$$p_2 = -\frac{a}{v_1^2} + \frac{T_2}{T_1}\left(p_1 + \frac{a}{v_1^2}\right)$$

$$p_2 = 4 \cdot 10^{6} \ Pa$$

$$v_2 = v_1 = 1{,}574 \cdot 10^{-2} \ \frac{m^3}{kg} \quad \rightarrow \tilde{v}_2 = 2{,}000$$

$$\tilde{T}_2 = \frac{T_2}{T_k} \quad \rightarrow \quad \tilde{T}_2 = 1$$

$$\tilde{p}_2 = \frac{p_2}{p_k} \quad \rightarrow \quad \tilde{p}_2 = 0{,}8496$$

c) ges: v_3, $\tilde{v}_3$

$$T_3 = T_2 = 190{,}6 \ K$$

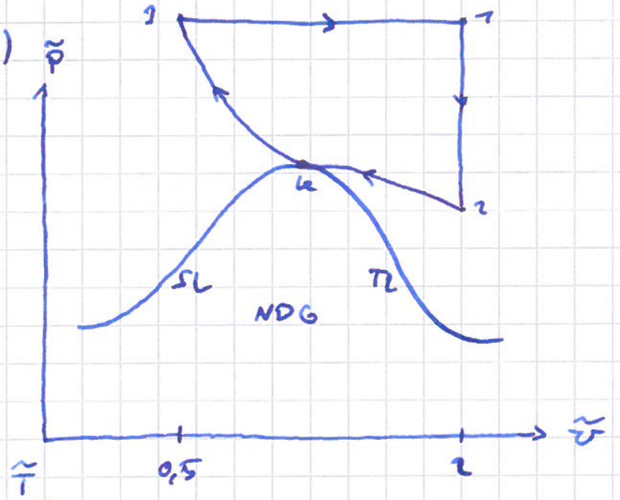
$$\text{I} \quad \frac{p_2 + \frac{a}{v_2^2}}{p_3 + \frac{a}{v_3^2}} = \frac{v_3 - b}{v_2 - b}$$

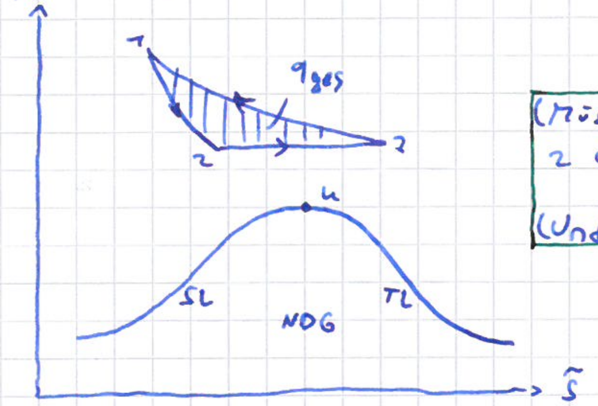
$$\text{II} \quad p_3 = \frac{v_2 - b}{v_3 - b}\left(p_2 + \frac{a}{v_2^2}\right) - \frac{a}{v_2^2}$$

$$p_3 = p_1 = 1{,}882 \cdot 10^{7} \ Pa$$

$$v_3 = 3{,}934 \cdot 10^{-3} \ \frac{m^3}{kg} \quad \rightarrow \quad \tilde{v}_3 = 0{,}4999$$

d)





(Müsste eigentlich zwischen 2 und 3 durch k gehen)

(Und müsste eigentlich gespiegelt sein)

e) ges: q_{12}, q_{23}, q_{31}, q_{ges}

$$q_{12} = c_v (T_2 - T_1) = -665{,}8\ kJ/kg$$

$$q_{23} = R T_2 \ln\left(\frac{v_1 - b}{v_2 - b}\right) = -227{,}5\ kJ/kg$$

$$q_{31} = \frac{a}{v_3} - \frac{a}{v_1} + c_v (T_1 - T_3) + p_3 (v_1 - v_3)$$

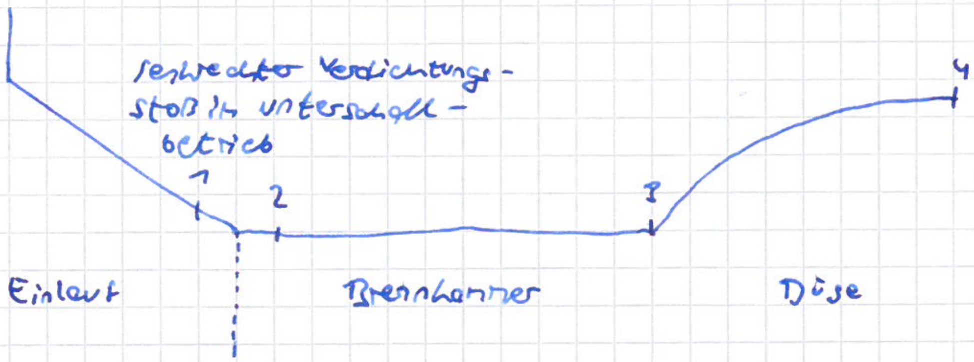
$$= 1047\ kJ/kg$$

$$q_{ges} = q_{12} + q_{23} + q_{31} = 154{,}1\ kJ/kg > 0$$

Es wurde dem Kreisprozess Wärme hinzugefügt

(3)

$p_\infty = 1100\ Pa$	$Ma_3 = 1$
$Ma_2 = 0{,}4$	$T_2 = 1217\ K$
$A_2 = 0{,}03\ m^2$	$\rho_3 = 3{,}349\ \frac{kg}{m^3}$
$T_{t3} = 2400\ K$	$A_4 = 1{,}4\ m^2$
Ideales Gas	$k = 1{,}33$
$c_p = 1150\ \frac{J}{kg\,K}$	



a) $Ma_2 = \sqrt{\frac{(k-1)(Ma_1^2-1)+k+1}{2k(Ma_1^2-1)+k+1}}$

$Ma_1 = 4{,}634$

$\frac{p_2}{p_1} = \frac{2kMa_1^2-k+1}{k+1}$

$= 24{,}37$

$\frac{T_2}{T_1} = \frac{(2kMa_1^2-k+1)(2+(k-1)Ma_1^2)}{(k+1)^2 Ma_1^2}$

$= 4{,}426$

$s_2 - s_1 = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$; $c_p = \frac{k}{k-1} R \rightarrow R = 285{,}3$

$= 799{,}6 \ \frac{J}{kgK}$

b) $T_{t,3} = T_{0,3}$

$\frac{T_{03}}{T_3} = 1 + \frac{k-1}{2} Ma_3^2$

$T_3 = 2060 \ K$

$p_3 = \rho_3 R T_3$

$= 19{,}68 \ bar$

$\dot{m} = \rho_3 c_3 A_3 = \rho_3 \sqrt{kRT_3} A_3$ (geht weil $Ma_3 = 1$)

$= 88{,}83 \ \frac{kg}{s}$

c) ges: $p_{t,3} = p_{03}$, Ma_4, p_4, T_4, c_4

$\frac{p_{03}}{p_3} = \left(\frac{T_{03}}{T}\right)^{\frac{k}{k-1}}$

$p_{t3} = 36{,}42 \ bar$

$A_3 = A^*$

$\frac{A_4}{A^*} = \frac{1}{Ma_4}\left(\frac{2}{k+1}\left(1+\frac{k-1}{2}Ma_4^2\right)\right)^{\frac{k+1}{2(k-1)}}$

$Ma_4 = 0{,}01250$ In Lösung mit gleichem Rechenweg 5,247...

$Ma_4 = 5{,}247$

$\frac{p_{03}}{p_4} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{k}{k-1}}$; $p_{t3} = p_{t4} \longrightarrow \Delta s = 0$

$p_4 = 3073 \ Pa$ 3-4 ist isentrop

$$\frac{p_{01}}{p_4} = \left(\frac{T_{04}}{T_{04}}\right)^{\frac{\kappa}{\kappa-1}} \quad ; \quad p_{04} = p_{03} \quad ; \quad T_{04} = T_{03}$$

$$\frac{p_4}{p_3} = \left(\frac{T_4}{T_3}\right)^{\frac{\kappa}{\kappa-1}}$$

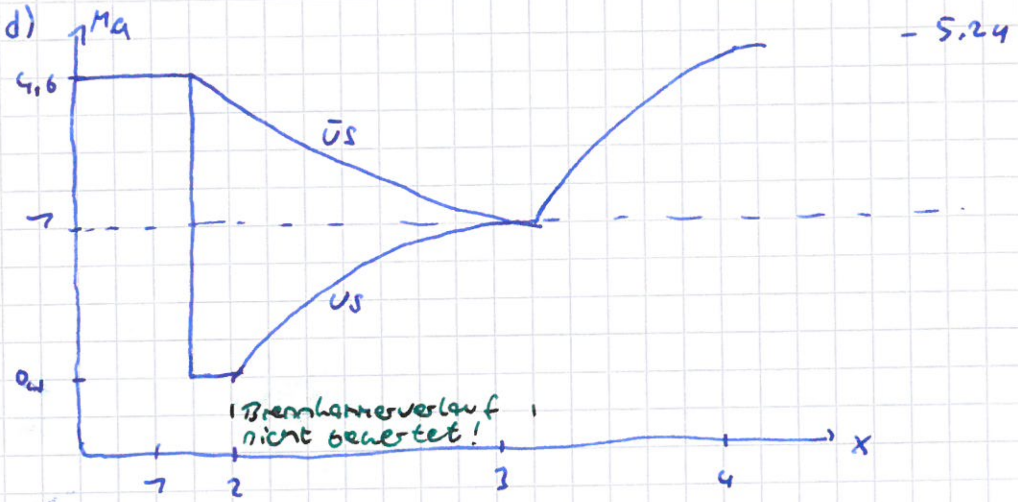
$T_4 = 433{,}3\ K$

$c_{s,4} = \sqrt{\kappa R T_4} = 405{,}5\ \frac{m}{s}$

$v_4 = Ma_4 \cdot c_{s,4} = 1725\ \frac{m}{s}$

keine "Angepasste Düse" weil $p_4 \neq p_\infty$

d)



e) $\dot{Q}_{23} = \dot{m}(h_{t3} - h_{t2})$

$= \dot{m}\left(\left(h_3 + \frac{c_3^2}{2}\right) - \left(h_2 + \frac{c_2^2}{2}\right)\right) = \left(h_3 - h_2 + \frac{c_3^2}{2} - \frac{c_2^2}{2}\right)\dot{m}$

$c_3 = Ma_3 \sqrt{\kappa R T_3} = 884{,}1\ \frac{m}{s}$

$c_2 = Ma_2 \sqrt{\kappa R T_2} = 217{,}8\ \frac{m}{s}$

$h_3 - h_2 = c_p (T_3 - T_2)$

$= 969450\ J/kg$

$\dot{Q}_{23} = 1{,}176 \cdot 10^8\ W$

$\rho_2 = \frac{\dot{m}}{c_2 A_2} = 10{,}89\ \frac{kg}{m^3}$

$c_2 : Ma_2 = \sqrt{\kappa \frac{p_2}{\rho_2}}$

$p_2 = 3{,}787 \cdot 10^6\ Pa$

f) $\frac{p_3}{p_2} = 0{,}5205 \quad ; \quad \frac{p_{t3}}{p_{t2}} = 0{,}8672$

$p_{t2} = 4{,}2 \cdot 10^6\ Pa$;

g) Wärmezufuhr wirkt auf Strömung wie Flächenkontraktion

$Ma > 1 \ : \ dq > 0 \rightarrow dMa < 0$

$M < 1 \ : \ dq > 0 \rightarrow dMa > 0$

(4) 1 $p_1 = 12$ bar

Adiabate Drosselung vollständig gesättigte Flüssigkeit

2 $p_2 = 3$ bar

Isobare Verdampfung in überkrit. Bereich

3

Irreversibel adiabate Kompression ; $\eta_{is,v} = 88{,}9\%$ $T_{4,rev} = 320\,K$

4 $p_4 = p_1 = 12$ bar ; $T_4 = 324\,K$

Isobare Verflüssigung, vollständig

1

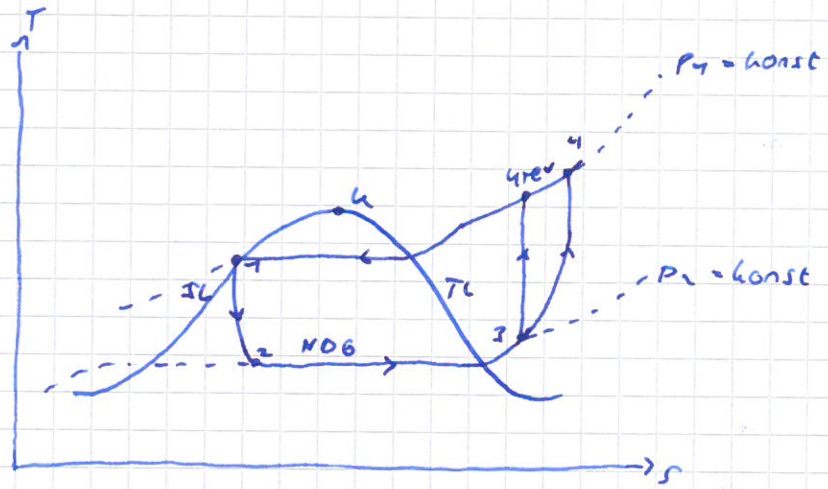
Realer Stoff

$T_{PR} = 295\,K$, $T_{10} = 305\,K$

$\dot{Q}_S = 120\,kW$, $\dot{Q}_P = 2\,kW$, $P_{el} = 5\,kW$

$\dot{m}_L = 0{,}15\,\frac{kg}{s}$, $\kappa = 1{,}4$, $R = 287\,\frac{J}{kg}$

a)



b) $T_1 = T(p = 12\,bar) = 307{,}5\,K$

$h_1 = h'(p = 12\,bar) = 297{,}7\,\frac{kJ}{kg}$ $h_2 = h_1$

$T_2 = T(p = 3\,bar) = 259{,}0\,K$

$x_2 : h = h_2' + x_2 (h_2'' - h_2')$

$x_2 = 0{,}3176$

c) $0 \overset{!}{=} Q_P + Q_S + Q_R + P_{el} + \dot{m}_L c_{p,L} (T_{10} - T_{PR})$

$c_{p,L} = R\left(\frac{\kappa}{\kappa - 1}\right) = 1005\,\frac{J}{kg\,K}$

$Q_R = -1{,}285 \cdot 10^5\,W$

d) ~~[illegible]~~ Interpolation

(5) $\dot{V}_U = 2\ \frac{l}{s}$ $t_U = 12°C$ $x_U = 11\ \frac{g}{kg\ tr.Luft}$

$t_u = 20°C$ $\varphi_u = 60\%$

M grade gesättigt $\dot{Q}_{zu} = 132{,}5\ W$ auf M'

a) Siehe h-x-Diagramm

b) $\dot{m}_v = \dot{V}_U \rho$

$\rho = \frac{(1+x)\,p}{(R_L + x_S R_D)T} = \frac{1{,}218}{\cancel{}}\ \frac{kg}{m^3}$ mit x in $\left[\frac{kg}{kg\ tr.Luft}\right]$

$\dot{m}_U = 2{,}436 \cdot 10^{-3}\ \frac{kg}{s}$

$\dot{m}_{U,tr} = \dot{m}_{U,L} - x_U \cdot \dot{m}_{U,L} = 2{,}409 \cdot 10^{-3}\ \frac{kg}{s}$

c) $l_U = 1{,}\cancel{2}3$ $l_u = \cancel{}\ 1{,}\cancel{2}6$

$l_1 = \frac{x_2 - x_{mix}}{x_2 - x_1} = \frac{\dot{m}_{L1}}{\dot{m}_{L1} + \dot{m}_{L2}}$

Ablesen ergibt: $x_M = 9{,}8\ \frac{g}{kg\ tr.Luft}$

$\frac{\dot{m}_{LU}}{\dot{m}_{LU} + \dot{m}_{Lu}} = \frac{\bar{l}_U}{l_U + l_u}$

$\dot{m}_{L,u} = 2{,}966 \cdot 10^{-3}\ \frac{kg}{s}$

$\dot{m}_{L,M} = \dot{m}_{Lu} + \dot{m}_{LU} = 5{,}376 \cdot 10^{-3}\ \frac{kg}{s}$

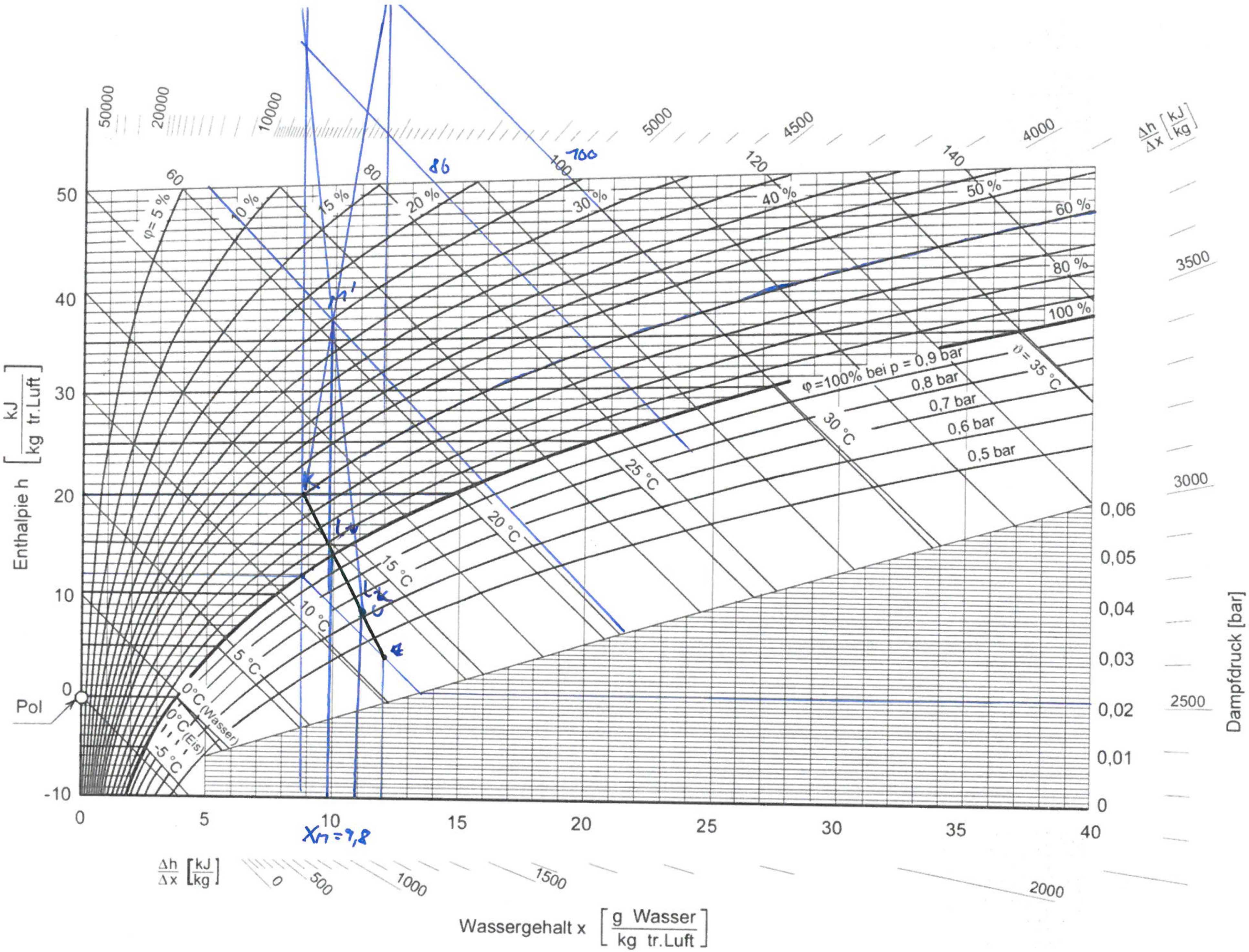
$h_{mix'} - h_{mix} = \frac{\dot{Q}}{\dot{m}_{Lmix}}$

$\dot{Q}_{zu} = 132{,}5\ W \rightarrow \dot{q}_{zu} = \frac{\dot{Q}_{zu}}{\dot{m}_{L,M}} = 24{,}65\ kJ/kg\ tr.Luft$

$h_M = 38\ \frac{kJ}{kg\ tr.Luft}$

$h_{M'} = 62{,}65\ \frac{kJ}{kg}$

d) Siehe Diagramm


Enthalpie h $\left[\frac{kJ}{kg\ tr.Luft}\right]$
Wassergehalt x $\left[\frac{g\ Wasser}{kg\ tr.Luft}\right]$
Dampfdruck [bar]
$\frac{\Delta h}{\Delta x}\left[\frac{kJ}{kg}\right]$
Pol
φ = 5 %
10 %
15 %
20 %
30 %
40 %
50 %
60 %
80 %
100 %
φ =100% bei p = 0,9 bar
0,8 bar
0,7 bar
0,6 bar
0,5 bar
ϑ = 35 °C
30 °C
25 °C
20 °C
15 °C
10 °C
5 °C
0°C (Wasser)
0°C (Eis)
-5 °C
50000
20000
10000
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0,06
0,05
0,04
0,03
0,02
0,01
86
100
$X_M = 9,8$

Thermo ~~F23~~ PRU F20

1

a) $U = S(V)$ (kanonisch)

b) $dU = \left(\frac{\partial U}{\partial V}\right)_S dV + \left(\frac{\partial U}{\partial S}\right)_V dS$

$V = \frac{1}{4}\left(\frac{\alpha S^2}{U}\right)^2$

$U = \frac{\alpha \cdot S^2}{2\sqrt{V}}$ und $U = \frac{\alpha^2 \cdot S^4}{V}$

$\left(\frac{\partial U}{\partial V}\right)_S = \frac{\alpha \cdot S}{\cancel{2}\sqrt{V}}$

$\left(\frac{\partial U}{\partial S}\right)_V = -\frac{\alpha S^2}{4V^{3/2}}$

Gibbsche Fundamentalgleichung

$dU = T dS - p dV$

$dU = +\underbrace{\frac{\alpha S^2}{4V^{3/2}}}\, dV + \underbrace{\frac{\alpha S}{\sqrt{V}}}\, dS$

$dU = \left(\frac{\partial U}{\partial S}\right)_V^{P} dS + \left(\frac{\partial U}{\partial V}\right)_S^{T} dV$

$T dS - p dV = \frac{\alpha S}{\sqrt{V}} dS + \frac{-\alpha S^2}{4V^{3/2}} dV$

$T = \frac{\alpha S}{\sqrt{V}}$ $\qquad P = \frac{\alpha S^2}{4V^{3/2}}$

$\rightarrow P(V,T) = \frac{1}{4}\frac{\alpha}{\sqrt{V^2}}\frac{T^2 V}{\alpha^2} = \frac{1}{4}\frac{T^2}{\alpha\sqrt{V}}$

$U(V,T) = \frac{1}{2}\frac{\alpha}{\sqrt{V}}\frac{T^2 V}{\alpha^2} = \frac{1}{2}\frac{T^2\sqrt{2}}{\alpha}$

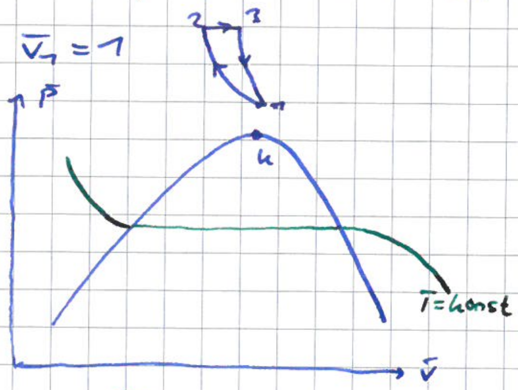
c)

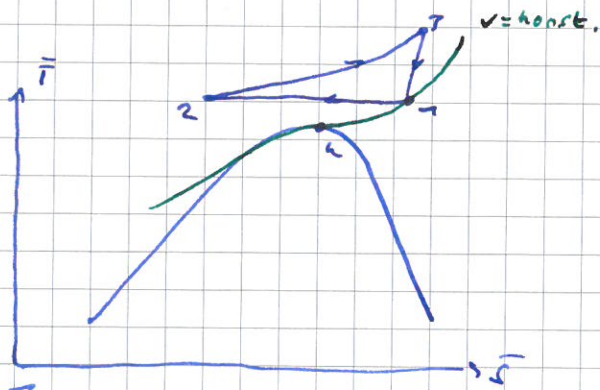
② Thermo Prüfung F20

a) $\left(p + \frac{a}{v^2}\right)(v-b) = RT$

$\left(\bar p + \frac{3}{\bar v^2}\right)(3\bar v - 1) = 8\bar T$

$\bar v_1 = 1$





b) $R = \frac{R_m}{M} = \cancel{0{,}1889}\ 188{,}9\ \frac{J}{kg\,K}$

$\bar p = \frac{p_1}{p_k} \rightarrow p_k = 7{,}385 \cdot 10^6$ MPa

$\bar T = \frac{T_1}{T_k} \rightarrow T_k = 304{,}2$ K

$\bar v = \frac{v_1}{v_k} \rightarrow v_k = 2{,}918 \cdot 10^{-3}$ m³/kg

$\cancel{pv = RT}$ $\quad \frac{3}{8} = \frac{p_k v_k}{R T_k}$

$v_1 = v_k = 2{,}918 \cdot 10^{-3}$ m³/kg

$a = 188{,}6\ \frac{m^5}{kg\,s^2}$; $b = 9{,}728 \cdot 10^{-4}\ \frac{m^3}{kg}$

c) $\cancel{p_1 = 2 - 2\frac{a}{v_1^2} \ldots \left(p_3 + \frac{a}{v_3^2}\right)\left(\frac{(v_3-b)}{(v_2-b)}\right)^{\frac{R}{c_v}+1}}$

$T_2 = T_1 = 380{,}3$ K

$p_2 = p_k \cdot \bar p_2 = 2{,}954 \cdot 10^7$ MPa

$p_2 = \frac{v_1 - b}{v_2 - b}\left(p_1 + \frac{a}{v_1^2}\right) - \frac{a}{v_2^2}$

$v_2 = 1{,}778 \cdot 10^{-3}\ \frac{m^3}{kg}$

d) $w_{v,23} = -80\ \frac{kJ}{kg} = -p_2(v_3 - v_2)$

$v_3 = 1{,}787 \cdot 10^{-3}\ \frac{m^3}{kg}$

$p_3 = p_2 = 2{,}954 \cdot 10^7$

Anders als in Musterlösung

T_3 über Therm. ZGL: $T_3 = 380{,}8$ K

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Thermo PRU P20

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Diagnostika · Praxisbedarf · Fachberatung

Dr. Wolf, Beckelmann & Partner GmbH

e) $\beta_2 = 1{,}02 \cdot 10^{-3} \frac{1}{K}$

$c_p = R + c_v = 8{,}069 \cdot 10^2$

$\delta_2 = 2{,}472 \cdot 10^{-7} \frac{Km^3}{kg}$ (quasi $\frac{K}{Pa}$) > 0; Abkühlung

1)

a) $\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$

$p^* = 0{,}5283 \cdot p_0 = 1{,}585$ bar

$\frac{T^*}{T_0} = \frac{2}{k+1}$

$T^* = 666{,}7\,K$

$Ma = \frac{v}{c_s} \Rightarrow v = 517{,}6 \frac{m}{s}$

$c_s = \sqrt{k \frac{p}{\rho}} \rightarrow \rho = 8{,}284 \cdot 10^{-1} \frac{kg}{m^3}$

$\dot{m} = \rho_1 c_1 A_1$

$A^* = 3{,}5 \cdot 10^{-4}\, m^2$

$A = \pi r^2 \rightarrow r = 1{,}055 \cdot 10^{-2}\, m$

$D^* = 2r \rightarrow D^* = 2{,}111 \cdot 10^{-2}\, m$

b) $Ma = 0{,}2$

$\frac{A}{A^*} = \frac{1}{Ma} \left(\frac{2}{k+1} \left(1 + \frac{k-1}{2} Ma^2\right)\right)^{\frac{k+1}{2(k-1)}}$

$A_1 = 1{,}037 \cdot 10^{-3}\, m^2$

c) $\frac{p_0}{p_4} = \left(\frac{T_0}{T_4}\right)^{\frac{k}{k-1}}$ $\quad p_4 = p_\infty$ weil angepasst

$T_4 = 584{,}5\,K$

$\frac{p_0}{p_4} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{k}{k-1}}$

$Ma_4 = 1{,}358$; $\frac{A_4}{A^*} = \frac{1}{Ma} \left(\frac{2}{k+1} \left(1 \; \frac{k-1}{2} Ma^2\right)\right)^{\frac{k+1}{2(k-1)}}$

$A_4 = 3{,}824 \cdot 10^{-4}\, m^2$

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F20

(1) $V_{OS} = 12\,m^3$; $p = 1{,}467$ bar ; $T = 150\,K$; nur N_2 ; O_2 Tank

ideale Gase c_{v,N_2} und c_{v,O_2} konst. $R_m = 8{,}314 \frac{J}{mol\,K}$

a)

$R_{N_2} = \frac{R_m}{M_{N_2}} = \underline{296{,}7 \frac{J}{kg\,K}}$

$pV = mRT \rightarrow m = \frac{pV}{RT} = \underline{39{,}56\,kg}$

$pV = nR_mT \Rightarrow n = \underline{1412\,mol}$

b) Volumenanteil 15% O_2 , 85% N_2

$\underline{\psi_{O_2} = 0{,}15}$; $\underline{\psi_{N_2} = 0{,}85}$

$\psi_i = \frac{n_i}{n}$

T=konst, p=konst

$p_{O_2} = 0{,}15 \cdot p = 0{,}2201$ bar

$\frac{n}{85} \cdot 100 =$ gesamt n

gesamt $n \cdot 0{,}15 = n_{O_2} = \underline{249{,}2\,mol}$

$M_{O_2} \cdot n_{O_2} = m_{O_2} = 7{,}974\,kg$

$pV = mRT$ mit $R_G = 290{,}5 \frac{J}{kg\,K}$

$V_{ges} = 14{,}12\,m^3$

$V_{Tank} = 2{,}14\,m^3$

c) $p_{O_2} = 0{,}15 \cdot p = 0{,}2201$ bar $p_{ON} = 0{,}85 \cdot p = 1{,}247$ bar

$S_2 - S_1 = \frac{1}{T} p_1 V_1 \ln\left(\frac{V}{V_1}\right) + \frac{1}{T} p_2 V_2 \ln\left(\frac{V}{V_2}\right)$

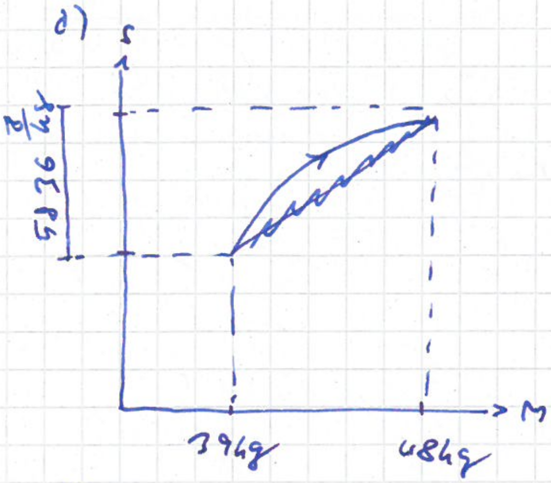
$= R_m \left(n \cdot \ln(n) - \sum_{k=1}^{k} n_k \ln(n_k)\right)$

$= 5836 \frac{J}{K}$

$R_G = 290{,}6 \frac{J}{kg\,K}$ (von vorhin)

$m_{OS} = 40{,}39\,kg$ (mit idealer Gasgleichung)

d)



$F = U - TS$

$F_2 - F_1 = -875{,}5\ \text{kJ}$

e) $p_{end} = 1{,}073\ \text{bar}$

$p_i V = m_i R_i T = n_i R_m T$

$n_{ges} = 974{,}7\ \text{mol}$

$n_{O_2,DS} = \dfrac{\psi \cdot p_1 \cdot V}{R_m \cdot T} = 211{,}7\ \text{Mol}$

$n_{N_2,end} = 974{,}7 - 211{,}7 = 763{,}0\ \text{mol}$

$p_{N_2,end} = \psi_{N_2} \cdot p_{ges} = 0{,}7930\ \text{bar}$ $(78{,}28\ \%)$

$p_{O_2,end} = 0{,}2200\ \text{bar}$

$m_{O_2,end} = 6{,}774\ \text{kg}$

$m_{N_2,ges} = 21{,}737\ \text{kg}$

f)

$Q_{12} = m\, c_p (T_2 - T_1)$

$c_p = 0{,}5922 =$?! ?!

(2) Sauerstoff, van der Waals-Gas

1 $\xrightarrow[\text{isotherm komprim.}]{}$ 2 $\xrightarrow[\text{isochor erwärmt}]{}$ 3 — adiabate Drosselung → 4₂ / → 4₃

ΔS_{12} abgegeben

c_v konst,

a)
$R = \dfrac{R_m}{M} = 259{,}8\ \dfrac{\text{J}}{\text{kgK}}$

$v_k = 6 \cdot 3 = 2{,}987 \cdot 10^{-3}\ \text{m}^3$

$p_k = \dfrac{a}{3 \cdot v_k^2} = 5{,}044 \cdot 10^6\ \text{Pa}$

$\dfrac{3}{8} = \dfrac{p_k v_k}{R\, T_k} \rightarrow T_k = 154{,}6\ \text{K}$

$v_1 = 0{,}7526\ \text{m}^3/\text{kg}$

b)

$T_2 = T_1 = 290\ K$

$\Delta s_{12} = s_2 - s_1 = R \ln\left(\frac{v_2 - b}{v_1 - b}\right)$

$v_2 = 3{,}201\ \frac{m^3}{kg}\ (\cdot 10^{-3})$

$p_2 = \frac{v_1 - b}{v_2 - b}\left(p_1 + \frac{a}{v_1^2}\right) - \frac{a}{v_2^2}$

$p_2 = 2{,}049 \cdot 10^7\ Pa$

$q_{12} = R T_1 \ln\left(\frac{v_2 - b}{v_1 - b}\right)$

$q_{12} = -439{,}3\ \frac{kJ}{kg}$

c) $v_3 = v_2 = 3{,}201 \cdot 10^{-3}\ \frac{m^3}{kg}$

$\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_2^2}}{T_2}$

$T_3 = 323{,}8\ K$

$q_{23} = c_v (T_3 - T_2)$

$p_3 = 2{,}497 \cdot 10^7\ Pa$

$\Delta s_{23} = c_v \ln\left(\frac{T_2}{T_1}\right) = 77{,}67\ \frac{J}{kgK}$

d)

$c_{p,i} - c_v = \frac{R}{1 - \frac{2a(v-b)^2}{RTv^3}}$

$c_{p,2} = 1204\ \frac{J}{kgK}$

$c_{p,3} = 1146\ \frac{J}{kgK}$

$\delta_{h,2\to 4} = -\frac{v}{c_p}\left(\frac{RTv^3 - 2a(v-b)^2 - T(v-b)Rv^2}{RTv^3 - 2a(v-b)^2}\right)$

$= 1{,}250 \cdot 10^{-6}\ \frac{Kms^2}{kg} > 0$ Abkühlung

$\delta_{h,3\to 4} = 8{,}786 \cdot 10^{-7}\ \frac{Kms^2}{kg} > 0$ Abkühlung

e)

$\delta_{h,2\to 4} = \frac{T_4 - T_2}{p_4 - p_2} \longrightarrow T_{4,2} = 263{,}9\ K$

$\delta_{h,3\to 4} = \frac{T_4 - T_3}{p_4 - p_3} \longrightarrow T_{4,3} = 301{,}9\ K$

Für Verflüssigung muss gelten $T_4 < T_k$

Weil beide Male $T_4 > T_k$ gilt, bleibt das Medium Gasförmig.

f)

③

Flughöhe 15 km

$p_{0,1} = 15950\ Pa$ $\quad p_1 = p_A = 11500\ Pa$ $\quad T_{0,1} = 239\ K$

$A_1 = 0{,}03\ m^2$ $\quad A_E = 0{,}045\ m^2$

Ideales Gas ; $R = 287 \frac{J}{kg\,K}$ $\quad \kappa = 1{,}4$; rev adiab., eindim.

a) $\frac{p_{01}}{p_1} = \left(1 + \frac{\kappa-1}{2} Ma^2\right)^{\frac{\kappa}{\kappa-1}}$

$Ma = 0{,}6999$

$\frac{T_{01}}{T_1} = 1 + \frac{\kappa-1}{2} Ma^2$

$T_1 = 217{,}7\ K$

b) $\frac{A}{A^*} = \frac{1}{Ma}\left(\frac{2}{\kappa+1}\left(1 + \frac{\kappa-1}{2} Ma^2\right)\right)^{\frac{\kappa+1}{2(\kappa-1)}}$

$A^* = 0{,}02747\ m^2$

$\dot m = \rho_1 c_1 A_1$

$c_1 = \sqrt{\kappa R T} = \sqrt{\kappa \frac{p}{\rho}}$

$c_1 = 295{,}8\ \frac{m}{s} \cdot 0{,}6999$

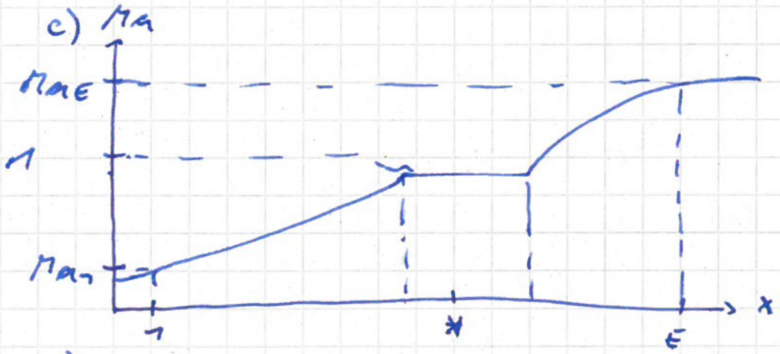
$\rho_1 = 0{,}1840\ \frac{kg}{m^3}$

$\dot m = 1{,}743\ \frac{kg}{s}$

$\frac{A_E}{A^*} = \frac{1}{Ma}\left(\frac{2}{\kappa+1}\left(1 + \frac{\kappa-1}{2} Ma^2\right)\right)^{\frac{\kappa+1}{2(\kappa-1)}}$

$Ma_E = 1{,}967$

c)



d) $\tilde{Ma}_1 = 1{,}446$, Flughöhe 20 km $\quad \tilde p_1 = \tilde p_a = 10300$

Vor dem Stoß: Strömung verzögert weil $Ma > 1$ und konvergente Düse
↳ Überschalldiffusor
Nach dem Stoß: Strömung beschleunigt weil $Ma < 1$ und konvergente Düse
↳ Unterschalldüse

e) $\tilde{Ma}_2 = 1{,}371$

$\tilde p_{01} = \tilde p_{02} = p_1 \cdot \left(1 + \frac{\kappa-1}{2} Ma_1^2\right)^{\frac{\kappa}{\kappa-1}} =$ 34990 Pa

H79

e) $\tilde{p}_{02} = 34990\ Pa = \tilde{p}_{01}$

$\frac{\tilde{p}_{02}}{\tilde{p}_2} = \left(1 + \frac{\kappa-1}{2} Ma_2^2\right)^{\frac{\kappa}{\kappa-1}}$

$\tilde{p}_2 = 11320\ Pa$

$\tilde{Ma}_3 = \sqrt{\frac{(\kappa-1)(Ma_2^2-1)+\kappa+1}{2\kappa(Ma_2^2-1)+\kappa+1}} = 0{,}7487$

$\frac{\tilde{p}_3}{\tilde{p}_2} = \frac{2\kappa Ma_2^2 - \kappa + 1}{\kappa+1}$

$\tilde{p}_3 = 21230\ Pa$

$\frac{\tilde{p}_{03}}{\tilde{p}_3} = \left(1 + \frac{\kappa-1}{2} Ma_3^2\right)^{\frac{\kappa}{\kappa-1}}$

$\tilde{p}_{03} = 33690\ Pa$

$\Delta \tilde{p}_{0,S} = \tilde{p}_{02} - \tilde{p}_{03} = 1298\ Pa$

$\Delta s = \tilde{s}_3 - \tilde{s}_2 = -R \cdot \ln\left(\frac{p_{03}}{p_{02}}\right) = 10{,}78\ \frac{J}{kgK}$

f)

$\tilde{Ma}_6 = Ma_E = 1{,}967$ weil gleiche Geometrie

$\tilde{T}_E = 1353\ K \neq \tilde{T}_5$

$\tilde{p}_{05} = \tilde{p}_{0E}$, $\tilde{p}_E = \tilde{p}_A = 10300\ Pa$

$\tilde{p}_{05} = 76560\ Pa$

$\tilde{T}_{0,E} = 2400\ K = \tilde{T}_{05}$

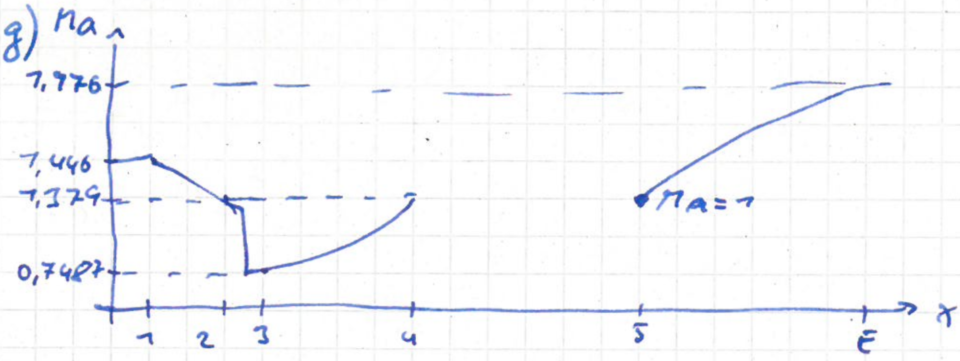
$\left(\frac{T_{05}}{T_5}\right)^{\frac{1}{\kappa-1}} = \left(1 + \frac{\kappa-1}{2} Ma_5^2\right)^{\frac{1}{\kappa-1}}$

~~Ma_5~~

$T_5 = 2000\ K$

$c_5 = \sqrt{\kappa R T_5} = 896{,}4\ \frac{m}{s}$

g)



5

$t_1 = 27°C$ $\varphi = 0,45$ $\dot{m}_W = 1,44 \frac{kg}{h}$ $t_W = 22°C$

$\dot{V}_H = 0,3 \frac{L}{s}$ $t_H = 37°C$ $\varphi_H = 0,9$

Ideale Gase $p = 1 bar$

a) $X_1 = 10,0 \frac{g}{kg}$

$X_H = 26,0 \frac{g}{kg}$

b) $\varphi = \frac{p_D}{p_S} = 0,9$??

c) Mit $\dot{m}_{WH} = 0,33 \frac{g}{s}$

1 a) Zusammensetzung I: 89,80 % H_2 ; 10,20 % He
$T = 245{,}0\,K$; $p = 2{,}500\,bar$

$V_1 = 0{,}5\,m^3$
Links: H_2 bei 245,0 K

$pV = n R_m T$ mit $n_{H_2} = n \cdot 0{,}8980$
und $n_{He} = n \cdot 0{,}1020$

$n_{H_2} = 55{,}11\,mol$
$n_{He} = 6{,}259\,mol$

b) $p_{H_2} = 2{,}5\,bar \cdot 0{,}8980 = 2{,}245\,bar$
$p_{He} = 2{,}5\,bar \cdot 0{,}1020 = 0{,}2550\,bar$

$R_{H_2} = 4124\,\frac{J}{kgK}$; $R_{He} = 2077\,\frac{J}{kgK}$

$m_{H_2} = 0{,}1111\,kg$; $m_{He} = 0{,}02506\,kg$

$\varepsilon_i = \frac{m_i}{m}$

$\varepsilon_{H_2} = 0{,}8160$; $\varepsilon_{He} = 0{,}1840$

Dichte = $\frac{Masse}{Volumen}$

$\rho_{H_2} = 0{,}2222\,\frac{kg}{m^3}$; $\rho_{He} = 0{,}005012\,\frac{kg}{m^3}$

c) $S_2 - S_1 = R_m \left[n \ln(n) - \sum_{k=1}^{k} n_k \ln(n_k) \right]$
$= 168{,}1\,\frac{J}{kg}$

d) Wärme Entzug (Isochorer Vorgang)

$Q_{12} = m \cdot c_v \cdot (T_2 - T_1)$

$R_i = c_{p_i} - c_{v_i}$

$c_{v,H_2} = 9926\,\frac{J}{kgK}$; $c_{v,He} = 3123\,\frac{J}{kgK}$

$c_{v,G} = \sum c_{v,H_2} \cdot \varepsilon_{H_2} + c_{v,He} \cdot \varepsilon_{He} = 8674\,\frac{J}{kgK}$

$Q_{12} = -17{,}12\,kJ$

e) ~~n_{H_2} = 12,62 bar ; n_{He} =~~
Helium Menge bleibt gleich!

$n_{He} = 6{,}259\,mol$ = 20 %

$n_{H_2} = 25{,}04\,mol$ = 80 %

$n_{ges} = 31{,}30\,mol$

$pV = n R_m T \rightarrow p_{ges} = 1{,}197\,bar$, $p_{H_2} = 0{,}9576\,bar$, $p_{He} = 0{,}2394\,bar$
Druck entspricht nicht der Neptun Atmosphäre!

f) $n_B = 30{,}07$ mol nur H_2

$M_{H_2} = 2{,}016 \frac{kg}{kmol}$

$c_{p,H_2} = 14050 \frac{J}{kg\,K}$

$T = 230$ K

p_B muss gleich auf beiden Seiten der Membran sein!

$p_B = 0{,}9576$ bar

$pV = n R_m T$

$V = 0{,}6004$ m³

$m_B = 0{,}06061$ kg

$m_A = m_{He} + m_{H_2} = n_{He} M_{He} + n_{H_2} M_{H_2} = 0{,}07554$ kg

~~[illegible]~~ $\frac{m_{H_2,B}}{m_{ges}} = 0{,}4452$

44,52 % der Gesamtmasse in B

2) Van der Waals-Gas

$p_k = 58{,}42$ bar $\quad v_k = 1{,}178 \cdot 10^{-3} \frac{m^3}{kg}$

$M = 131{,}3 \frac{g}{mol}$

$c_v = 94{,}60 \frac{J}{kg\,K}$

$R_m = 8{,}314 \frac{J}{mol\,K}$

$\bar{p} = \frac{p}{p_k}$; gleich für $\bar{v}$ und $\bar{T}$

$v_1 = 4{,}712 \cdot 10^{-3} \frac{m^3}{kg} \qquad T_1 = 435{,}0$ K

$\downarrow$ rev. adiabat

$v_2 = 2 \cdot v_k = 2{,}356 \cdot 10^{-3} \frac{m^3}{kg}$

$\downarrow$ Isochor

$p_3 < p_2$

$\downarrow$ isotherme Expansion

Ausgangszustand

a) $R = \frac{R_m}{M} = 63{,}32 \frac{J}{kg\,K}$

$\frac{3}{8} = \frac{p_k v_k}{R T_k} \rightarrow T_k = 289{,}8$ K

$a = 3 p_k v_k^2 = 24{,}32 \frac{m^5}{kg\,s^2}$; $\quad b = \frac{v_k}{3} = 3{,}927 \cdot 10^{-4}$

b) Thermische Zustandsgleichung für vdW-Gase:

$$\left(p_1 + \frac{a}{v_1^2}\right)(v_1 - b) = R \cdot T_1$$

$$p_1 = 52{,}82 \text{ bar}$$

$$v_2 = 2{,}356 \cdot 10^{-3} \frac{m^3}{kg}$$

$$p_2 = -\frac{a}{v_2^2} + \left(p_1 + \frac{a}{v_1^2}\right)\left(\frac{v_1 - b}{v_2 - b}\right)^{\frac{R}{c_v}+1} = 194{,}0 \text{ bar}$$

$$T_2 = T_1 \left(\frac{v_1 - b}{v_2 - b}\right)^{\frac{R}{c_v}} = 737{,}4 \text{ K}$$

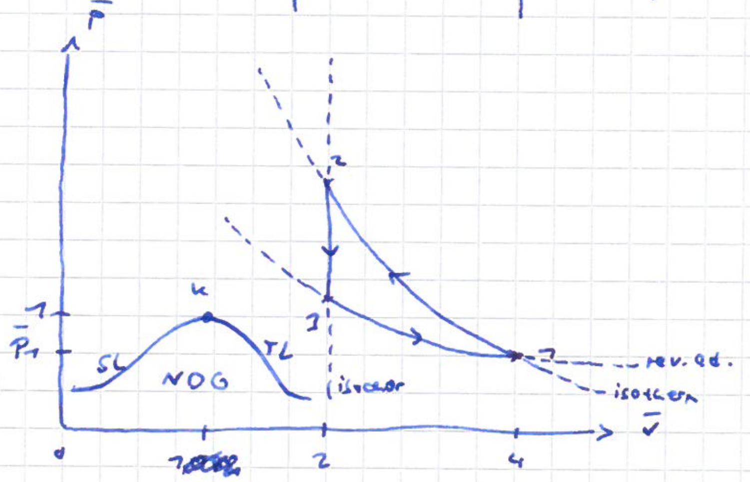
c) $v_3 = v_2 = 2{,}356 \cdot 10^{-3} \frac{m^3}{kg}$

Von 3 auf 1 isotherm $\rightarrow$ $T_3 = T_1 = 435{,}0$ K

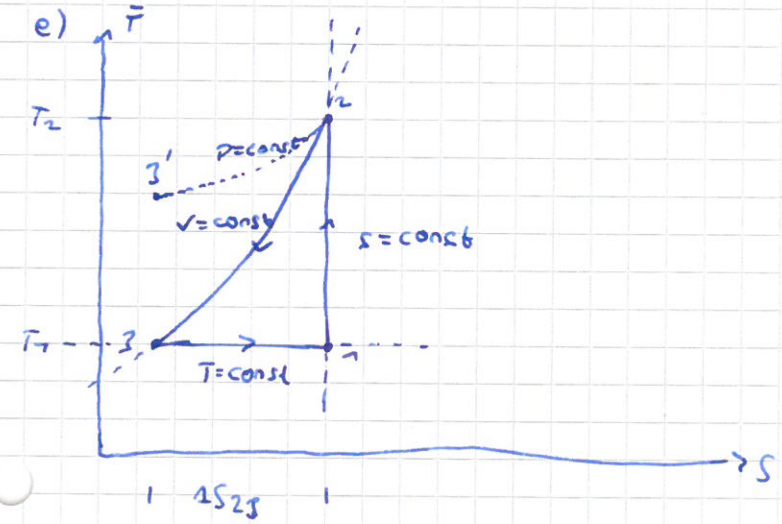
$$p_3 = \frac{T_3}{T_2}\left(p_2 + \frac{a}{v_2^2}\right) - \frac{a}{v_2^2} = 96{,}47 \text{ bar}$$

d)

	$\bar{p}_i$	$\bar{v}_i$
Zustand 1	0,9047	4,000
2	3,327	2,000
3	1,657	2,000



e)



f)

$W_{v,ges} = W_{v,12} + W_{v,23} + W_{v,31}$

$W_{v,12} = \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) \qquad = 23450\ J$

$W_{v,23} = 0 \qquad = 0\ J$

$W_{v,31} = -RT_3 \ln\left(\frac{v_1-b}{v_3-b}\right) + \frac{a}{v_3} - \frac{a}{v_1} \qquad = -16560\ J$

$W_{v,ges} = 6{,}890\ kJ$

$q_{23} = c_v(T_3 - T_2) = -28{,}67\ \frac{kJ}{kg}$

$q_{31} = RT_3 \cdot \ln\left(\frac{v_1-b}{v_3-b}\right) = 21{,}72\ \frac{kJ}{kg}$

$q_{ges} = -6{,}890\ \frac{kJ}{kg}$

$q_{ges} + W_{v,ges} = 0 \longrightarrow$ 1. Hauptsatz erfüllt!

g) $2 \rightarrow 3$ jetzt isobar

$p_3' = p_2 = 144{,}0\ bar$

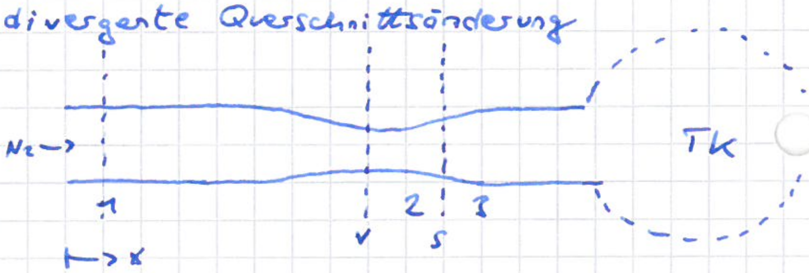
$\Delta s_{23}' = \Delta s_{23} = c_v \ln\left(\frac{T_3'}{T_2}\right) + R \ln\left(\frac{v_3'-b}{v_2-b}\right) = -49{,}93\ \frac{J}{kgK}$

$T_3' = T_2 \frac{v_3'-b}{v_2-b} + \frac{a}{R}(v_3'-b)\left(\frac{1}{v_3'^2} - \frac{1}{v_2^2}\right)$

$T_3' = 572{,}9\ K$

$v_3' = 1{,}694 \cdot 10^{-3}\ \frac{m^3}{kg}$

(3) Stickstoff (N_2), konvergent-divergente Querschnittsänderung



$p_0 = 20\ bar \qquad T_0 = 298{,}15\ K$

$d_1 = 10\ mm \qquad c_1 = 228\ m/s$

Ventil kritisch durchströmt

isentrope Verzögerung auf 0 m/s in Testkammer

ideales Gas, $\kappa = 1{,}4$, $R = 296{,}8\ \frac{J}{kgK}$

Strömung eindimensional, rev. adiabat

a) $c_p = \frac{\kappa}{\kappa-1} R = 1039\ \frac{J}{kgK}$

$c_{S1} = \sqrt{\kappa R T_1} \rightarrow T_1 = ???$ Weiter mit Wert aus Lösung: $T_1 = 273{,}1\ K$

$c_{S1} = 336{,}9\ m/s$

$Ma_1 = \frac{c_1}{c_{S,1}} = 0{,}6768$

$\frac{p_0}{p_1} = \left(\frac{T_0}{T}\right)^{\frac{\kappa}{\kappa-1}} \longrightarrow p_1 = 14{,}77\ bar$

$c_1 = \sqrt{\kappa \cdot \frac{p_1}{\rho_1}} \rightarrow \rho_1 = \ldots$

$\downarrow$

$\dot{m} = \cancel{0{,}3251\ \frac{kg}{s}}$???

$3{,}241\ \frac{kg}{s}$

b) Verdichtungsstoß nach Ventil bei $Ma_2 = 2{,}444$

$A = \pi \cdot r^2$ ~~[illegible]~~

$= 7{,}854 \cdot 10^{-5}\ m^2$

$$\frac{A}{A^*} = \frac{1}{Ma}\left(\frac{2}{k+1}\left(1 + \frac{k-1}{2} Ma^2\right)\right)^{\frac{k+1}{2(k-1)}} \quad \text{mit } Ma = 0{,}6768$$

$A^* = 7{,}067\ m^2$

für Durchmesser beim Stoß in obige Gleichung $Ma_2 = 2{,}444$ und A^* einsetzen

$A_2 = 1{,}767 \cdot 10^{-4}\ m^2$

$d_2 = 15\ mm$

$$Ma_3 = \sqrt{\frac{(k-1)(Ma_2^2 - 1) + k + 1}{2k\,(Ma_2^2 - 1) + k + 1}} = 0{,}5785$$

$$\frac{p_3}{p_2} = \frac{2k\,Ma_2^2 - k + 1}{k+1} = 6{,}802$$

c) $$\frac{p_{0,th}}{p_3} = \left(1 + \frac{k-1}{2} Ma_3^2\right)^{\frac{k}{k-1}} = 1{,}207 \quad \text{I}$$

$$\frac{p_0}{p_2} = \left(1 + \frac{k-1}{2} Ma_2^2\right)^{\frac{k}{k-1}} = 15{,}66 \quad \text{II}$$

$$\frac{p_3}{p_2} = 6{,}802 \quad \text{III}$$

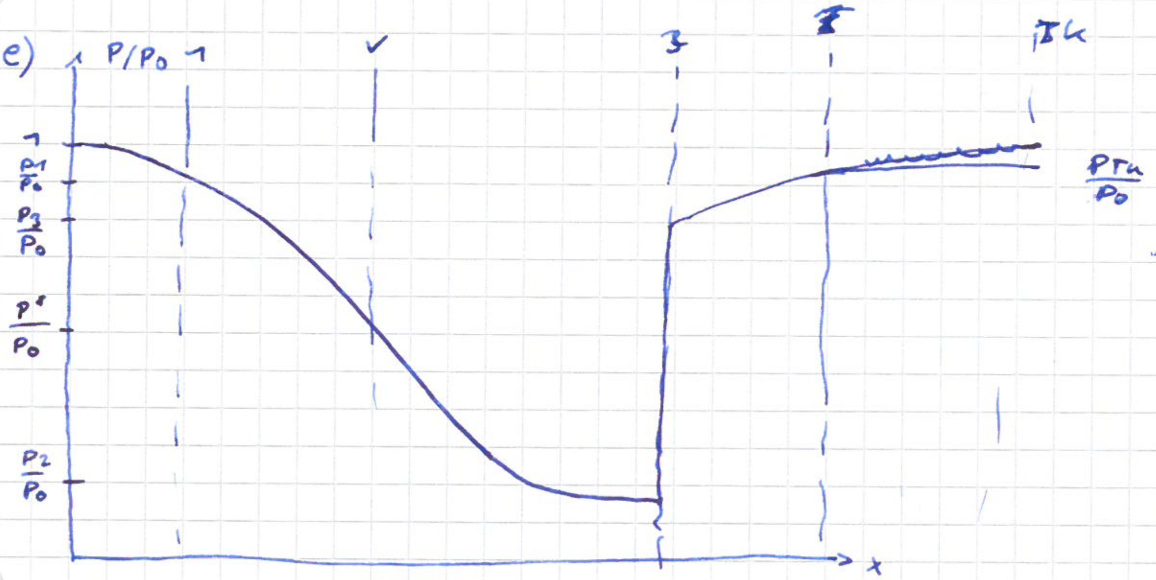
Lineares Gleichungssystem lösen

$\longrightarrow p_{0,th} = 10{,}43$ bar , $p_2 = 1{,}277$ bar , $p_3 = 8{,}687$ bar

d) $$s_2 - s_1 = c_p \ln\left(\frac{T_3}{T_2}\right) - R \ln\left(\frac{p_3}{p_2}\right) = -R \ln\left(\frac{p_{0,Th}}{p_0}\right)$$

$p_{0,Th} = 10{,}42$ ✓

e)



f) f ist sehr komisch, Leonie fragen wie sie das gemacht hat

(4) $T_{max} = 350\,K$

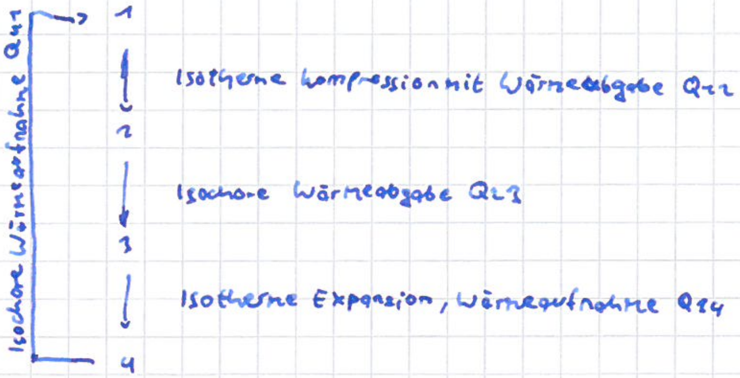
$T_{min} = 70\,K$

$p_{min} = 9{,}874\,bar$

$V_{max} = 1{,}569 \cdot 10^{-6}\,m^3$

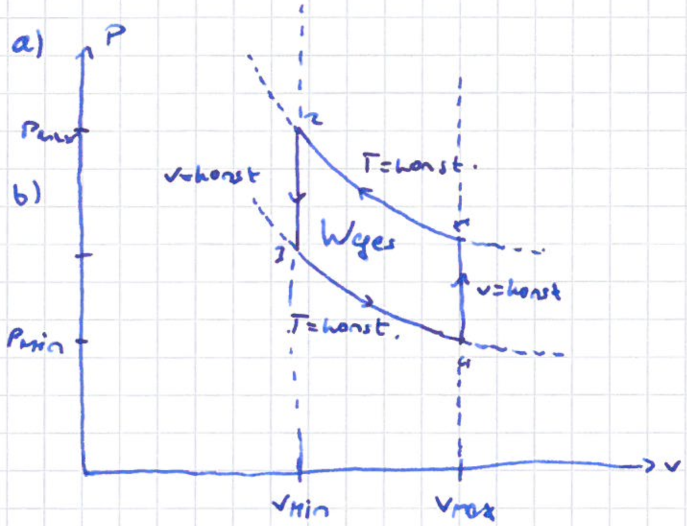
$Q_{34} = 0{,}04\,J$

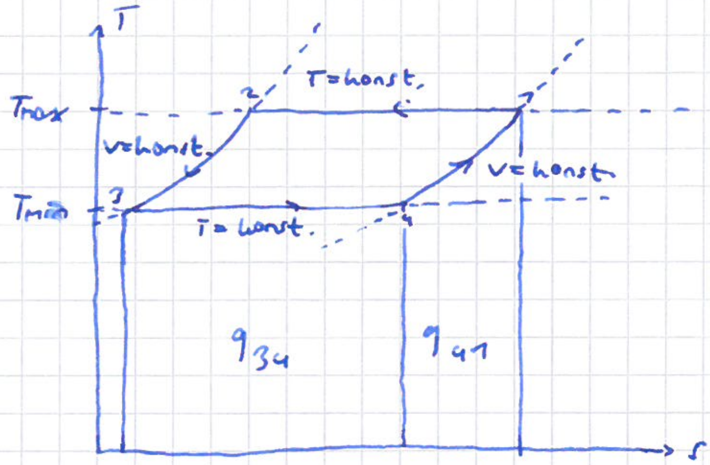
Ideales Gas



a)

b)





c)

$T_4 = T_{min} = 70\,K = T_3$

$p_4 = p_{min} = 9{,}874\,bar$

$c_p = \frac{\kappa}{\kappa-1} R = 519{,}3\,\frac{J}{kgK}$ $\quad c_v = 311{,}6\,\frac{J}{kgK}$ $\quad c_n = \frac{n-\kappa}{n-1} c_v$

$R = \frac{R_m}{M} = 207{,}7\,\frac{J}{kgK}$

$\cancel{H_4 - H_3 = Q_{34} = m \cdot c_p \cdot (T_4 - T_3)}$

$pv = RT \rightarrow v_4 = 1{,}472 \cdot 10^{-2}\,\frac{m^3}{kg}$ $\quad 1{,}472 \cdot 10^{-1}\,\frac{m^3}{kg}$ eigentlich?

$m = 1{,}066 \cdot 10^{-5}\,kg$

$q_{34} = p_3 v_3 \ln\left(\frac{p_3}{p_4}\right) = \frac{Q_{34}}{m} = 3752\,\frac{J}{kg}$

d)

Zustand	T [K]	p [bar]	v [m^3/kg]
1	350	49,39	0,1472
2	350	50,66	0,1435
3	70	10,13	0,1435
4	70	9,874	0,1472

4

d) $pv = RT$

$p_1 = 49{,}39$ bar

I $\frac{p_3}{p_4} = \frac{v_4}{v_3} \quad \longrightarrow \quad p_3 = \frac{v_4}{v_3} \cdot p_4$

II $q_{34} = p_3 v_3 \ln\left(\frac{p_3}{p_4}\right)$

Lineares Gleichungssystem lösen

$\frac{p_2}{T_2} = \frac{p_3}{T_3} \quad \longrightarrow \quad p_3 = \frac{p_2}{T_2} \cdot T_3 \qquad$ I

$s_4 - s_3 = s_2 - s_1$

$\rightarrow R \cdot \ln\left(\frac{p_3}{p_4}\right) = R \cdot \ln\left(\frac{p_1}{p_2}\right) \qquad$ II

$p_2 = 49{,}38$ bar $\qquad p_3 = 9{,}876$ bar $\quad$ ↯

eigentl. $= 50{,}66$ bar $\qquad = 10{,}13$ bar

e) c_v bereits berechnet.

$c_v = 3176 \ \frac{J}{kgK}$

$Q_{41} = c_v(T_1 - T_4) \cdot m = 9{,}301 \ J$

$Q_{23} = c_v(T_3 - T_2) \cdot m = -9{,}301 \ J$

$Q_{ab} = Q_{12} = m \cdot p_1 v_1 \ln\left(\frac{p_1}{p_2}\right) = -0{,}1968 \ J$

f) $\varepsilon = \frac{q_{34}}{w_{ges}} \qquad \varepsilon_{Carnot} = \frac{T_{min}}{T_{max} - T_{min}}$

$q_{zu} = q_{34}$

$w_{ges} = -q_{34} - q_{12}$

$\varepsilon = \frac{RT_3 \ln\left(\frac{p_3}{p_4}\right)}{-RT_3 \ln\left(\frac{p_3}{p_4}\right) - RT_1 \ln\left(\frac{p_1}{p_2}\right)} = \frac{T_3}{-T_3 + T_1} = \frac{T_{min}}{T_{max} - T_{min}} = \varepsilon_{Carnot}$

5

$\dot{m}_{ein} = 6 \ kg/s \qquad t_1 = 25\,°C \qquad \varphi_1 = 50\%$

$\dot{Q}_K = 5 \ kW \qquad \dot{m}_{DG} = 45 \ \frac{g}{s} \qquad t_D = 100\,°C$

$x_2 = 18 \ g/kg_{tr.Luft}$

ideale Gase, $p = 1$ bar

a) $x_1 = 10{,}00 \ \frac{g}{kg_{tr.Luft}}$

$h_1 = 50{,}00 \ \frac{kJ}{kg}$

$\dot{m}_{tr,ein} = \dot{m}_{ein} - \dot{m}_{ein} \cdot x_1 = 5{,}940 \ \frac{kg}{s}$

b) ~~H_D ...~~

$H_2 - H_1 = m\, c_p\, (T_2 - T_1)$

$h_1 = 50{,}00 \frac{kJ}{kg}$

Wie lässt sich der Rest ablesen?

$h_D = 2692 \frac{kJ}{kg}$

$t_2 = 27{,}1\,°C$

$h_2 = 73{,}0 \frac{kJ}{kg}$

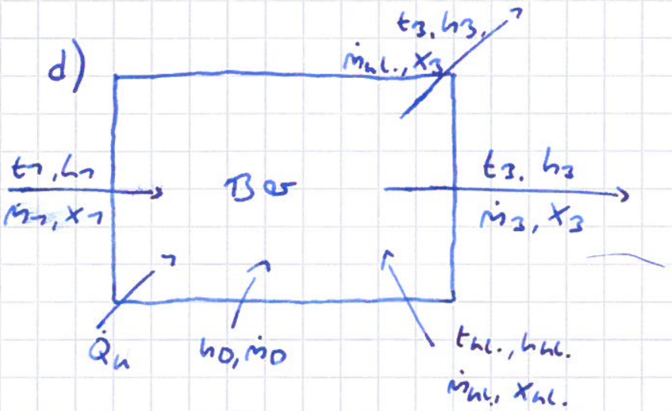
c) $t_3 = 22\,°C$ $\varphi_3 = 60\%$

$\dot{m}_{Klima} =$

$x_{Klima} = 2\ g/kg\ tr.\ Luft$

Ablesen: $h_3 = 47 \frac{kJ}{kg}$; $x_3 = 10\ g/kg\ tr.\ Luft$

d)



e) $\dot{m}_{tr,1} = \dot{m}_{tr,3}$ (Massenbilanz)

$\dot{m}_1 x_1 + \dot{m}_D + \dot{m}_{tr,Kl} \cdot x_{Kl} = \dot{m}_{3,tr} \cdot x_3 + \dot{m}_{Kl,tr} x_3$ (Wasserbilanz)

$\dot{m}_{tr,1} h_1 + \dot{m}_D h_D + \dot{m}_{tr,Kl} \cdot h_3 + \dot{Q}_K = \dot{m}_{tr3} \cdot h_3 + \dot{m}_{tr,Kl} h_3$

f)

Thermo F18

(1) $V_{max} = 4L$

$T_{angebrochen}$: 293K

T_{voll} : 287K

$T_1 = 293K$

$V_1 = 3L$

$p_1 = 0{,}75$ bar

$p_2 = 5$ bar

a) $R_{H2} = \frac{R_m}{M_{H2}} = 4124 \frac{J}{kgK}$ ✓

$m_{H2} = M_{H2} \cdot n_{H2}$

$p_1V_1 = n_{H2} R_m T_1$

$n_{H2} = 9{,}236 \cdot 10^{-2}$ mol ✓

$m_{H2} = 186{,}2$ g (✓)

b) $p_{H2}, p_{CH4}, \psi_{H2}, \psi_{CH4}$ in 2

$p_{H2} = 0{,}75$ bar ✓

$p_{CH4} = 4{,}25$ bar ✓

$\psi_{H2} = \frac{p_{H2}}{p_{ges}} = 0{,}15$ ✓

$\psi_{CH4} = \frac{p_{CH4}}{p_{ges}} = 0{,}85$ ✓

c) $n_{CH4} = \psi_{CH4} \cdot n$

$n = n_{H2} / \psi_{H2}$

$n_{CH4} = 5{,}234 \cdot 10^{-1}$ mol ✓

$m_{H2} = M_{H2} \cdot n_{H2} = 186{,}2$ g

$m_{CH4} = M_{CH4} \cdot n_{CH4} = 8395$ g (✓)

$m_{ges} = 858{,}2$ g (✓)

$\varepsilon_{H2} = \frac{m_{H2}}{m_{ges}} = 0{,}02170$ ✓

$\varepsilon_{CH4} = \frac{m_{CH4}}{m_{ges}} = 0{,}9782$ ✓

$c_{v,ges} = c_{vH2} \cdot \varepsilon_{H2} + c_{vCH4} \cdot \varepsilon_{CH4}$

$= 1745 \frac{J}{kgK}$ (✓)

d) /

e) $p_{G,3} = 8{,}3\ \text{bar}$ $\quad m_{CH_4,3} = 15{,}20 \cdot 10^{-3}\ \text{kg}$

f)

g)

(2) Van der Waals-Gas

rev. adia.

1 $\quad \bar{p}_1 = 0{,}75\ \cancel{\text{bar}} \quad \bar{v}_1 = 3{,}5$

$\downarrow$ isochore Druckerhöhung

2 $\quad \bar{p}_2 = 1{,}2$

$\downarrow$ isobar Volumen verringert

3

a) $R = \frac{R_m}{M} = 188{,}9\ \frac{\text{J}}{\text{kgK}}$ ✓

$\frac{3}{8} = \frac{p_k v_k}{R\, T_k}$

$v_k = 2{,}92 \cdot 10^{-2}\ \text{m}^3$ (✓)

$a = 3 p_k v_k^2 = 1887\ \frac{\text{m}^5}{\text{kg s}^2}$ (✓)

$b = \frac{v_k}{3} = 0{,}3244\ \frac{\text{m}^3}{\text{kg}}$ ✗

b) $\bar{T}_1,\ \bar{T}_2,\ \bar{v}_2,\ p_1,\ p_2$

$\left(\bar{p}_1 + \frac{3}{\bar{v}_1^2}\right)(3\bar{v}_1 - 1) = 8\,\bar{T}_1$

$\bar{T}_1 = 1{,}181$ ✓

$\bar{v}_2 = \bar{v}_1 = 3{,}5$ ✓

$\bar{p}_2 = 1{,}2$

$\bar{T}_2$ auf gleiche Weise wie $\bar{T}_1$

$\bar{T}_2 = 1{,}716$ ✓

$T_1 = \bar{T}_1 \cdot T_k = 359{,}2\ \text{K}$ ✓

$T_2 = \bar{T}_2 \cdot T_k = 521{,}9\ \text{K}$ ✓

$p_1 = \bar{p}_1 \cdot p_k = 5{,}533\ \text{MPa}$ ✓

$p_2 = \bar{p}_2 \cdot p_k = 8{,}853\ \text{MPa}$ ✓

$v_1 = \bar{v}_1 \cdot v_k = 1{,}022 \cdot 10^{-1}\ \text{m}^3$ (✓)

$v_2 = v_1 = 1{,}022 \cdot 10^{-1}\ \text{m}^3$ (✓)

Thermo F18

c) $\beta, \varkappa, \gamma, c_p$ in 2

$$\beta = \frac{(v_2-b)Rv_2^2}{RT_2v_2^3-2a(v_2-b)^2} = 5{,}406\cdot 10^{-3}\ K^{-1}$$ x

$$\gamma = \frac{(v_2-b)^2v_2^2}{RT_2v_2^3-2a(v_2-b)^2} = 1{,}362\cdot 10^{-3}\ K^{-1}$$ x

$$\varkappa = \frac{Rv_2^2}{RT_2v_2^3-a(v_2-b)} = -6{,}359\cdot 10^{-6}\ K^{-1}$$ x

$$c_p - c_v = \frac{T_2 v_2 \beta^2}{\varkappa}$$

$$c_p = 372{,}9\ \frac{J}{kgK}$$ (✓)

d) v_3, T_3

$$p_3 = p_2 = 8{,}853\ MPa$$ ✓

LGS:

$$I:\ \frac{RT_2}{v_2-b} - \frac{a}{v_2^2} = \frac{RT_3}{v_3-b} - \frac{a}{v_3^2}$$

$$II:\ T_3 = T_2\frac{v_3-b}{v_2-b} + \frac{a}{R}(v_3-b)\left(\frac{1}{v_3^2}-\frac{1}{v_2^2}\right)$$

$$T_3 = 383{,}8\ K$$ (✓)

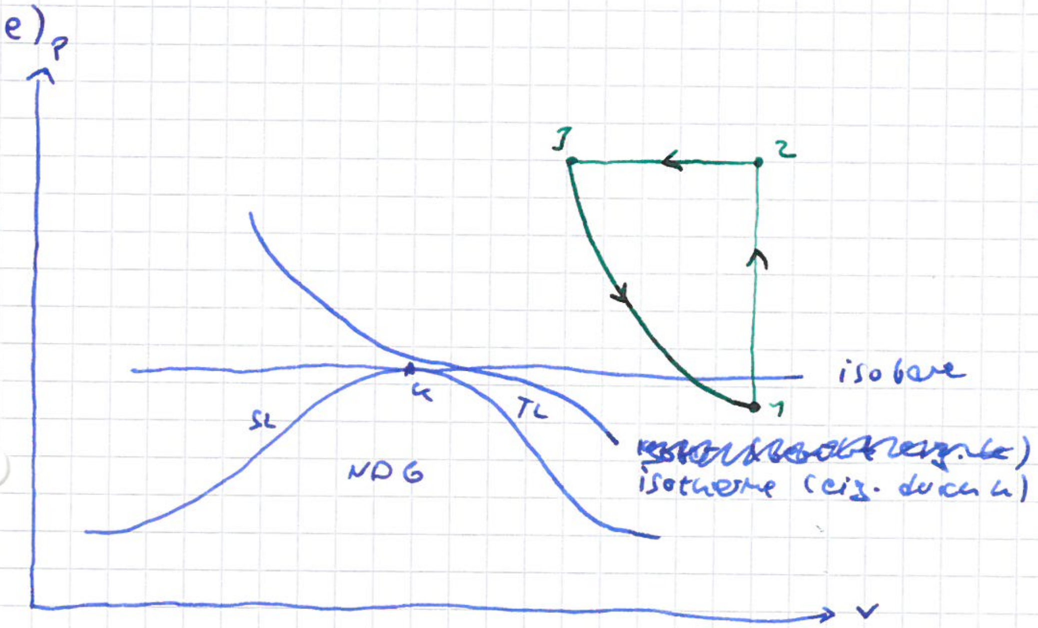
$$v_3 = 0{,}1455\ m^3$$ (✓)

Rev. ad. auf Zustand 1

$$T_1 = T_3\left(\frac{v_3-b}{v_1-b}\right)^{\frac{R}{c_v}}$$

$$\left(p_3+\frac{a}{v_3^2}\right)(v_3-b)^{\frac{c_v+R}{c_v}} = \left(p_1+\frac{a}{v_1^2}\right)(v_1-b)^{\frac{c_v+R}{c_v}}$$

e)



✓

f) q_{12} q_{23} q_{31} q_{ges}

$q_{12} = c_v(T_2 - T_1) = 1{,}005 \cdot 10^5 \frac{J}{kg}$ ✓

$q_{23} = \frac{a}{v_2} - \frac{a}{v_3} + c_v(T_3 - T_2) + p_2(v_3 - v_2) = -4{,}752 \cdot 10^4 \frac{J}{kg}$ (✓)

$q_{31} = 0 \frac{kJ}{kg}$ ✓

$q_{ges} = q_{12} + q_{23} + q_{31} = 5{,}903 \cdot 10^4 \frac{J}{kg}$ (✓)

3) $Ma_{11} = 3$ $T_{0,11} = 1699\,K$ $\Delta \dot{S} = 22{,}3 \frac{kJ}{Ks}$

$H = 15\,km$ $p_\infty = 12044{,}6\,Pa$ $R = 287 \frac{J}{kgK}$ $\kappa = 1{,}4$

a) T_{12}, V_{12}

$Ma_{12} = \sqrt{\frac{(\kappa-1)(Ma_{11}^2 - 1) + \kappa + 1}{2\kappa(Ma_{11}^2 - 1) + \kappa + 1}}$

$= 0{,}4752$ ✓

$\frac{T_{12}}{T_{11}} = \frac{(2\kappa Ma_{11}^2 - \kappa + 1)(2 + (\kappa-1)Ma_{11}^2)}{(\kappa+1)^2 Ma_{11}^2}$ mit $T_{11} = 606{,}8\,K$ ✓

$T_{12} = 1626\,K$ ✓

$c_{s12} = \sqrt{\kappa R T} = 808{,}3 \frac{m}{s}$ ✓

$V_{12} = Ma_{12} \cdot c_{s12} = 384{,}1 \frac{m}{s}$ ✓

b) $\dot{m}_{12} = \rho_{11} c_{11} A_{11} = \rho_{12} c_{12} A_{12}$

$\dot{m}_{12} = \frac{\dot{S}_{12}}{\Delta s_{12}}$?

$c_v = \frac{1}{\kappa-1} R = 717{,}5 \frac{J}{kgK}$

$s_{12} = c_v \cdot \ln\left(\frac{T_{12}}{T_{11}}\right) + R \ln\left(\frac{v_{12}}{v_{11}}\right) = 179{,}8 \frac{kJ}{K}$

$\dot{m} = 14{,}34 \frac{kg}{s}$ (✓)

c) $\dot{m}_{Br} = 6\,kg/s$ $T_{0,2} = 1400\,K$ $A^* = 0{,}02\,m^2$

$\frac{T^*}{T_{0,2}} = \frac{2}{\kappa+1}$

$T^* = 1167\,K$ ✓

$Ma^* = 1$

$c^* = \sqrt{\kappa R T^*} = 684{,}8 \frac{m}{s} = c^*$ ✓

$\dot{m}^* = \dot{m} + \dot{m}_{Br} = 20{,}34 \frac{kg}{s}$ (✓)

$\dot{m}^* = \rho^* c^* A^*$ $\rho^* = 1{,}485 \frac{kg}{m^3}$ ✗

Thermo F18

$c_s^* = \sqrt{\kappa \frac{p^*}{\rho^*}}$

$p^* = 4{,}974$ bar ×

d) Ma_3, A_3

$p_3 = p_\infty$

$\dot{m} = \rho_3 c_3 A_3 = \frac{p_3 c_3 A_3}{R T_3}$

$\frac{p^*}{p_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$

$p_0 = \cancel{\ldots}\ 9{,}415$ bar ×

$\frac{p_0}{p_3} = \left(1 + \frac{\kappa-1}{2} Ma_3^2\right)^{\frac{\kappa}{\kappa-1}}$ (✓)

$Ma_3 = 3{,}517$ (✓)

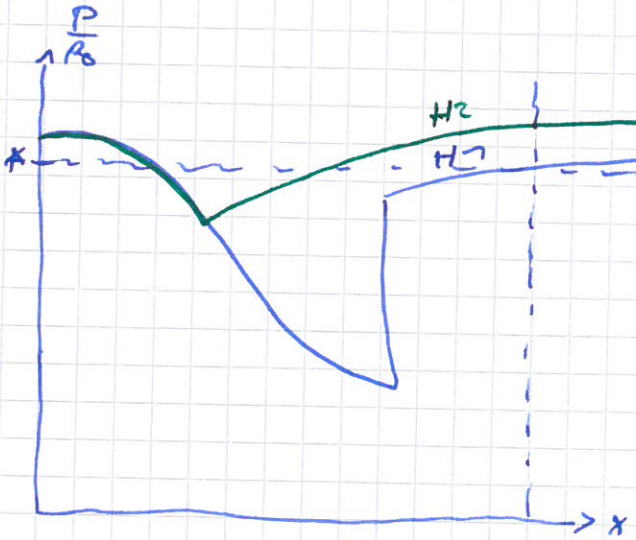
$\frac{A_3}{A^*} = \frac{1}{Ma}\left(\frac{2}{\kappa+1}\left(1 + \frac{\kappa-1}{2} Ma_3^2\right)\right)^{\frac{\kappa+1}{2(\kappa-1)}}$

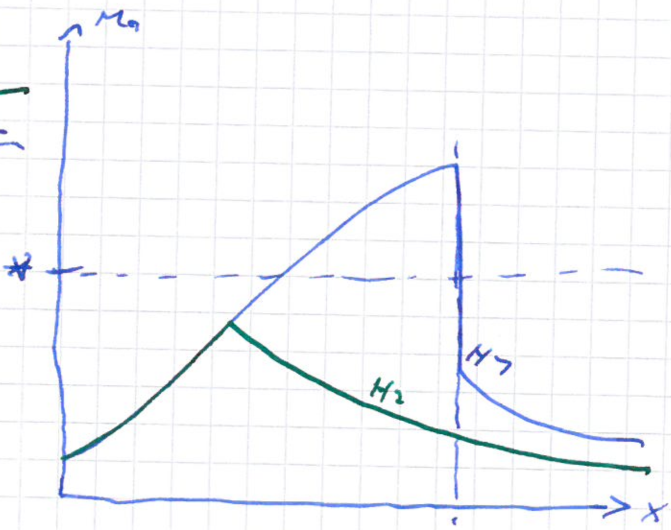
$A_3 = 0{,}138\ m^2$ (✓)

e) $H = 17{,}7$ km $\quad p_\infty = 7868{,}4$ Pa

$Ma_3 = 4{,}4 \quad A^*$ und T_0 bleiben gleich

f)





(✓)

5

$\dot{m}_{FL,4S} = 200 \frac{g}{s}$

$\dot{m}_{FL,8S} = 400 \frac{g}{s}$

$\vartheta_{U,4S} = 4°C$

$\varphi_{U,4S} = 80\%$

$\dot{Q}_{4S} = 115\,W \quad \cdot 4$

$\dot{m}_{W,4S} = 0{,}025\, g/s \quad \cdot 4$

$t_{W,4S} = 20°C$

a) $X_{U,4S} = 4 \frac{g}{kg\ tr.Luft}$ ✓

$h_{U,4S} = 15 \frac{kJ}{kg\ tr.Luft}$ ✓

b) $\dot{m}_{L,4S} = \dot{m}_{FL,4S} - m_{FL,4S} \cdot X_{U,4S} = 199{,}8 \frac{g}{s}$ ✓

$115\,W = 6{,}9 \cdot \frac{kJ}{min} = 0{,}115 \frac{kJ}{s}$ $\qquad \dot{Q}_{4S} = 4 \cdot 0{,}115 \frac{kJ}{s} = 0{,}46 \frac{kJ}{s}$

$q_{4S} = \frac{\dot{Q}_{4S}}{\dot{m}_{L,4S}} = 2{,}302 \frac{kJ}{kg}$ ✓

c) $\frac{\Delta h}{\Delta x} = c_W\, t_{ein} = 1159 \frac{kJ}{kg}$