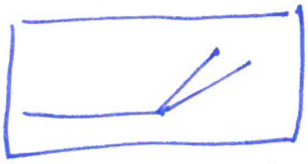
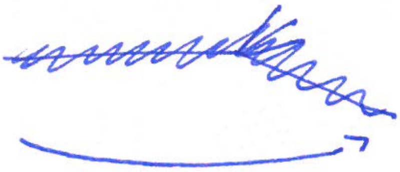
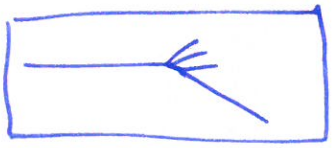
FACHSCHAFT LUFT- UND RAUMFAHRTTECHNIK UNIVERSITÄT STUTTGART





$\frac{V_2}{V_1} < 1$ $\frac{Ma_2}{Ma_1} < 1$ $\frac{\rho_2}{\rho_1} > 1$ $\frac{p_2}{p_1} > 1$

$\frac{p_{02}}{p_{01}} < 1$ $\frac{T_2}{T_1} > 1$ $\frac{T_{02}}{T_{01}} = 1$ $\frac{s_2}{s_1} > 1$



$\frac{V_2}{V_1} > 1$ $\frac{Ma_2}{Ma_1} > 1$ $\frac{\rho_2}{\rho_1} < 1$ $\frac{p_2}{p_1} < 1$

$\frac{p_{02}}{p_{01}} = 1$ $\frac{T_2}{T_1} < 1$ $\frac{T_{02}}{T_{01}} = 1$ $\frac{s_2}{s_1} = 1$

Strömungslehre II

Formelsammlung

Der Fachschaft zur Verfügung gestellt von Robert John

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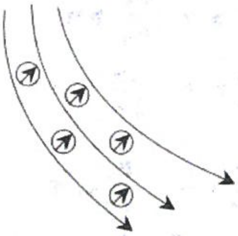
Wirbeltheorie

Drehungsfrei

$\operatorname{rot}(\underline{v}_1) = 0$ | $\operatorname{div}(\rho\underline{v}_1) \neq 0$ | $\underline{v}_1 = \nabla(\phi)$

ϕ = Skalares Geschwindigkeitspotential



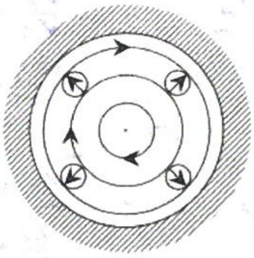
Volumenstrom

$$Q = \underline{v} \cdot \underline{A}$$

Drehungsbehaftet

$\operatorname{rot}(\underline{v}_2) \neq 0$ | $\operatorname{div}(\rho\underline{v}_2) = 0$ | $\rho\underline{v}_2 = \rho_b \operatorname{rot}(\psi)$

ψ = Vektorielles Wirbelpotential



Wirbelstrom

$$\underline{\omega} \cdot \underline{A}$$

Wirbelstärke

$$\underline{\omega} = \frac{1}{2}\operatorname{rot}(\underline{v})$$

Zirkulation $(+ : \circlearrowleft)$

$$\Gamma = \oint_S \underline{v} \cdot d\underline{s} = \oint_S v\cos(\alpha)ds$$ (7.20

$$= \oiint_A \operatorname{rot}(\underline{v}) \cdot \underline{n}dA = \oiint_A \underline{\Omega} \cdot \underline{n}dA$$

(7.20b) (

Rechte-Han

Laplace-Operator

$$\Delta(\underline{\psi}) = \begin{pmatrix} \frac{\partial^2\psi_x}{\partial x^2} + \frac{\partial^2\psi_x}{\partial y^2} + \frac{\partial^2\psi_x}{\partial z^2} \\ \frac{\partial^2\psi_y}{\partial x^2} + \frac{\partial^2\psi_y}{\partial y^2} + \frac{\partial^2\psi_y}{\partial z^2} \\ \frac{\partial^2\psi_z}{\partial x^2} + \frac{\partial^2\psi_z}{\partial y^2} + \frac{\partial^2\psi_z}{\partial z^2} \end{pmatrix}$$ (7.1

Rotation

Allgemein

(7.11)

$$\underline{\Omega} = 2\underline{\omega} = \operatorname{rot}(\underline{v})$$

Dichtebeständig

Allgemein

$$\underline{\Omega} = \nabla(\operatorname{div}(\underline{\psi})) - \Delta(\underline{\psi})$$

(7.12)

Ebene Platte

$$\Omega_z = -\Delta(\psi_z)$$

(7.14)

Kartesisch

$$\underline{\Omega} = \begin{pmatrix} \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \end{pmatrix}$$

Zylindrisch

$$\underline{\Omega} = \begin{pmatrix} \frac{1}{r}\frac{\partial v_z}{\partial \varphi} - \frac{\partial v_\varphi}{\partial z} \\ \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \\ \frac{1}{r}\left(\frac{\partial}{\partial r}(r \cdot v_\varphi) - \frac{\partial v_r}{\partial \varphi}\right) \end{pmatrix}$$ (7.16

Wirbelsätze

1. Helmholtzscher Wirbelsatz

$$\operatorname{div}(\underline{\omega}) = \operatorname{div}(\underline{\Omega}) = 0$$ (7.22)

Wirbeltransportgleichung

Allgemein

$$\frac{D}{Dt}(\underline{\omega}) = \underline{\omega} \cdot \nabla(\underline{v}) + \nu\Delta(\underline{\omega})$$ (7.31)

2D/Rotationssymmetrisch

$$\frac{D\omega}{Dt} = \frac{\partial\omega}{\partial t} + \underline{v} \cdot \nabla(\omega) = \nu\Delta(\omega)$$ (7.32a)

Homogenes Fluid | Reibungsbehaftet | U Existiert

2. Helmholtzscher Wirbelsatz

Reibungsfrei & Barotrop

$$\frac{D}{Dt}\left(\frac{\underline{\omega}}{\rho}\right) = \frac{\underline{\omega}}{\rho} \cdot \nabla(\underline{v})$$ (7.29b)

2D/Rotationssym. & Inkompr.

$$\frac{D\omega}{Dt} = \frac{\partial\omega}{\partial t} + \underline{v} \cdot \nabla(\omega) = 0$$

aus (7.30)

Satz von Thomson/Kelvin

$$\frac{D\Gamma}{Dt} = -\oint_{S(t)} \frac{dp}{\rho}$$

Reibungsfrei

Barotrop/Inkompressibel

U Existiert

Biot-Savart Gesetz $(+ : \circlearrowleft)$ RHR !

Vektorform

$$d\underline{v} = \frac{\Gamma}{4\pi}\frac{\underline{a} \times d\underline{s}}{a^3}$$

Betragsform

$$dv = \frac{\Gamma}{4\pi}\frac{\sin(\alpha)ds}{a^2}$$

Einseitig Unendlich

$$v = \frac{\Gamma}{4\pi r}(1 + \cos(\alpha_1))$$

$$\alpha_2 = \pi$$

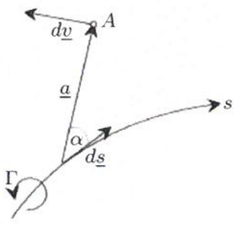
Gerade Wirbellinie

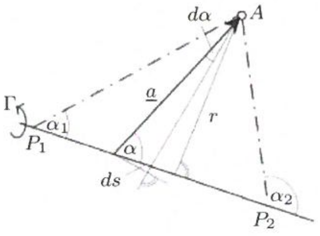
$$dv = \frac{\Gamma}{4\pi r}\sin(\alpha)d\alpha$$

Beidseitig Unendlich

$$v = \frac{\Gamma}{2\pi r}$$

$$\alpha_1 = 0$$





Arctan2

$$\operatorname{arctan2}(x,y) = \begin{cases} \arctan\left(\frac{y}{x}\right) & \text{für } x > 0 & \textbf{I,IV} \\ \arctan\left(\frac{y}{x}\right) + \pi & \text{für } x < 0, y > 0 & \textbf{II} \\ \pm\pi & \text{für } x < 0, y = 0 & \\ \arctan\left(\frac{y}{x}\right) - \pi & \text{für } x < 0, y < 0 & \textbf{III} \\ +\frac{\pi}{2} & \text{für } x = 0, y > 0 & \\ -\frac{\pi}{2} & \text{für } x = 0, y < 0 & \end{cases}$$

$$\text{arctan2(x,-y)} = -\operatorname{arctan2}(x,y)$$

$$\operatorname{arctan2}(-x,y) = \begin{cases} -\operatorname{arctan2}(x,y) + \operatorname{sign}(y)\pi & \text{für } y \neq 0 \\ 0 & \text{für } y = 0 \land x < 0 \\ \pm\pi & \text{für } y = 0 \land x > 0 \end{cases}$$

Stromlinie: $\Psi = \text{konst.}$

Isotache: $|\underline{v}| = \text{konst.}$

(Bernoulli)

Isobare: $p + \frac{\rho}{2}\underline{v}^2 + \rho \cdot g \cdot z = p_0 = p_\infty$; $\rho = \text{konst.}$

Isokline: Steigung $\tan(\alpha) = \frac{v}{u} = \text{konst.} \rightarrow v = \tan(\alpha) \cdot u$
$= k_\alpha \cdot u$

Potentialströmungen

Geschwindigkeitspotential

Definiton

$$\underline{v} = \nabla(\phi) \quad (7.36)$$

Inkompressibel

$$\Delta(\phi) = 0 \quad (7.40b)$$

Grundgleichung

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla(\phi))^2 + \frac{p}{\rho} + U = F(t) \quad (7.44)$$

Zirkulation

$$\Gamma_{1\rightarrow 2} = \int_1^2 \underline{v} \cdot d\underline{s} = \phi_2 - \phi_1 \quad (7.39$$

Kartesisch

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (7.51)$$

Zylinderkoordinaten

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial r^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

Randbedingungen

Feste Wand

$$\frac{\partial \phi}{\partial n} = 0 \quad (7.46)$$

Bewegte Wand

$$\frac{\partial \phi}{\partial n} = \underline{u} \cdot \underline{n} \quad (7.47)$$

Im Unendlichen

$$\nabla(\phi) = \underline{v}_\infty$$

aus (7.50)

Ebene Geschwindigkeiten (7.37)

Kartesisch

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Polarkoordinaten

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \phi}$$

$$v_\varphi = \frac{1}{r}\frac{\partial \phi}{\partial \phi} = -\frac{\partial \psi}{\partial r}$$

Geschwindigkeitsbeziehungen (7.54b)

Eben (Cauchy-Riemann)

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Rotationssymmetrisch

$$v_r = \frac{\partial \phi}{\partial r} = -\frac{1}{r}\frac{\partial \psi}{\partial z}$$

$$v_z = \frac{\partial \phi}{\partial z} = \frac{1}{r}\frac{\partial \psi}{\partial r}$$

Resultierende Geschw.

$$w = \frac{\partial \psi}{\partial n} = \sqrt{u^2 + v^2} = \sqrt{v_r^2 + v_\varphi^2}$$

Massendurchsatz

mittl. geschw.

$$\dot{m}_{1\rightarrow 2} = \rho t \int_1^2 d\psi = \rho t\,(\psi_2 - \psi_1)$$

$$\frac{\dot{V}}{t} = |\psi_2(x,y) - \psi_1(x,y)| = b \cdot \langle v \rangle$$

tiefe

Breite

Potentiallinie

$$\phi = \text{konst.} \qquad \frac{dy}{dx} = -\frac{u}{v}$$

Stromlinie

$$\psi = \text{konst.} \qquad \frac{dy}{dx} = \frac{v}{u}$$

$\nabla(\phi) \cdot \nabla(\psi) = 0 \Rightarrow$ Potentiallinie $\perp$ Stromlinie (7.58)

Komplexe Strömung

Komplexes Geschwindigkeitspotential

$$F(z) = F(x+iy) = \phi(x,y) + i\psi(x,y) \quad (7.68)$$

Geschwindigkeiten

$$w = u + iv \quad (7.66)$$

$$w_* = u - iv = \frac{dF(z)}{dz} \quad (7.69)$$

Druckverteilung

$$p = p_0 - \frac{\rho}{2}a^2 r^{2(n-1)} \quad (7.74)$$

Ebene Translationsströmung

$$F(z) = a \cdot z = (a_1 - ia_2) \cdot z$$

$$\phi = a_1 x + a_2 y$$

$$\psi = a_1 y - a_2 x$$

$$u = a_1$$

$$v = a_2$$

Ebene Winkel- & Eckenströmung

$$F(z) = \frac{a}{n}z^n = \frac{a}{n}r^n(\cos(n\phi) + i\sin(n\phi)) \quad (7$$

$$\phi = \frac{a}{n}r^n\cos(n\phi)$$

$$\psi = \frac{a}{n}r^n\sin(n\phi) \quad (7.71)$$

$$|w| = |az^{n-1}| = |a|r^{n-1} \quad (7.72)$$

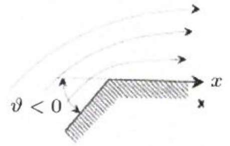
Feste Wände

$$\phi_{k,n} = k\frac{\pi}{n}$$

$$\varepsilon = \frac{\pi}{n}$$

$$\vartheta = \pi - \varepsilon = \pi\frac{n-1}{n}$$

$\frac{1}{2} < n < 1$

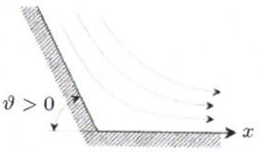


Konvexe Ecke

$$|w| \sim \frac{1}{r^{n-1}}$$

$r = 0 \Rightarrow$ Eckenumströmung mit $|w| = \infty$

$1 < n < 2$

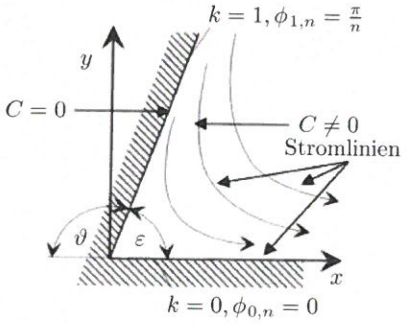


Konkave Ecke

$$|w| \sim r^{n-1}$$

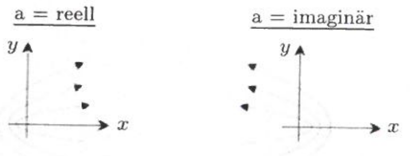
$r = 0 \Rightarrow$ Staupunkt

$n > 2$



$n = \frac{1}{2}$

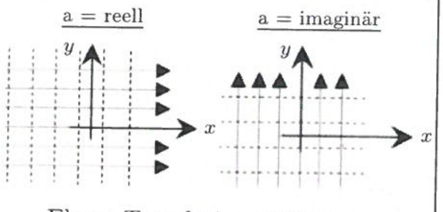
$$F(z) = 2a\sqrt{z} = 2a\sqrt{r}\left(\cos\left(\frac{\phi}{2}\right) + i\sin\left(\frac{\phi}{2}\right)\right)$$


a = reell
a = imaginär

Ebene Randumströmung

$n = 1$

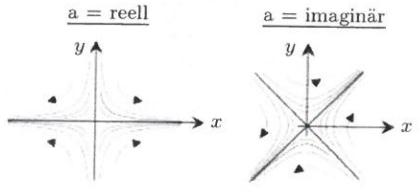
$$F(z) = a \cdot z$$


a = reell
a = imaginär

Ebene Translationsströmung

$n = 2$

$$F(z) = \frac{a}{2}z^2$$


a = reell
a = imaginär

Ebene Staupunktströmung

Stromfkt. ist Imaginärteil der komplexen Potentialfkt

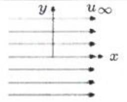
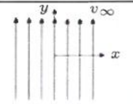
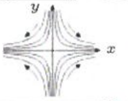
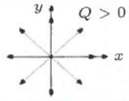
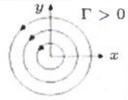
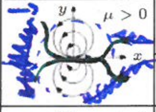
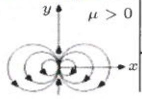
Potentialströmungen

Komplexe Strömung

Singularitätentabelle

Für Konturlinie

$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi}$ $v_\varphi = -\frac{\partial\psi}{\partial r}$

Bezeichnung	Stromlinienbild	Komplexe Potentialfunktion	Skalare Potentialfunktion	Skalare Stromfunktion	Geschwindigkeitskomponenten u	v	$=\frac{\partial\phi}{\partial r}$	$=\frac{1}{r}\frac{\partial\phi}{\partial\varphi}$
		$F(z)$	$\phi(x,y)$	$\psi(x,y)$	$v_x(x,y)$	$v_y(x,y)$	$v_r(x,y)$	$v_\varphi(x,y)$
		$F(r,\phi)$	$\phi(r,\phi)$	$\psi(r,\phi)$	$v_x(r,\phi)$	$v_y(r,\phi)$	$v_r(r,\phi)$	$v_\varphi(r,\phi)$
Translationsströmung in x-Richtung		$u_\infty z$	$u_\infty x$	$u_\infty y$	u_∞	0	$u_\infty\frac{x}{\sqrt{x^2+y^2}}$	$-u_\infty\frac{y}{\sqrt{x^2+y^2}}$
		$u_\infty re^{i\varphi}$	$u_\infty r\cos(\phi)$	$u_\infty r\sin(\phi)$	u_∞	0	$u_\infty\cos(\phi)$	$-u_\infty\sin(\phi)$
Translationsströmung in y-Richtung		$-iv_\infty z$	$v_\infty y$	$-v_\infty x$	0	v_∞	$v_\infty\frac{y}{\sqrt{x^2+y^2}}$	$v_\infty\frac{x}{\sqrt{x^2+y^2}}$
		$-iv_\infty re^{i\varphi}$	$v_\infty r\sin(\phi)$	$-v_\infty r\cos(\phi)$	0	v_∞	$v_\infty\sin(\phi)$	$v_\infty\cos(\phi)$
Staupunkt-, Eckenströmung $a\in\mathbb{R}^+$		$\frac{a}{2}z^2$	$\frac{a}{2}\left(x^2-y^2\right)$	axy	ax	$-ay$	$a\frac{x^2-y^2}{\sqrt{x^2+y^2}}$	$-a\frac{2xy}{\sqrt{x^2+y^2}}$
		$\frac{a}{2}r^2e^{i2\varphi}$	$\frac{a}{2}r^2\cos(2\phi)$	$\frac{a}{2}r^2\sin(2\phi)$	$ar\cos(\phi)$	$-ar\sin(\phi)$	$ar\cos(2\phi)$	$-ar\sin(2\phi)$
Quelle, Senke Ergiebigkeit $Q\neq 0$		$\frac{Q}{2\pi}\ln(z)$	$\frac{Q}{2\pi}\ln\left(\sqrt{x^2+y^2}\right)$	$\frac{Q}{2\pi}\arctan\left(\frac{y}{x}\right)$	$\frac{Q}{2\pi}\frac{x}{x^2+y^2}$	$\frac{Q}{2\pi}\frac{y}{x^2+y^2}$	$\frac{Q}{2\pi}\frac{1}{\sqrt{x^2+y^2}}$	0
		$\frac{Q}{2\pi}\left(\ln(r)+i\phi\right)$	$\frac{Q}{2\pi}\ln(r)$	$\frac{Q}{2\pi}\phi$	$\frac{Q}{2\pi}\frac{\cos(\varphi)}{r}$	$\frac{Q}{2\pi}\frac{\sin(\varphi)}{r}$	$\frac{Q}{2\pi}\frac{1}{r}$	0
Potentialwirbel Zirkulation $\Gamma\neq 0$		$-i\frac{\Gamma}{2\pi}\ln(z)$	$\frac{\Gamma}{2\pi}\arctan\left(\frac{y}{x}\right)$	$-\frac{\Gamma}{2\pi}\ln\left(\sqrt{x^2+y^2}\right)$	$-\frac{\Gamma}{2\pi}\frac{y}{x^2+y^2}$	$\frac{\Gamma}{2\pi}\frac{x}{x^2+y^2}$	0	$\frac{\Gamma}{2\pi}\frac{1}{\sqrt{x^2+y^2}}$
		$-\frac{\Gamma}{2\pi}\left(i\ln(r)-\phi\right)$	$\frac{\Gamma}{2\pi}\phi$	$-\frac{\Gamma}{2\pi}\ln(r)$	$-\frac{\Gamma}{2\pi}\frac{\sin(\varphi)}{r}$	$\frac{\Gamma}{2\pi}\frac{\cos(\varphi)}{r}$	0	$\frac{\Gamma}{2\pi}\frac{1}{r}$
Dipol Dipolachse: x-Achse		$-\frac{\mu}{2\pi}\frac{1}{z}$	$-\frac{\mu}{2\pi}\frac{x}{x^2+y^2}$	$\frac{\mu}{2\pi}\frac{y}{x^2+y^2}$	$\frac{\mu}{2\pi}\frac{x^2-y^2}{\left(x^2+y^2\right)^2}$	$\frac{\mu}{2\pi}\frac{2xy}{\left(x^2+y^2\right)^2}$	$\frac{\mu}{2\pi}\frac{x}{\sqrt{\left(x^2+y^2\right)^3}}$	$\frac{\mu}{2\pi}\frac{y}{\sqrt{\left(x^2+y^2\right)^3}}$
		$-\frac{\mu}{2\pi}\frac{1}{r}e^{-i\varphi}$	$-\frac{\mu}{2\pi}\frac{\cos(\varphi)}{r}$	$\frac{\mu}{2\pi}\frac{\sin(\varphi)}{r}$	$\frac{\mu}{2\pi}\frac{\cos(2\varphi)}{r^2}$	$\frac{\mu}{2\pi}\frac{\sin(2\varphi)}{r^2}$	$\frac{\mu}{2\pi}\frac{\cos(\varphi)}{r^2}$	$\frac{\mu}{2\pi}\frac{\sin(\varphi)}{r^2}$
Dipol Dipolachse: y-Achse		$-i\frac{\mu}{2\pi}\frac{1}{z}$	$-\frac{\mu}{2\pi}\frac{y}{x^2+y^2}$	$-\frac{\mu}{2\pi}\frac{x}{x^2+y^2}$	$\frac{\mu}{2\pi}\frac{2xy}{\left(x^2+y^2\right)^2}$	$-\frac{\mu}{2\pi}\frac{x^2-y^2}{\left(x^2+y^2\right)^2}$	$\frac{\mu}{2\pi}\frac{y}{\sqrt{\left(x^2+y^2\right)^3}}$	$-\frac{\mu}{2\pi}\frac{x}{\sqrt{\left(x^2+y^2\right)^3}}$
		$-i\frac{\mu}{2\pi}\frac{1}{r}e^{-i\varphi}$	$-\frac{\mu}{2\pi}\frac{\sin(\varphi)}{r}$	$-\frac{\mu}{2\pi}\frac{\cos(\varphi)}{r}$	$\frac{\mu}{2\pi}\frac{\sin(2\varphi)}{r^2}$	$-\frac{\mu}{2\pi}\frac{\cos(2\varphi)}{r^2}$	$\frac{\mu}{2\pi}\frac{\sin(\varphi)}{r^2}$	$-\frac{\mu}{2\pi}\frac{\cos(\varphi)}{r^2}$

$w'(z) = \frac{dF(z)}{dz}$

$w'(z_{sp}) = 0$

$$\boxed{M = -\mu\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]}$$ VZW

$$E = Q\left[\frac{\mathrm{m}^2}{\mathrm{s}}\right]$$

Potentialwirbel im Uhrzeigersinn an Profilspitze um Auftrieb über Druckunterschied zu simulieren.

Kreiszylinder

Kutta-Joukowskischer Auftriebssatz

$$F_y = F_A = b\rho u_\infty\Gamma \qquad (7.98)$$

⇒ Nur Wirbel trägt zum Auftrieb bei

Druckverteilung

$$c_p(\phi) = \frac{p_w(\phi)-p_\infty}{\frac{\varrho}{2}u_\infty^2} = 1-\left(\frac{v_{\varphi w}}{u_\infty}\right)^2 = 1-\left(4\sin^2(\phi)+\frac{2\Gamma\sin(\phi)}{\pi a u_\infty}+\left(\frac{\Gamma}{2\pi a u_\infty}\right)^2\right)$$

$c_{p,max} = 1$ | $c_{p,min} = -3$

Inkompressibel

Strömungsbild Bestimmen

$$\boxed{\psi(x,y)}$$

↓

$$\boxed{u(x,y), v(x,y)}$$

↓

$$\boxed{\left.\begin{array}{l} u = 0 \\ v = 0 \end{array}\right\} \Rightarrow \text{Staupunkte } x_{0,i}, y_{0,i}}$$

↓

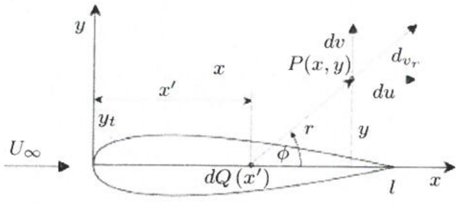
$$\boxed{\psi_{0,i} = \psi(x_{0,i}, y_{0,i})}$$

↓

$$\boxed{\psi_{S,i}(x,y) = \psi_{0,i}}$$

Tropfentheorie

Geschwindigkeiten



$$U(x,y) = U_\infty + u(x,y)$$

Allgemein (7.109)

$$U(x,y) = U_\infty + \frac{1}{2\pi} \int_0^l q(x') \frac{x - x'}{(x-x')^2 + y^2} dx'$$

$$V(x,y) = \frac{1}{2\pi} \int_0^l q(x') \frac{y}{(x-x')^2 + y^2} dx'$$

Dünnes Profil (Körperoberfläche)

$$U(x) = U_\infty \left(1 + \frac{1}{\pi} \oint_0^l \frac{\frac{dy_t}{dx}(x')}{x - x'} dx' \right) \quad \text{(7.1}$$

$$V(x) = \frac{q(x)}{2} \qquad (7.117)$$

Kinematische Verträg.

$$\frac{dy_t}{dx} = \frac{V(x)}{U(x)} \qquad (7.112)$$

Druckverteilung

$$c_p = 1 - \frac{U^2 + V^2}{U_\infty^2}$$

(7.113)

Quell- & Senkenverteilung

Allgemein (7.110)

$$q(x) = 2 \frac{d}{dx} \left((U_\infty + u)\, y_t \right)$$

Dünnes Profil (7.114

$$q(x) = 2U_\infty \frac{dy_t(x)}{dx}$$

$U_{Abs} = \sqrt{U^2 + V^2}$

Riegels-Korrektur

Druckverteilung

$C_{p,max} = 1$

$$c_p = 1 - \left(\frac{U_k}{U_\infty} \right)^2$$

(7.121)

Definition (7.119)

$$\kappa(x) = \frac{1}{\cos(\theta)} = \sqrt{1 + \tan^2(\theta)} = \sqrt{1 + \left(\frac{dy_t}{dx} \right)^2}$$

Geschwindigkeit (7.120)

$$U_k(x) = \frac{U}{\kappa(x)} = \frac{U_\infty}{\kappa(x)} \left(1 + \frac{1}{\pi} \oint_0^l \frac{\frac{dy_t}{dx}(x')}{x - x'} dx' \right)$$

Fourier-Ansatz

Definition

(7.124)

$$Y_t = \frac{1}{2} \sum_{\nu=1}^{n} b_\nu \sin(\nu\phi)$$

(7.125)

$$X = \frac{1}{2}(1 + \cos(\phi))$$

Geschwindigkeit (7.129)

$$U_k(\phi) = \frac{U_\infty}{\kappa(\phi)} \left(1 + \sum_{\nu=1}^{n} \nu b_\nu \frac{\sin(\nu\phi)}{\sin(\phi)} \right)$$

Riegels-Faktor (7.130)

$$\kappa(\phi) = \sqrt{1 + \left(\frac{dY_t}{dX} \right)^2} = \sqrt{1 + \left(\frac{dY_t}{d\phi} \cdot \frac{d\phi}{dX} \right)^2}$$

Bernoulli: $P_\infty + \frac{\rho}{2} U_\infty^2 = P_s + \frac{\rho}{2} U_s^2$

Skeletttheorie

Geschwindigkeiten

Allgemein (7.143)

$$u(x,y) = \frac{1}{2\pi} \int\limits_0^l \gamma(x') \frac{y - y_s(x')}{(x-x')^2 + \left(y - y_s(x')^2\right) dx'}$$

$$v(x,y) = -\frac{1}{2\pi} \int\limits_0^l \gamma(x') \frac{x - x'}{(x-x')^2 + \left(y - y_s(x')^2\right) dx'}$$

Wirbel Auf x-Achse (7.147)

Gesamt

$$U(X) = U_\infty + \frac{\gamma(X)}{2} \operatorname{sign}(Y - Y_s)$$

$$V(X) = U_\infty \alpha - \frac{1}{2\pi} \oint\limits_0^1 \frac{\gamma(X')}{X - X'} dX'$$

Induziert In y-Richtung

$$v(X) = -\frac{1}{2\pi} \oint\limits_0^1 \frac{\gamma(X')}{X - X'} dX'$$

Auftrieb

$$A = \rho U_\infty b \Gamma$$

(7.150)

Moment

$$M = -b\rho U_\infty \int\limits_0^l \gamma(x) x dx$$

(7.153a)

Druck

Oberseite

$$p_o(x) = p_\infty - \frac{\rho}{2} U_\infty \gamma(x)$$

(7.149a)

Unterseite

$$p_u(x) = p_\infty + \frac{\rho}{2} U_\infty \gamma(x)$$

(7.149b)

Druckbeiwert

$$\Delta C_p = 2 \frac{\gamma(x)}{U_\infty}$$

(7.152)

Auftriebsbeiwert (7.151)

$$C_a = \frac{2}{U_\infty} \int\limits_0^1 \gamma(X) dX = \int\limits_0^1 \Delta C_p(X) dX$$

Momentenbeiwert (7.153b)

$$C_m = -\frac{2}{U_\infty} \int\limits_0^1 \gamma(X) X dX = -\int\limits_0^1 \Delta C_p(X) X dX$$

Skelettlinie

$$Y_s(X) = \alpha X - \frac{1}{2\pi} \int\limits_0^1 \frac{\gamma(X')}{U_\infty} \ln\left|\frac{X - X'}{X'}\right| dX' \tag{7.155}$$

$$\alpha = \frac{1}{2\pi} \int\limits_0^1 \frac{\gamma(X')}{U_\infty} \ln\left|\frac{1 - X'}{X'}\right| dX' \tag{7.156}$$

Ansatz Von Birnbaum-Ackermann & Glauert

Zirkulationsverteilung

(7.157)

$$\gamma(\phi) = 2U_\infty \left(A_0 \tan\left(\frac{\phi}{2}\right) + \sum_{n=1}^{\infty} A_n \sin(n\phi)\right)$$

(7.163)

$$A_0 = \alpha - \frac{1}{\pi} \int\limits_0^\pi \frac{dY_s(X)}{dX} d\phi$$

(7.163)

$$A_n = -\frac{2}{\pi} \int\limits_0^\pi \frac{dY_s(X)}{dX} \cos(n\phi) d\phi$$

Induzierte Geschwindigkeit (7.161)

$$\frac{v(\phi)}{U_\infty} = -\left(A_0 + \sum_{n=1}^{N} A_n \cos(n\phi)\right)$$

Kinematische Strömungsbedingung

$$\alpha - \frac{dY_s(X)}{dX} = A_0 + \sum_{n=1}^{N} A_n \cos(n\phi)$$

Auftriebsbeiwert

$$C_a = \pi(2A_0 + A_1)$$

(7.164 / 7.165)

Momentenbeiwert

$$C_m = -\frac{\pi}{4}(2A_0 + 2A_1 + A_2)$$

(7.166)

Druckpunktslage

(7.167)

$$\frac{x_A}{l} = -\frac{C_m}{C_a}$$

Überlagerung

$$\frac{U}{U_\infty} = 1 + \left(\frac{U_1}{U_\infty} - 1\right) + \left(\frac{U_2}{U_\infty} - 1\right) + \left(\frac{U_3}{U_\infty} - 1\right)$$

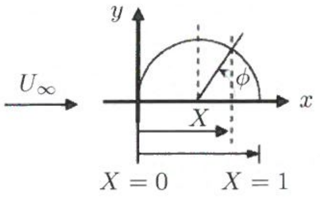
Bed.: $\frac{\alpha}{\pi}, \frac{f}{l}, \frac{d}{l} \ll 1$

Skelettlinie

$$Y_s(X) = \alpha X - A_0 X - \int\limits_0^X \sum_{n=1}^{N} A_n \cos(n\phi) dX \tag{7.162b}$$

Transformation

$$X = \frac{1}{2}(1 + \cos(\phi)) \tag{7.158a}$$



Energiesatz

Thermische Zustandsgleichung

$$\boxed{pV = mRT}$$

$$\boxed{p = \rho RT}$$ (1.5)

Enthalpie

$$h = e + pv = c_pT$$ (8.9) (8.23d)

Innere Energie

$$e = c_vT$$ (8.23c)

Entropie

$$ds = \frac{d_{q,rev}}{T} + ds_{irr}$$ (8.1

1. Hauptsatz

(8.1) (8.11)

$$de = dq + dw = Tds - pdv$$

$$dq = dh - vdp$$

Maxwell-Beziehung

$$\frac{\partial e}{\partial v}\bigg|_T = T\frac{\partial p}{\partial T}\bigg|_v - p$$ (8.14)

Spezifische Wärmekapazität

Konstantes Volumen

$$c_v = \frac{\partial e}{\partial T}\bigg|_v = \frac{R}{\kappa - 1}$$ (8.21a) (8.23b)

Konstanter Druck

$$c_p = \frac{\partial h}{\partial T}\bigg|_p = R\frac{\kappa}{\kappa - 1}$$ (8.21b) (8.23a)

Zusammenhang

$$R = c_p - c_v$$ (8.22)

Isentropenexponent

$$\kappa = \frac{c_p}{c_v}$$ (8.23e)

Einatoming: $\kappa = \frac{5}{3}$

Zweiatoming: $\kappa = \frac{7}{5}$

Entropieänderung

$$\Delta s = R\ln\left(\frac{\rho_1}{\rho 2}\left(\frac{T_2}{T_1}\right)^{\frac{1}{\kappa-1}}\right)$$

Isentropenbeziehungen

$$\frac{p}{\rho^\kappa} = \text{konst.}$$ (9.18)

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\kappa-1}}$$ (8.28a)

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\kappa = \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}}$$ (8.28b)

Energiegleichung

Nicht-Konservativ (8.39)

$$\rho\frac{D}{Dt}\left(e + \frac{V^2}{2}\right) = \rho\dot{q} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - \frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z}$$
$$+ \frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{xy})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(v\tau_{zy})}{\partial z} + \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial(w\tau_{yz})}{\partial y} + \frac{\partial(w\tau_{zz})}{\partial z} + \rho\underline{f}\cdot\underline{V}$$

Konservativ (8.51)

$$\frac{\partial}{\partial t}\left(\rho\left(e + \frac{V^2}{2}\right)\right) + \nabla\cdot\left(\rho\left(e + \frac{V^2}{2}\right)\underline{V}\right) = \rho\dot{q} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - \frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z}$$
$$+ \frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{xy})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(v\tau_{zy})}{\partial z} + \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial(w\tau_{yz})}{\partial y} + \frac{\partial(w\tau_{zz})}{\partial z} + \rho\underline{f}\cdot\underline{V}$$

Energiegleichung Der Laminaren Grenzschicht

Allgemein (8.77)

$$\rho u\frac{\partial}{\partial x}\left(c_pT + \frac{u^2}{2}\right) + \rho v\frac{\partial}{\partial y}\left(c_pT + \frac{u^2}{2}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\eta u\frac{\partial u}{\partial y}\right)$$

Pr=1

Definition (8.78)

$$\rho u\frac{\partial}{\partial x}\left(c_pT + \frac{u^2}{2}\right) + \rho v\frac{\partial}{\partial y}\left(c_pT + \frac{u^2}{2}\right) = \frac{\partial}{\partial y}\left(\eta\frac{\partial}{\partial y}\left(c_pT + \frac{u^2}{2}\right)\right)$$

$\frac{dp}{dx}$ Beliebig (8.79)

$$T + \frac{u^2}{2c_p} = T_{0_\infty} \Rightarrow r = 1$$

$\frac{dp}{dx} = 0$ (8.84)

$$T(u) = T_W + \frac{u}{u_e}(T_{0_e} - T_W) - \underbrace{\frac{u_e^2}{2c_p}\left(\frac{u}{u_e}\right)^2}_{\approx 0 \text{ für } Ma \ll 1}$$

Recovery-Faktor

(8.76a)

$$r = \frac{2}{(\kappa-1)Ma_e^2}\left(\frac{T_{aw}}{T_e} - 1\right) \approx \begin{cases}\sqrt{\text{Pr}} & \text{für laminare Grenzschicht}\\ \sqrt[3]{\text{Pr}} & \text{für turbulente Grenzschicht}\end{cases}$$

Recovery-Temperatur

$$T_r = T_e + r\frac{u_e^2}{2c_p}$$ (8.81)

Prandtl-Zahl

$$\text{Pr} = \frac{\eta c_p}{k}$$

Reynolds-Analogie

(8.89) Definition

$$\dot{q}_W = c_p\frac{T_r - T_W}{u_e}\tau_W$$

Stanton-Zahl

(8.91) (8.92, 8.93)

$$\text{St} = \frac{\dot{q}_W}{\rho_e u_e c_p (T_r - T_W)} \approx \begin{cases}\frac{c_f}{2} & \text{für Pr=1}\\ \frac{c_f}{2}\text{Pr}^{-\frac{2}{3}} & \text{~~für Pr=1~~}\end{cases}$$

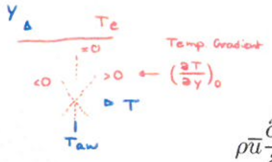
korr: für Pr > 0.5

(8.90) (8.94)

$$c_f = \frac{2\tau_W}{\rho_e u_e^2} = \frac{0,664}{\sqrt{\text{Re}_x}}$$

(8.94)

$$\text{Re}_x = \frac{\rho u_e x}{\eta}$$



Energiegleichung Der Turbulenten Grenzschicht ($Ma \ll 1$)

Allgemein (8.104)

$$\rho\overline{u}\frac{\partial\overline{T}}{\partial x} + \rho\overline{v}\frac{\partial\overline{T}}{\partial y} = \frac{k}{c_p}\frac{\partial^2\overline{T}}{\partial y^2} - \rho\frac{\partial\left(\overline{v'T'}\right)}{\partial y}$$

recovery-Faktor:

$$r = \frac{T_{aw} - T_e}{u_e^2/2 \cdot c_p}$$

$$\frac{T_{aw}}{T_e} = 1 + r\cdot\frac{(\kappa-1)}{2}\cdot Ma_e^2$$

Lösung

$$\overline{T} = T_W + (T_{0_e} - T_W)\frac{\overline{u}}{u_e}$$ (8.106)

Gasdynamik

$1 bar = 10^5 Pa = 10^5 \frac{N}{m^2}$

Massenerhaltung

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho A V)}{\partial x} = 0$$

(9.4 b)

Impulssatz

$$\rho \frac{DV}{Dt} = -\frac{\partial p}{\partial x}$$

(9.5)

Energieerhaltung

$$\frac{Dh_0}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial t}$$

(9.8)

Entropieänderung

$$\frac{Ds}{Dt} = 0$$

(9.10)

Schallgeschwindigkeit

$$c = \sqrt{\kappa R T} = \sqrt{\kappa \frac{p}{\rho}}$$

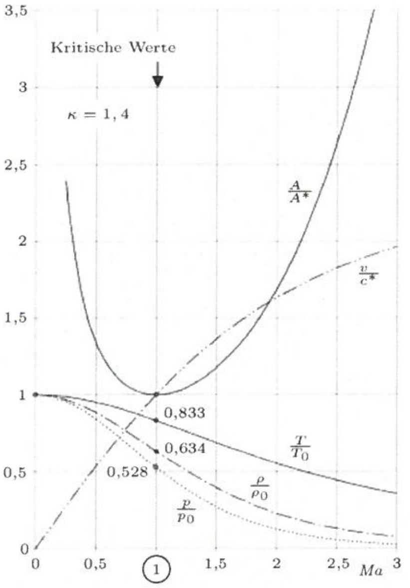
Kritisch

(9.22a)

$$c_L = c^* = \sqrt{\frac{2}{\kappa+1}} c_0 = \sqrt{2\frac{\kappa}{\kappa+1} R T_0}$$

(9.21b) !

$$\left(\frac{V}{c^*}\right)^2 = \frac{\kappa+1}{\kappa-1}\left(1 - \frac{T}{T_0}\right) = \frac{\kappa+1}{\frac{2}{Ma^2} + \kappa - 1}$$



Stationäre Stromröhre

Massenerhaltung

$$\dot{m} = \rho A V = \text{konst.}$$

(9.12b)

Impulssatz

$$\frac{V^2}{2} + \int \frac{1}{\rho} dp = \text{konst.}$$

(9.14a)

Energiegleichung

$$h_0 = h + \frac{V^2}{2} = \text{konst.}$$

(9.15b)

Adiabate Strömungsvorgänge

Größere Umlenkung im Schrägen Stoß → Größerer Totaldruckverl.

Ruhezustand

Max. Temp, bleibt vor Brennkammer gleich

$$\frac{T_0}{T} = 1 + \frac{\kappa-1}{2} Ma^2$$

(9.19)

Bleibt nach Brennkammer gleich

(9.20 a)

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\kappa}{\kappa-1}} = \left(1 + \frac{\kappa-1}{2} Ma^2\right)^{\frac{\kappa}{\kappa-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\kappa-1}} = \left(1 + \frac{\kappa-1}{2} Ma^2\right)^{\frac{1}{\kappa-1}}$$

(9.20 b)

$$h_0 = h + \frac{V^2}{2} \Leftrightarrow T_0 = T + \frac{V^2}{2c_p}$$

(8.60)

Kritischer Zustand

Mit Ma = 1:

$$\frac{T^*}{T_0} = \frac{2}{\kappa+1}$$

→ aus (9.19)

$$\frac{p^*}{p_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$$

→ aus (9.20a

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}$$

→ aus (9.20

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\frac{2}{\kappa+1}\left(1 + \frac{\kappa-1}{2} Ma^2\right)\right]^{\frac{\kappa+1}{2(\kappa-1)}}$$

B1 (9.26)

$$\dot{m} = \frac{Ma}{\left(1 + \frac{\kappa-1}{2} Ma^2\right)^{\frac{\kappa+1}{2(\kappa-1)}}} \sqrt{\frac{\kappa}{R T_0}} p_0 A = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{2(\kappa-1)}} \sqrt{\frac{\kappa}{R T_0}} p_0 A^*$$

(9.24)

Machzahl

$$Ma = \frac{V}{c}$$

Lavalzahl

(9.23 a)

$$La = \frac{V}{c^*} = \sqrt{\frac{\kappa+1}{2 + (\kappa-1) Ma^2}} Ma$$

(9.23 b)

$$L_{a_{max}} = \sqrt{\frac{\kappa+1}{\kappa-1}}$$

Luft: $L_{a_{max}} = 2,449$

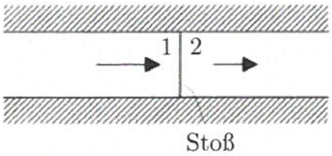
Stationäre Senkrechte Verdichtungsstöße

Erhaltungssätze

$$\rho_1 V_1 = \rho_2 V_2$$

$$\rho_1 V_1^2 + p_1 = \rho_2 V_2^2 + p_2$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$



Entropiezunahme

$$s_2 - s_1 = R \ln\left(\frac{\rho_1}{\rho_2}\left(\frac{T_2}{T_1}\right)^{\frac{1}{\kappa-1}}\right)$$

$$= R \ln\left(\frac{p_{01}}{p_{02}}\right)$$

(9.40)

Stoßbeziehungen

$$\frac{p_2}{p_1} = \frac{2\kappa Ma_1^2 - \kappa + 1}{\kappa + 1}$$

(9.38 c) f938c(P1,P2,k,Ma)

$$\frac{\rho_2}{\rho_1} = \frac{(\kappa+1) Ma_1^2}{2 + (\kappa-1) Ma_1^2}$$

(9.38 b) f938b(ρ1,ρ2,k,Ma)

(9.38 a) f938a(T1,T2,k,Ma)

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \frac{V_2^2}{V_1^2} = \frac{\left(2\kappa Ma_1^2 - \kappa + 1\right)\left(2 + (\kappa-1) Ma_1^2\right)}{(\kappa+1)^2 Ma_1^2}$$

$$Ma_2 = \sqrt{\frac{(\kappa-1)(Ma_1^2 - 1) + \kappa + 1}{2\kappa\left(Ma_1^2 - 1\right) + \kappa + 1}}$$

(9.35) f935(Ma1,Ma2,k)

$$\frac{p_{02}}{p_{01}} = \left[\frac{\left(\frac{\kappa+1}{2}\right)^{\kappa+1} Ma_1^{2\kappa}}{\left(1 + \frac{\kappa-1}{2} Ma_1^2\right)^{\kappa}\left(\kappa Ma_1^2 - \frac{\kappa-1}{2}\right)}\right]^{\frac{1}{\kappa-1}}$$

Anhang B2

(9.39 b) f939b(P01,P02,k,Ma)

Rayleigh-Pitot-Formel

$$\frac{p_{pitot}}{p_1} = \left[\frac{\left(\frac{\kappa+1}{2}\right)^{\kappa+1} Ma_1^{2\kappa}}{\left(\kappa Ma_1^2 - \frac{\kappa-1}{2}\right)}\right]^{\frac{1}{\kappa-1}}$$

(9.41) f941(P1,Ppitot,k,Ma)

Rankine-Hugoniot-Beziehung

$$e_2 - e_1 = \frac{p_1 + p_2}{2}(v_1 - v_2)$$

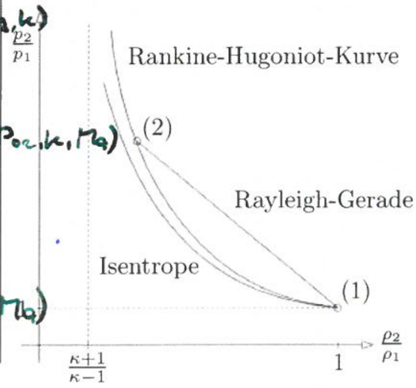
(9.5

$$\frac{p_2}{p_1} = \frac{(\kappa+1)\rho_2 - (\kappa-1)\rho_1}{(\kappa+1)\rho_1 - (\kappa-1)\rho_2}$$

(9.5

Rayleigh-Gerade

$$\frac{p_2 - p_1}{v_2 - v_1} = -\left(\frac{u_1}{v_1}\right)^2$$



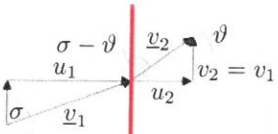
Gasdynamik

Antrieb, Widerstand
Beispiel: H2O, Aufgabe 2c)

Schräge Verdichtungsstöße

Stoßbeziehungen

"Statischer Druck"



(aus 9.43)

$$Ma_{1_n} = Ma_1 \sin(\sigma)$$

$$Ma_{2_n} = Ma_2 \sin(\sigma - \theta)$$

(9.47a) f947a(P1,P2,k,Ma,σ)

$$\frac{p_2}{p_1} = 1 + \frac{2\kappa}{\kappa+1}\left(Ma_1^2 \sin^2(\sigma) - 1\right)$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\kappa+1)Ma_1^2\sin^2(\sigma)}{2+(\kappa-1)Ma_1^2\sin^2(\sigma)}$$

(9.44) f944(ρ1,ρ2,k,Ma,σ)

(9.74b mit 9.47a und 9.44)

$$\frac{T_2}{T_1} = \frac{V_2^2}{V_1^2} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} = \frac{\left(2\kappa Ma_1^2\sin^2(\sigma) - \kappa + 1\right)\left(2+(\kappa-1)Ma_1^2\sin^2(\sigma)\right)}{(\kappa+1)^2 Ma_1^2 \sin^2(\sigma)}$$

~~[illegible]~~(T1,T2,k,Ma,σ) f944

$$\Delta = 1 - \frac{1 - p_{02}/p_{00}}{1 - p_{01}/p_{00}}$$

f959(Ma1,Ma2,k,σ,ϑ)
(9.59)
XLVii

$$\frac{Ma_1}{Ma_2} = \left[\frac{1+\frac{2\kappa}{\kappa+1}\left(Ma_1^2\sin^2(\sigma)-1\right)}{1-\frac{2}{\kappa+1}\left(1-\frac{1}{Ma_1^2\sin^2(\sigma)}\right)}\right]^{\frac{1}{2}} \frac{\sin(\sigma-\vartheta)}{\sin(\sigma)}$$

$\frac{p_{04}}{p_{01}} = \frac{p_{04}}{p_{03}} \cdot \frac{p_{03}}{p_{02}} \cdot \frac{p_{02}}{p_{01}}$; Maximaler Druckverlust für $\sigma = 90°$

(9.58)
XLVi

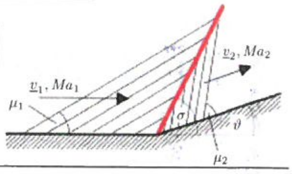
$$\frac{p_{02}}{p_{01}} = \left[\frac{\left(\frac{\kappa+1}{2}\right)^{\kappa+1}\left(Ma_1\sin(\sigma)\right)^{2\kappa}}{\left(1+\frac{\kappa-1}{2}\left(Ma_1\sin(\sigma)\right)^2\right)^{\kappa}\left(\kappa\left(Ma_1\sin(\sigma)\right)^2 - \frac{\kappa-1}{2}\right)}\right]^{\frac{1}{\kappa-1}}$$

f958(P01,P02,k,Ma,σ)

Machsche Welle

(9.67)

$$\mu = \arcsin\left(\frac{1}{Ma}\right) \leq \sigma \leq \frac{\pi}{2}$$



Umlenkwinkel

f949(k,Ma,σ,ϑ)
(9.49) ~~[illegible]~~
XLV

$$\tan(\vartheta) = \cot(\sigma)\frac{Ma_1^2\sin^2(\sigma)-1}{1+\left(\frac{\kappa+1}{2}-\sin^2(\sigma)\right)Ma_1^2}$$

Entropiezunahme (Ideales Gas)

(9.47c)

$$s_2 - s_1 = c_v \ln\left(\frac{p_2}{p_1}\left(\frac{\rho_1}{\rho_2}\right)^{\kappa}\right)$$

Schräge Stoßexpansion

(9.77a) f977a(T1,T2,k,Ma1,Ma2)

$$\frac{T_2}{T_1} = \frac{1+\frac{\kappa-1}{2}Ma_1^2}{1+\frac{\kappa-1}{2}Ma_2^2}$$

(9.77b) f977b(P1,P2,k,Ma1,Ma2)

$$\frac{p_2}{p_1} = \left(\frac{1+\frac{\kappa-1}{2}Ma_1^2}{1+\frac{\kappa-1}{2}Ma_2^2}\right)^{\frac{\kappa}{\kappa-1}}$$

Öffnungskegel

$\mu_1 = \arcsin\left(\frac{1}{Ma_1}\right)$ rechnerg. möglich

$$\delta = \mu_2 - \mu_1 + \Delta\vartheta_{12}$$

$\mu_2 = \arcsin\left(\frac{1}{Ma_2}\right)$

~~[illegible]~~

(9.75)

$$\nu\left(Ma_2\right) = \nu\left(Ma_1\right) + |\vartheta_{12}|$$

Prandtl-Meyer-Funktion

(9.74) f974(V,k,Ma) Expansions-Fächer

Umlenkwinkel dann ~~[illegible]~~ $\nu_2 - \nu_1 = \Delta\vartheta_{12}$

$$\nu(Ma) = \sqrt{\frac{\kappa+1}{\kappa-1}}\arctan\left(\sqrt{\frac{\kappa-1}{\kappa+1}\left(Ma^2-1\right)}\right) - \arctan\left(\sqrt{Ma^2-1}\right) [°]$$

Wenn ν > 130,454° -> unmöglich -> Grenzfall betrachten.
↳ $\mu_2 = 0°$
↳ $Ma_2 = \infty$

Instationäre Eindimensionale Strömung

Kolbengeschwindigkeit

$$\frac{V_k'}{c_1} = \frac{2}{\kappa+1}\frac{Ma_s^2-1}{Ma_s}$$

Machzahl

$$Ma_2' = \frac{Ma_s^2-1}{\sqrt{\left(1+\frac{\kappa-1}{2}Ma_s^2\right)\left(\kappa Ma_s^2 - \frac{\kappa-1}{2}\right)}}$$

Stoßbeziehungen

$$\frac{p_2'}{p_1'} = \frac{2\kappa Ma_s^2 - \kappa + 1}{\kappa+1}$$

$$\frac{p_{02}'}{p_1'} = \left(\frac{\kappa+1}{2\kappa Ma_s^2 - \kappa+1}\right)^{\frac{1}{\kappa-1}}\left(Ma_s^2\frac{2(\kappa-1)Ma_s^2+(3-\kappa)}{2+(\kappa-1)Ma_s^2}\right)^{\frac{\kappa}{\kappa-1}}$$

$$\frac{T_2'}{T_1'} = \frac{\left(2\kappa Ma_s^2-\kappa+1\right)\left(2+(\kappa-1)Ma_s^2\right)}{(\kappa+1)^2Ma_s^2}$$

$$\frac{T_{02}'}{T_1} = \frac{2}{\kappa+1}\left((\kappa-1)Ma_s^2 + \frac{3-\kappa}{2}\right)$$

$Ma_s > 7$

$$Ma_2 \approx \sqrt{\frac{\kappa-1}{2\kappa}}$$

$$\frac{p_{02}}{p_{01}} \approx \left(\frac{\kappa+1}{2\kappa}\left(\frac{\kappa+1}{\kappa-1}\right)\frac{1}{Ma_s^2}\right)^{\frac{1}{\kappa-1}}$$

$$\frac{T_2}{T_1} \approx 2\kappa\frac{(\kappa-1)}{(\kappa+1)^2}Ma_s^2$$

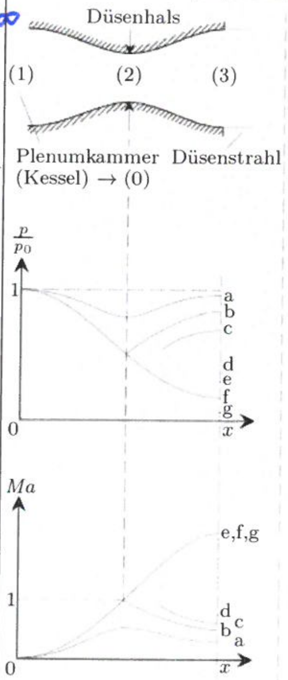
$$Ma_2' \approx \sqrt{\frac{2}{\kappa(\kappa-1)}}$$

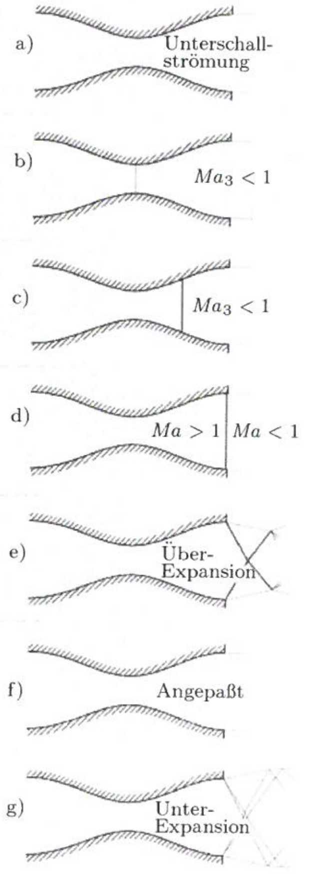
$$\frac{p_{02}'}{p_{01}'} = \frac{p_{02}'}{p_1'} = \frac{p_{02}'}{p_1} \approx \left(\frac{\kappa+1}{\kappa}\right)^{\frac{1}{\kappa-1}}Ma_s^2$$

$$\frac{T_{02}'}{T_{01}'} = \frac{T_{02}'}{T_1'} = \frac{T_{02}'}{T_1} \approx 2\frac{(\kappa-1)}{\kappa+1}Ma_s^2$$

$$\frac{p_2}{p_1} = \frac{p_2'}{p_1'} \approx \frac{2\kappa}{\kappa+1}Ma_s^2$$

Düsenströmung





Anhang D.

Isentrope Strömung (Ideales Gas; $\kappa = 1,4$)

Für "Vor der Düse" Aufgaben z.B. $\frac{A}{A^*}$ in der Düse mit
[illegible] hier vergleichen und danach die restlichen
[illegible] berechnen.

Ma	La	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{A}{A^*}$
0	0	1.00000	1.00000	1.00000	∞
.01	.0110	.99993	.99995	.99998	57.87384
.02	.0219	.99972	.99980	.99992	28.94213
.03	.0329	.99937	.99955	.99982	19.30054
.04	.0438	.99888	.99920	.99968	14.48149
.05	.0548	.99825	.99875	.99950	11.59144
.06	.0657	.99748	.99820	.99928	9.66591
.07	.0766	.99658	.99755	.99902	8.29153
.08	.0876	.99553	.99681	.99872	7.26161
.09	.0985	.99435	.99596	.99838	6.46134
.10	.1094	.99303	.99502	.99800	5.82183
.11	.1204	.99158	.99398	.99759	5.29923
.12	.1313	.98998	.99284	.99713	4.86432
.13	.1422	.98826	.99160	.99663	4.49686
.14	.1531	.98640	.99027	.99610	4.18240
.15	.1639	.98441	.98884	.99552	3.91034
.16	.1748	.98228	.98731	.99491	3.67274
.17	.1857	.98003	.98569	.99425	3.46351
.18	.1965	.97765	.98398	.99356	3.27793
.19	.2074	.97514	.98218	.99283	3.11226
.20	.2182	.97250	.98028	.99206	2.96352
.21	.2290	.96973	.97829	.99126	2.82929
.22	.2398	.96685	.97620	.99041	2.70760
.23	.2506	.96383	.97403	.98953	2.59681
.24	.2614	.96070	.97177	.98861	2.49556
.25	.2722	.95745	.96942	.98765	2.40271
.26	.2829	.95408	.96698	.98666	2.31729
.27	.2936	.95060	.96446	.98563	2.23847
.28	.3043	.94700	.96185	.98456	2.16555
.29	.3150	.94329	.95916	.98346	2.09793
.30	.3257	.93947	.95638	.98232	2.03507
.31	.3364	.93554	.95352	.98114	1.97651
.32	.3470	.93150	.95058	.97993	1.92185
.33	.3576	.92736	.94756	.97868	1.87074
.34	.3682	.92312	.94446	.97740	1.82288
.35	.3788	.91877	.94128	.97609	1.77797
.36	.3893	.91433	.93803	.97473	1.73578
.37	.3999	.90979	.93470	.97335	1.69609
.38	.4104	.90516	.93130	.97193	1.65870
.39	.4209	.90043	.92782	.97048	1.62343
.40	.4313	.89561	.92427	.96899	1.59014
.41	.4418	.89071	.92066	.96747	1.55867
.42	.4522	.88572	.91697	.96592	1.52891
.43	.4626	.88065	.91322	.96434	1.50072
.44	.4729	.87550	.90940	.96272	1.47401
.45	.4833	.87027	.90551	.96108	1.44867
.46	.4936	.86496	.90157	.95940	1.42463
.47	.5038	.85958	.89756	.95769	1.40180
.48	.5141	.85413	.89349	.95595	1.38010
.49	.5243	.84861	.88936	.95418	1.35947
.50	.5345	.84302	.88517	.95238	1.33984
.51	.5447	.83737	.88093	.95055	1.32117
.52	.5548	.83165	.87663	.94869	1.30339
.53	.5649	.82588	.87228	.94681	1.28645
.54	.5750	.82005	.86788	.94489	1.27032
.55	.5851	.81417	.86342	.94295	1.25495
.56	.5951	.80823	.85892	.94098	1.24029
.57	.6051	.80224	.85437	.93898	1.22633
.58	.6150	.79621	.84978	.93696	1.21301
.59	.6249	.79013	.84514	.93491	1.20031
.60	.6348	.78400	.84045	.93284	1.18820
.61	.6447	.77784	.83573	.93073	1.17665
.62	.6545	.77164	.83096	.92861	1.16565
.63	.6643	.76540	.82616	.92646	1.15515
.64	.6740	.75913	.82132	.92428	1.14515
.65	.6837	.75283	.81644	.92208	1.13562
.66	.6934	.74650	.81153	.91986	1.12654
.67	.7031	.74014	.80659	.91762	1.11789
.68	.7127	.73376	.80162	.91535	1.10965
.69	.7223	.72735	.79661	.91306	1.10182
.70	.7318	.72093	.79158	.91075	1.09437
.71	.7413	.71448	.78652	.90841	1.08729
.72	.7508	.70803	.78143	.90606	1.08057
.73	.7602	.70155	.77632	.90369	1.07419
.74	.7696	.69507	.77119	.90129	1.06814
.75	.7789	.68857	.76604	.89888	1.06242
.76	.7883	.68207	.76086	.89644	1.05700
.77	.7975	.67556	.75567	.89399	1.05188
.78	.8068	.66905	.75046	.89152	1.04705
.79	.8160	.66254	.74523	.88903	1.04251
.80	.8251	.65602	.73999	.88652	1.03823
.81	.8343	.64951	.73474	.88400	1.03422
.82	.8t133	.64300	.72947	.88146	1.03046
.83	.8524	.63650	.72419	.87890	1.02696
.84	.8614	.63000	.71891	.87633	1.02370
.85	.8704	.62351	.71361	.87374	1.02067
.86	.8793	.61703	.70831	.87114	1.01787
.87	.8882	.61057	.70300	.86852	1.01530
.88	.8970	.60412	.69768	.86589	1.01294
.89	.9058	.59768	.69236	.86324	1.01080
.90	.9146	.59126	.68704	.86059	1.00886
.91	.9233	.58486	.68172	.85791	1.00713
.92	.9320	.57848	.67640	.85523	1.00560
.93	.9406	.57211	.67108	.85253	1.00426
.94	.9493	.56578	.66576	.84982	1.00311
.95	.9578	.55946	.66044	.84710	1.00215
.96	.9663	.55317	.65513	.84437	1.00136
.97	.9748	.54691	.64982	.84162	1.00076
.98	.9832	.54067	.64452	.83887	1.00034
.99	.9916	.53446	.63923	.83611	1.00008

Anhang B.1

Isentrope Strömung (Ideales Gas; $\kappa = 1,4$)

Ma	La	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{A}{A^*}$	μ	ν
1.00	1.0000	.52828	.63394	.83333	1.00000	90.000	.000
1.01	1.0083	.52213	.62866	.83055	1.00008	81.931	.045
1.02	1.0166	.51602	.62339	.82776	1.00033	78.635	.126
1.03	1.0248	.50994	.61813	.82496	1.00074	76.138	.229
1.04	1.0330	.50389	.61289	.82215	1.00131	74.058	.351
1.05	1.0411	.49787	.60765	.81934	1.00203	72.247	.487
1.06	1.0492	.49189	.60243	.81651	1.00291	70.630	.637
1.07	1.0573	.48595	.59723	.81368	1.00394	69.160	.797
1.08	1.0653	.48005	.59203	.81085	1.00512	67.808	.968
1.09	1.0733	.47418	.58686	.80800	1.00645	66.553	1.148
1.10	1.0812	.46835	.58170	.80515	1.00793	65.380	1.336
1.11	1.0891	.46257	.57655	.80230	1.00955	64.277	1.532
1.12	1.0970	.45682	.57143	.79944	1.01131	63.235	1.735
1.13	1.1048	.45111	.56632	.79657	1.01322	62.246	1.944
1.14	1.1126	.44545	.56123	.79370	1.01527	61.306	2.160
1.15	1.1203	.43983	.55616	.79083	1.01745	60.408	2.381
1.16	1.1280	.43425	.55112	.78795	1.01978	59.550	2.607
1.17	1.1356	.42872	.54609	.78506	1.02224	58.727	2.839
1.18	1.1432	.42322	.54108	.78218	1.02484	57.936	3.074
1.19	1.1508	.41778	.53610	.77929	1.02757	57.176	3.314
1.20	1.1583	.41238	.53114	.77640	1.03044	56.443	3.558
1.21	1.1658	.40702	.52620	.77350	1.03344	55.735	3.806
1.22	1.1732	.40171	.52129	.77061	1.03657	55.052	4.057
1.23	1.1806	.39645	.51640	.76771	1.03983	54.391	4.312
1.24	1.1879	.39123	.51154	.76481	1.04323	53.751	4.569
1.25	1.1952	.38606	.50670	.76190	1.04675	53.130	4.830
1.26	1.2025	.38093	.50189	.75900	1.05041	52.528	5.093
1.27	1.2097	.37586	.49710	.75610	1.05419	51.943	5.359
1.28	1.2169	.37083	.49234	.75319	1.05810	51.375	5.627
1.29	1.2240	.36585	.48761	.75029	1.06214	50.823	5.898
1.30	1.2311	.36091	.48290	.74738	1.06630	50.285	6.170
1.31	1.2382	.35603	.47822	.74448	1.07060	49.761	6.445
1.32	1.2452	.35119	.47358	.74158	1.07502	49.251	6.721
1.33	1.2522	.34640	.46895	.73867	1.07957	48.753	7.000
1.34	1.2591	.34166	.46436	.73577	1.08424	48.268	7.279
1.35	1.2660	.33697	.45980	.73287	1.08904	47.795	7.561
1.36	1.2729	.33233	.45526	.72997	1.09396	47.332	7.843
1.37	1.2797	.32773	.45076	.72707	1.09902	46.880	8.128
1.38	1.2864	.32319	.44628	.72418	1.10419	46.439	8.413
1.39	1.2932	.31869	.44184	.72128	1.10950	46.007	8.699
1.40	1.2999	.31424	.43742	.71839	1.11493	45.585	8.987
1.41	1.3065	.30984	.43304	.71550	1.12048	45.171	9.276
1.42	1.3131	.30549	.42869	.71262	1.12616	44.767	9.565
1.43	1.3197	.30119	.42436	.70973	1.13197	44.371	9.855
1.44	1.3262	.29693	.42007	.70685	1.13790	43.983	10.146
1.45	1.3327	.29272	.41581	.70398	1.14396	43.603	10.438
1.46	1.3392	.28856	.41158	.70111	1.15015	43.230	10.730
1.47	1.3456	.28445	.40739	.69824	1.15646	42.865	11.023
1.48	1.3520	.28039	.40322	.69537	1.16290	42.507	11.317
1.49	1.3583	.27637	.39909	.69251	1.16947	42.155	11.611
1.50	1.3646	.27240	.39498	.68966	1.17617	41.810	11.905
1.51	1.3708	.26848	.39091	.68680	1.18299	41.472	12.200
1.52	1.3770	.26461	.38688	.68396	1.18994	41.140	12.495
1.53	1.3832	.26078	.38287	.68112	1.19702	40.813	12.790
1.54	1.3894	.25700	.37890	.67828	1.20423	40.493	13.086
1.55	1.3955	.25326	.37496	.67545	1.21157	40.178	13.381
1.56	1.4015	.24957	.37105	.67262	1.21904	39.868	13.677
1.57	1.4075	.24593	.36717	.66980	1.22664	39.564	13.973
1.58	1.4135	.24233	.36332	.66699	1.23438	39.265	14.269
1.59	1.4195	.23878	.35951	.66418	1.24224	38.971	14.564
1.60	1.4254	.23527	.35573	.66138	1.25023	38.682	14.860
1.61	1.4313	.23181	.35198	.65858	1.25836	38.398	15.156
1.62	1.4371	.22839	.34827	.65579	1.26663	38.118	15.452
1.63	1.4429	.22501	.34458	.65301	1.27502	37.843	15.747
1.64	1.4487	.22168	.34093	.65023	1.28355	37.572	16.043
1.65	1.4544	.21840	.33731	.64746	1.29222	37.305	16.338
1.66	1.4601	.21515	.33372	.64470	1.30102	37.043	16.633
1.67	1.4657	.21195	.33017	.64194	1.30996	36.784	16.928
1.68	1.4713	.20879	.32664	.63919	1.31904	36.530	17.222
1.69	1.4769	.20567	.32315	.63645	1.32825	36.279	17.516
1.70	1.4825	.20259	.31969	.63371	1.33761	36.032	17.810
1.71	1.4880	.19956	.31626	.63099	1.34710	35.789	18.103
1.72	1.4935	.19656	.31287	.62827	1.35673	35.549	18.396
1.73	1.4989	.19361	.30950	.62556	1.36651	35.312	18.689
1.74	1.5043	.19070	.30617	.62285	1.37643	35.080	18.981
1.75	1.5097	.18782	.30287	.62016	1.38649	34.850	19.273
1.76	1.5150	.18499	.29959	.61747	1.39670	34.624	19.565
1.77	1.5203	.18219	.29635	.61479	1.40705	34.400	19.855
1.78	1.5256	.17944	.29315	.61212	1.41755	34.180	20.146
1.79	1.5308	.17672	.28997	.60945	1.42819	33.963	20.436
1.80	1.5360	.17404	.28682	.60680	1.43898	33.749	20.725
1.81	1.5411	.17140	.28370	.60415	1.44992	33.538	21.014
1.82	1.5463	.16879	.28061	.60151	1.46101	33.329	21.302
1.83	1.5514	.16622	.27756	.59888	1.47225	33.124	21.590
1.84	1.5564	.16369	.27453	.59626	1.48365	32.921	21.877
1.85	1.5614	.16120	.27153	.59365	1.49519	32.720	22.163
1.86	1.5664	.15873	.26857	.59104	1.50689	32.523	22.449
1.87	1.5714	.15631	.26563	.58845	1.51875	32.328	22.734
1.88	1.5763	.15392	.26272	.58586	1.53076	32.135	23.019
1.89	1.5812	.15156	.25984	.58329	1.54293	31.945	23.303
1.90	1.5861	.14924	.25699	.58072	1.55526	31.757	23.586
1.91	1.5909	.14695	.25417	.57816	1.56774	31.571	23.869
1.92	1.5957	.14470	.25138	.57561	1.58039	31.388	24.151
1.93	1.6005	.14247	.24861	.57307	1.5932	31.207	24.432
1.94	1.6052	.14028	.24588	.57054	1.60617	31.028	24.712
1.95	1.6099	.13813	.24317	.56802	1.61931	30.852	24.992
1.96	1.6146	.13600	.24049	.56551	1.63261	30.677	25.271
1.97	1.6192	.13390	.23784	.56301	1.64608	30.505	25.549
1.98	1.6239	.13184	.23521	.56051	1.65972	30.335	25.827
1.99	1.6284	.12981	.23262	.55803	1.67352	30.166	26.104

Anhang B.1

Isentrope Strömung (Ideales Gas; $\kappa = 1,4$)

Ma	La	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{A}{A^*}$	μ	ν
2.00	1.6330	.12780	.23005	.55556	1.68750	30.000	26.380
2.01	1.6375	.12583	.22751	.55309	1.70165	29.836	26.655
2.02	1.6420	.12389	.22499	.55064	1.71597	29.673	26.930
2.03	1.6465	.12197	.22250	.54819	1.73047	29.512	27.203
2.04	1.6509	.12009	.22004	.54576	1.74514	29.353	27.476
2.05	1.6553	.11823	.21760	.54333	1.75999	29.196	27.748
2.06	1.6597	.11640	.21519	.54092	1.77502	29.041	28.020
2.07	1.6640	.11460	.21281	.53851	1.79022	28.888	28.290
2.08	1.6683	.11282	.21045	.53611	1.80561	28.736	28.560
2.09	1.6726	.11107	.20811	.53373	1.82119	28.585	28.829
2.10	1.6769	.10935	.20580	.53135	1.83694	28.437	29.097
2.11	1.6811	.10766	.20352	.52898	1.85288	28.290	29.364
2.12	1.6853	.10599	.20126	.52663	1.86901	28.145	29.631
2.13	1.6895	.10434	.19902	.52428	1.88533	28.001	29.896
2.14	1.6936	.10273	.19681	.52194	1.90184	27.859	30.161
2.15	1.6977	.10113	.19463	.51962	1.91854	27.718	30.425
2.16	1.7018	$.99562\cdot 10^{-1}$	.19247	.51730	1.93543	27.578	30.688
2.17	1.7059	$.98017\cdot 10^{-1}$	.19033	.51499	1.95252	27.441	30.951
2.18	1.7099	$.96495\cdot 10^{-1}$	.18821	.51269	1.96981	27.304	31.212
2.19	1.7139	$.94997\cdot 10^{-1}$	.18612	.51041	1.98729	27.169	31.473
2.20	1.7179	$.93522\cdot 10^{-1}$	.18405	.50813	2.00497	27.036	31.732
2.21	1.7219	$.92070\cdot 10^{-1}$	.18200	.50586	2.02286	26.903	31.991
2.22	1.7258	$.90640\cdot 10^{-1}$	.17998	.50361	2.04094	26.773	32.249
2.23	1.7297	$.89232\cdot 10^{-1}$	.17798	.50136	2.05923	26.643	32.507
2.24	1.7336	$.87846\cdot 10^{-1}$	.17600	.49912	2.07773	26.515	32.763
2.25	1.7374	$.86482\cdot 10^{-1}$	.17404	.49689	2.09643	26.388	33.018
2.26	1.7412	$.85139\cdot 10^{-1}$	.17211	.49468	2.11535	26.262	33.273
2.27	1.7450	$.83817\cdot 10^{-1}$	.17020	.49247	2.13447	26.138	33.527
2.28	1.7488	$.82515\cdot 10^{-1}$	.16830	.49027	2.15381	26.014	33.780
2.29	1.7526	$.81234\cdot 10^{-1}$	.16643	.48809	2.17336	25.892	34.032
2.30	1.7563	$.79973\cdot 10^{-1}$	.16458	.48591	2.19313	25.771	34.283
2.31	1.7600	$.78731\cdot 10^{-1}$	.16275	.48374	2.21311	25.652	34.533
2.32	1.7637	$.77509\cdot 10^{-1}$	.16095	.48158	2.23332	25.533	34.782
2.33	1.7673	$.76306\cdot 10^{-1}$	.15916	.47944	2.25375	25.416	35.031
2.34	1.7709	$.75122\cdot 10^{-1}$	.15739	.47730	2.27440	25.300	35.279
2.35	1.7745	$,73957\cdot 10^{-1}$	.15564	.47517	2.29527	25.184	35.525
2.36	1.7781	$.72810\cdot 10^{-1}$	.15391	.47306	2.31638	25.070	35.771
2.37	1.7817	$.71681\cdot 10^{-1}$	.15221	.47095	2.33771	24.957	36.016
2.38	1.7852	$.70570\cdot 10^{-1}$	.15052	.46885	2.35927	24.845	36.261
2.39	1.7887	$.69476\cdot 10^{-1}$	.14885	.46676	2.38107	24.734	36.504
2.40	1.7922	$.68400\cdot 10^{-1}$	.14720	.46468	2.40310	24.624	36.746
2.41	1.7956	$.67340\cdot 10^{-1}$	.14556	.46262	2.42536	24.515	36.988
2.42	1.7991	$.66297\cdot 10^{-1}$	.14395	.46056	2.44787	24.407	37.229
2.43	1.8025	$.65271\cdot 10^{-1}$	.14235	.45851	2.47061	24.301	37.469
2.44	1.8059	$.64261\cdot 10^{-1}$	.14078	.45647	2.49360	24.195	37.708
2.45	1.8092	$.63267\cdot 10^{-1}$	.13922	.45444	2.51683	24.090	37.946
2.46	1.8126	$.62289\cdot 10^{-1}$	.13768	.45242	2.54031	23.985	38.183
2.47	1.8159	$.61326\cdot 10^{-1}$	.13615	.45041	2.56403	23.882	38.419
2.48	1.8192	$.60378\cdot 10^{-1}$	.13465	.44841	2.58801	23.780	38.655
2.49	1.8225	$.59446\cdot 10^{-1}$	.13316	.44642	2.61223	23.679	38.890
2.50	1.8257	$.58528\cdot 10^{-1}$	.13169	.44444	2.63672	23.578	39.124
2.51	1.8290	$.57625\cdot 10^{-1}$	.13023	.44247	2.66145	23.479	39.356
2.52	1.8322	$.56736\cdot 10^{-1}$	.12879	.44051	2.68645	23.380	39.589
2.53	1.8354	$.55861\cdot 10^{-1}$	.12737	.43856	2.71171	23.282	39.820
2.54	1.8386	$.55000\cdot 10^{-1}$	.12597	.43662	2.73722	23.185	40.050
2.55	1.8417	$.54153\cdot 10^{-1}$	.12458	.43469	2.76301	23.089	40.280
2.56	1.8448	$.53319\cdot 10^{-1}$	.12321	.43277	2.78906	22.993	40.508
2.57	1.8-!79	$.52499\cdot 10^{-1}$	.12185	.43085	2.81538	22.899	40.736
2.58	1.8510	$.51692\cdot 10^{-1}$	.12051	.42895	2.84197	22.805	40.963
2.59	1.8541	$.50897\cdot 10^{-1}$	.11918	.42706	2.86883	22.712	41.189
2.60	1.8571	$.50115\cdot 10^{-1}$	.11787	.42517	2.89597	22.620	41.415
2.61	1.8602	$.49346\cdot 10^{-1}$	.11658	.42330	2.92339	22.528	41.639
2.62	1.8632	$.48589\cdot 10^{-1}$	.11530	.42143	2.95108	22.438	41.863
2.63	1.8662	$.47844\cdot 10^{-1}$	.11403	.41957	2.97906	22.348	42.085
2.64	1.8691	$.47110\cdot 10^{-1}$	.11278	.41773	3.00733	22.259	42.307
2.65	1.8721	$.46389\cdot 10^{-1}$	.11154	.41589	3.03588	22.170	42.528
2.66	1.8750	$.45679\cdot 10^{-1}$	.11032	.41406	3.06472	22.082	42.749
2.67	1.8779	$.44980\cdot 10^{-1}$	.10911	.41224	3.09385	21.995	42.968
2.68	1.8808	$.44293\cdot 10^{-1}$	.10792	.41043	3.12327	21.909	43.187
2.69	1.8837	$.43616\cdot 10^{-1}$	.10674	.40863	3.15299	21.823	43.405
2.70	1.8865	$.42950\cdot 10^{-1}$	.10557	.40684	3.18301	21.738	43.621
2.71	1.8894	$.42295\cdot 10^{-1}$	.10442	.40505	3.21332	21.654	43.838
2.72	1.8922	$.41650\cdot 10^{-1}$	.10328	.40328	3.24395	21.571	44.053
2.73	1.8950	$.41016\cdot 10^{-1}$	.10215	.40151	3.27487	21.488	44.267
2.74	1.8978	$.40391\cdot 10^{-1}$	.10104	.39976	3.30611	21.405	44.481
2.75	1.9005	$.39777\cdot 10^{-1}$	$.99939\cdot 10^{-1}$	.39801	3.33765	21.324	44.694
2.76	1.9033	$.39172\cdot 10^{-1}$	$.98852\cdot 10^{-1}$	.39627	3.36951	21.243	44.906
2.77	1.9060	$.38577\cdot 10^{-1}$	$.97777\cdot 10^{-1}$	.39454	3.40168	21.162	45.117
2.78	1.9087	$.37992\cdot 10^{-1}$	$.96714\cdot 10^{-1}$	.39282	3.43417	21.083	45.327
2.79	1.9114	$.37415\cdot 10^{-1}$	$.95664\cdot 10^{-1}$	.39111	3.46698	21.003	45.537
2.80	1.9140	$.36848\cdot 10^{-1}$	$.94627\cdot 10^{-1}$	.38941	3.50012	20.925	45.746
2.81	1.9167	$.36290\cdot 10^{-1}$	$.93601\cdot 10^{-1}$	.38771	3.53358	20.847	45.954
2.82	1.9193	$.35741\cdot 10^{-1}$	$.92587\cdot 10^{-1}$	.38603	3.56736	20.770	46.161
2.83	1.9219	$.35201\cdot 10^{-1}$	$.91585\cdot 10^{-1}$	.38435	3.60148	20.693	46.367
2.84	1.9245	$.34669\cdot 10^{-1}$	$.90595\cdot 10^{-1}$	.38268	3.63593	20.617	46.573
2.85	1.9271	$.34146\cdot 10^{-1}$	$.89616\cdot 10^{-1}$	.38103	3.67071	20.541	46.778
2.86	1.9297	$.33631\cdot 10^{-1}$	$.88648\cdot 10^{-1}$	.37937	3.70584	20.466	46.982
2.87	1.9323	$.33124\cdot 10^{-1}$	$.87692\cdot 10^{-1}$	.37773	3.74130	20.391	47.185
2.88	1.9348	$.32625\cdot 10^{-1}$	$.86747\cdot 10^{-1}$	.37610	3.77711	20.318	47.388
2.89	1.9373	$.32135\cdot 10^{-1}$	$.85813\cdot 10^{-1}$	.37447	3.81326	20.244	47.589
2.90	1.9398	$.31652\cdot 10^{-1}$	$.84889\cdot 10^{-1}$	.37286	3.84976	20.171	47.790
2.91	1.9423	$.31176\cdot 10^{-1}$	$.83977\cdot 10^{-1}$	.37125	3.88662	20.099	47.990
2.92	1.9448	$.30708\cdot 10^{-1}$	$.83075\cdot 10^{-1}$	.36965	3.92382	20.027	48.190
2.93	1.9472	$.30248\cdot 10^{-1}$	$.82184\cdot 10^{-1}$	.36806	3.96139	19.956	48.388
2.94	1.9497	$.29795\cdot 10^{-1}$	$.81302\cdot 10^{-1}$	.36647	3.99931	19.885	48.586
2.95	1.9521	$.29349\cdot 10^{-1}$	$.80432\cdot 10^{-1}$	.36490	4.03760	19.815	48.783
2.96	1.9545	$.28910\cdot 10^{-1}$	$.79571\cdot 10^{-1}$	.36333	4.07625	19.745	48.980
2.97	1.9569	$.28479\cdot 10^{-1}$	$.78720\cdot 10^{-1}$	.36177	4.11527	19.676	49.175
2.98	1.9593	$.28054\cdot 10^{-1}$	$.77879\cdot 10^{-1}$	.36022	4.15466	19.607	49.370
2.99	1.9616	$.27635\cdot 10^{-1}$	$.77048\cdot 10^{-1}$	.35868	4.19442	19.539	49.564
3.00	1.9640	$.27224\cdot 10^{-1}$	$.76226\cdot 10^{-1}$	.35714	4.23457	19.471	49.757
3.10	1.9866	$.23449\cdot 10^{-1}$	$.68517\cdot 10^{-1}$	.34223	4.65731	18.819	51.650
3.20	2.0079	$.20228\cdot 10^{-1}$	$.61654\cdot 10^{-1}$	.32808	5.12096	18.210	53.470
3.30	2.0278	$.17477\cdot 10^{-1}$	$.55541\cdot 10^{-1}$	.31466	5.62865	17.640	55.222
3.40	2.0466	$.15125\cdot 10^{-1}$	$.50093\cdot 10^{-1}$	.30193	6.18370	17.105	56.908
3.50	2.0642	$.13111\cdot 10^{-1}$	$.45233\cdot 10^{-1}$	.28986	6.78962	16.602	58.530
3.60	2.0808	$.11385\cdot 10^{-1}$	$.40894\cdot 10^{-1}$	.27840	7.45011	16.128	60.091
3.70	2.0964	$.99029\cdot 10^{-2}$	$.37017\cdot 10^{-1}$	.26752	8.16906	15.680	61.595
3.80	2.1111	$.86290\cdot 10^{-2}$	$,33549\cdot 10^{-1}$	.25720	8.95058	15.258	63.044
3.90	2.1250	$.75321\cdot 10^{-2}$	$.30445\cdot 10^{-1}$	.24740	9.79897	14.857	64.440
4.00	2.1381	$.65861\cdot 10^{-2}$	$.27662\cdot 10^{-1}$	.23810	10.71874	14.478	65.785
4.10	2.1505	$.57690\cdot 10^{-2}$	$.25164\cdot 10^{-1}$	.22925	11.71464	14.117	67.082
4.20	2.1622	$.50621\cdot 10^{-2}$	$.22921\cdot 10^{-1}$	.22085	12.79163	13.774	68.333
4.30	2.1732	$.44494\cdot 10^{-2}$	$.20903\cdot 10^{-1}$	.21286	13.95489	13.448	69.541
4.40	2.1837	$.39176\cdot 10^{-2}$	$.19087\cdot 10^{-1}$	.20525	15.20985	13.137	70.706
4.50	2.1936	$.34553\cdot 10^{-2}$	$.17449\cdot 10^{-1}$	.19802	16.56218	12.840	71.832
4.60	2.2030	$.30526\cdot 10^{-2}$	$.15971\cdot 10^{-1}$	.19113	18.01777	12.556	72.919
4.70	2.2119	$.27012\cdot 10^{-2}$	$.14635\cdot 10^{-1}$	.18457	19.58280	12.284	73.970
4.80	2.2204	$.23943\cdot 10^{-2}$	$.13427\cdot 10^{-1}$	.17832	21.26368	12.025	74.986
4.90	2.2284	$.21256\cdot 10^{-2}$	$.12333\cdot 10^{-1}$	.17235	23.06709	11.776	75.969
5.00	2.2361	$.18900\cdot 10^{-2}$	$.11340\cdot 10^{-1}$	.16667	25.00000	11.537	76.920
6.00	2.2953	$.63336\cdot 10^{-3}$	$.51936\cdot 10^{-2}$	.12195	53.17979	9.594	84.955
7.00	2.3333	$.24156\cdot 10^{-3}$	$.26088\cdot 10^{-2}$	$.92593\cdot 10^{-1}$	104.14290	8.213	90.973
8.00	2.3591	$.10243\cdot 10^{-3}$	$.14135\cdot 10^{-2}$	$.72464\cdot 10^{-1}$	190.10940	7.181	95.625
9.00	2.3772	$.47386\cdot 10^{-4}$	$.81504\cdot 10^{-3}$	$.58140\cdot 10^{-1}$	327.18940	6.379	99.318
10.00	2.3905	$.23563\cdot 10^{-4}$	$.49482\cdot 10^{-3}$	$.47619\cdot 10^{-1}$	535.93770	5.739	102.316
∞	2.4495	0	0	0	∞	0	130.454

Anhang B.2

Senkrechter Stoß (Ideales Gas; $\kappa = 1,4$)

Ma_1	Ma_2	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{02}}{p_{01}}$
1.00	1.00000	1.00000	1.00000	1.00000	1.00000
1.01	.99013	1.02345	1.01669	1.00664	1.00000
1.02	.98052	1.04713	1.03344	1.01325	.99999
1.03	.97115	1.07105	1.05024	1.01981	.99997
1.04	.96203	1.09520	1.06709	1.02634	.99992
1.05	.95313	1.11958	1.08398	1.03284	.99985
1.06	.94445	1.14420	1.10092	1.03931	.99975
1.07	.93598	1.16905	1.11790	1.04575	.99961
1.08	.92771	1.19413	1.13492	1.05217	.99943
1.09	.91965	1.21945	1.15199	1.05856	.99920
1.10	.91177	1.24500	1.16908	1.06494	.99893
1.11	.90408	1.27078	1.18621	1.07129	.99860
1.12	.89656	1.29680	1.20338	1.07763	.99821
1.13	.88922	1.32305	1.22057	1.08396	.99777
1.14	.88204	1.34953	1.23779	1.09027	.99726
1.15	.87502	1.37625	1.25504	1.09658	.99669
1.16	.86816	1.40320	1.27231	1.10287	.99605
1.17	.86145	1.43038	1.28961	1.10916	.99535
1.18	.85488	1.45780	1.30693	1.11544	.99457
1.19	.84846	1.48545	1.32426	1.12172	.99372
1.20	.84217	1.51333	1.34161	1.12799	.99280
1.21	.83601	1.54145	1.35898	1.13427	.99180
1.22	.82999	1.56980	1.37636	1.14054	.99073
1.23	.82408	1.59838	1.39376	1.14682	.98958
1.24	.81830	1.62720	1.41116	1.15309	.98836
1.25	.81264	1.65625	1.42857	1.15937	.98706
1.26	.80709	1.68553	1.44599	1.16566	.98568
1.27	.80164	1.71505	1.46341	1.17195	.98422
1.28	.79631	1.74480	1.48084	1.17825	.98268
1.29	.79108	1.77478	1.49827	1.18456	.98107
1.30	.78596	1.80500	1.51569	1.19087	.97937
1.31	.78093	1.83545	1.53312	1.19720	.97760
1.32	.77600	1.86613	1.55055	1.20353	.97575
1.33	.77116	1.89705	1.56796	1.20988	.97382
1.34	.76641	1.92820	1.58538	1.21624	.97182
1.35	.76175	1.95958	1.60278	1.22261	.96974
1.36	.75718	1.99120	1.62018	1.22900	.96758
1.37	.75269	2.02305	1.63757	1.23540	.96534
1.38	.74829	2.05513	1.65494	1.24181	.96304
1.39	.74396	2.08745	1.67231	1.24825	.96065
1.40	.73971	2.12000	1.68965	1.25469	.95819
1.41	.73554	2.15278	1.70699	1.26116	.95566
1.42	.73144	2.18580	1.72430	1.26764	.95306
1.43	.72741	2.21905	1.74160	1.27414	.95039
1.44	.72345	2.25253	1.75888	1.28066	.94765
1.45	.71956	2.28625	1.77613	1.28720	.94484
1.46	.71574	2.32020	1.79337	1.29376	.94196
1.47	.71198	2.35438	1.81058	1.30035	.93901
1.48	.70829	2.38880	1.82777	1.30695	.93600
1.49	.70466	2.42345	1.84493	1.31357	.93293
1.50	.70109	2.45833	1.86207	1.32022	.92979
1.51	.69758	2.49345	1.87918	1.32688	.92659
1.52	.69413	2.52880	1.89626	1.33357	.92332
1.53	.69073	2.56438	1.91331	1.34029	.92000
1.54	.68739	2.60020	1.93033	1.34703	.91662
1.55	.68410	2.63625	1.94731	1.35379	.91319
1.56	.68087	2.67253	1.96427	1.36057	.90970
1.57	.67769	2.70905	1.98119	1.36738	.90615
1.58	.67455	2.74580	1.99808	1.37422	.90255
1.59	.67147	2.78278	2.01493	1.38108	.89890
1.60	.66844	2.82000	2.03175	1.38797	.89520
1.61	.66545	2.85745	2.04852	1.39488	.89145
1.62	.66251	2.89513	2.06526	1.40182	.88765
1.63	.65962	2.93305	2.08196	1.40879	.88381
1.64	.65677	2.97120	2.09863	1.41578	.87992
1.65	.65396	3.00958	2.11525	1.42280	.87599
1.66	.65119	3.04820	2.13183	1.42985	.87201
1.67	.64847	3.08705	2.14836	1.43693	.86800
1.68	.64579	3.12613	2.16486	1.44403	.86394
1.69	.64315	3.16545	2.18131	1.45117	.85985
1.70	.64054	3.20500	2.19772	1.45833	.85572
1.71	.63798	3.24478	2.21408	1.46552	.85156
1.72	.63545	3.28480	2.23040	1.47274	.84736
1.73	.63296	3.32505	2.24667	1.47999	.84312
1.74	.63051	3.36553	2.26289	1.48727	.83886
1.75	.62809	3.40625	2.27907	1.49458	.83457
1.76	.62570	3.44720	2.29520	1.50192	.83024
1.77	.62335	3.48838	2.31128	1.50929	.82589
1.78	.62104	3.52980	2.32731	1.51669	.82151
1.79	.61875	3.57145	2.34329	1.52412	.81711
1.80	.61650	3.61333	2.35922	1.53158	.81268
1.81	.61428	3.65545	2.37510	1.53907	.80823
1.82	.61209	3.69780	2.39093	1.54659	.80376
1.83	.60993	3.74038	2.40671	1.55415	.79927
1.84	.60780	3.78320	2.42244	1.56173	.79476
1.85	.60570	3.82625	2.43811	1.56935	.79023
1.86	.60363	3.86953	2.45373	1.57700	.78569
1.87	.60159	3.91305	2.46930	1.58468	.78113
1.88	.59957	3.95680	2.48481	1.59239	.77655
1.89	.59758	4.00078	2.50027	1.60014	.77196
1.90	.59562	4.04500	2.51568	1.60791	.76736
1.91	.59368	4.08945	2.53103	1.61572	.76274
1.92	.59177	4.13413	2.54632	1.62357	.75812
1.93	.58988	4.17905	2.56156	1.63144	.75349
1.94	.58802	4.22420	2.57675	1.63935	.74884
1.95	.58619	4.26958	2.59188	1.64729	.74420
1.96	.58437	4.31520	2.60695	1.65527	.73954
1.97	.58258	4.36105	2.62196	1.66328	.73488
1.98	.58082	4.40713	2.63692	1.67132	.73021
1.99	.57907	4.45345	2.65182	1.67939	.72555

Anhang B.2

Senkrechter Stoß (Ideales Gas; $\kappa = 1,4$)

Ma_1	Ma_2	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{02}}{p_{01}}$
2.00	.57735	4.50000	2.66667	1.68750	.72087
2.01	.57565	4.54678	2.68145	1.69564	.71620
2.02	.57397	4.59380	2.69618	1.70382	.71153
2.03	.57232	4.64105	2.71085	1.71203	.70685
2.04	.57068	4.68853	2.72546	1.72027	.70218
2.05	.56906	4.73625	2.74001	1.72855	.69751
2.06	.56747	4.78419	2.75451	1.73686	.69284
2.07	.56589	4.83238	2.76895	1.74520	.68817
2.08	.56433	4.88080	2.78332	1.75359	.68351
2.09	.56280	4.92944	2.79764	1.76200	.67886
2.10	.56128	4.97833	2.81190	1.77045	.67420
2.11	.55978	5.02744	2.82610	1.77893	.66956
2.12	.55829	5.07679	2.84024	1.78745	.66492
2.13	.55683	5.12638	2.85432	1.79601	.66029
2.14	.55538	5.17619	2.86834	1.80459	.65567
2.15	.55395	5.22624	2.88231	1.81322	.65105
2.16	.55254	5.27653	2.89621	1.82187	.64645
2.17	.55115	5.32704	2.91005	1.83057	.64185
2.18	.54977	5.37779	2.92383	1.83930	.63727
2.19	.54840	5.42878	2.93756	1.84806	.63270
2.20	.54706	5.47999	2.95122	1.85686	.62814
2.21	.54572	5.53144	2.96482	1.86569	.62359
2.22	.54441	5.58313	2.97836	1.87456	.61905
2.23	.54311	5.63504	2.99185	1.88347	.61453
2.24	.54182	5.68719	3.00527	1.89241	.61002
2.25	.54055	5.73958	3.01863	1.90138	.60553
2.26	.53930	5.79219	3.03193	1.91040	.60105
2.27	.53805	5.84504	3.04518	1.91944	.59659
2.28	.53683	5.89813	3.05836	1.92853	.59214
2.29	.53561	5.95144	3.07148	1.93764	.58771
2.30	.53441	6.00499	3.08455	1.94680	.58330
2.31	.53322	6.05878	3.09755	1.95599	.57890
2.32	.53205	6.11279	3.11049	1.96522	.57452
2.33	.53089	6.16704	3.12338	1.97448	.57015
2.34	.52974	6.22153	3.13620	1.98378	.56581
2.35	.52861	6.27624	3.14896	1.99311	.56148
2.36	.52749	6.33119	3.16167	2.00248	.55718
2.37	.52638	6.38638	3.17431	2.01189	.55289
2.38	.52528	6.44179	3.18690	2.02133	.54862
2.39	.52419	6.49744	3.19943	2.03081	.54437
2.40	.52312	6.55333	3.21189	2.04033	.54014
2.41	.52206	6.60944	3.22430	2.04988	.53594
2.42	.52100	6.66579	3.23665	2.05947	.53175
2.43	.51996	6.72238	3.24894	2.06910	.52758
2.44	.51894	6.77919	3.26117	2.07876	.52344
2.45	.51792	6.83624	3.27335	2.08846	.51931
2.46	.51691	6.89353	3.28546	2.09819	.51521
2.47	.51592	6.95104	3.29751	2.10796	.51113
2.48	.51493	7.00879	3.30951	2.11777	.50707
2.49	.51395	7.06677	3.32145	2.12762	.50303
2.50	.51299	7.12499	3.33333	2.13750	.49902
2.51	.51203	7.18344	3.34515	2.14742	.49502
2.52	.51109	7.24212	3.35692	2.15737	.49105
2.53	.51015	7.30104	3.36863	2.16736	.48711
2.54	.50923	7.36019	3.38028	2.17739	.48318
2.55	.50831	7.41957	3.39187	2.18746	.47928
2.56	.50741	7.47919	3.40341	2.19756	.47540
2.57	.50651	7.53904	3.41488	2.20770	.47155
2.58	.50562	7.59912	3.42631	2.21788	.46772
2.59	.50474	7.65944	3.43767	2.22809	.46391
2.60	.50387	7.71999	3.44898	2.23834	.46012
2.61	.50301	7.78077	3.46023	2.24863	.45636
2.62	.50216	7.84179	3.47143	2.25895	.45263
2.63	.50131	7.90304	3.48257	2.26932	.44891
2.64	.50048	7.96452	3.49365	2.27971	.44522
2.65	.49965	8.02624	3.50468	2.29015	.44156
2.66	.49883	8.08819	3.51565	2.30062	.43792
2.67	.49802	8.15037	3.52657	2.31114	.43430
2.68	.49722	8.21279	3.53743	2.32168	.43071
2.69	.49642	8.27544	3.54824	2.33227	.42714
2.70	.49563	8.33832	3.55899	2.34289	.42359
2.71	.49485	8.40144	3.56969	2.35355	.42007
2.72	.49408	8.46479	3.58033	2.36425	.41657
2.73	.49332	8.52837	3.59092	2.37498	.41310
2.74	.49256	8.59219	3.60146	2.38575	.40965
2.75	.49181	8.65624	3.61194	2.39656	.40623
2.76	.49107	8.72052	3.62237	2.40741	.40283
2.77	.49033	8.78504	3.63274	2.41829	.39945
2.78	.48960	8.84979	3.64306	2.42922	.39610
2.79	.48888	8.91477	3.65333	2.44017	.39277
2.80	.48817	8.97999	3.66355	2.45117	.38946
2.81	.48746	9.04544	3.67371	2.46221	.38618
2.82	.48676	9.11112	3.68383	2.47328	.38293
2.83	.48606	9.17704	3.69388	2.48439	.37970
2.84	.48538	9.24319	3.70389	2.49553	.37649
2.85	.48469	9.30957	3.71385	2.50672	.37330
2.86	.48402	9.37619	3.72375	2.51794	.37014
2.87	.48335	9.44304	3.73361	2.52920	.36700
2.88	.48269	9.51012	3.74341	2.54050	.36389
2.89	.48203	9.57744	3.75316	2.55183	.36080
2.90	.48138	9.64499	3.76286	2.56321	.35773
2.91	.48074	9.71277	3.77251	2.57462	.35469
2.92	.48010	9.78079	3.78211	2.58606	.35167
2.93	.47946	9.84904	3.79166	2.59755	.34867
2.94	.47884	9.91752	3.80117	2.60907	.34570
2.95	.47821	9.98624	3.81062	2.62064	.34275
2.96	.47760	10.05519	3.82002	2.63223	.33982
2.97	.47699	10.12437	3.82937	2.64387	.33692
2.98	.47638	10.19379	3.83868	2.65555	.33404
2.99	.47578	10.26344	3.84793	2.66726	.33118
3.00	.47519	10.33333	3.85714	2.67901	.32834
3.50	.45115	14.12500	4.26087	3.31505	.21295
4.00	.43496	18.50000	4.57143	4.04688	.13876
4.50	.42355	23.45833	4.81188	4.87509	.09170
5.00	.41523	29.00000	5.00000	5.80000	.06172
6.00	.40416	41.83333	5.26829	7.94059	.02965
7.00	.39736	57.00000	5.44444	10.46939	.01535
8.00	.39289	74.50000	5.56522	13.38672	.00849
9.00	.38980	94.33334	5.65116	16.69273	.00496
10.00	.38758	116.50000	5.71429	20.38750	.00304
∞	.37796	∞	6.00000	∞	0

Anhang

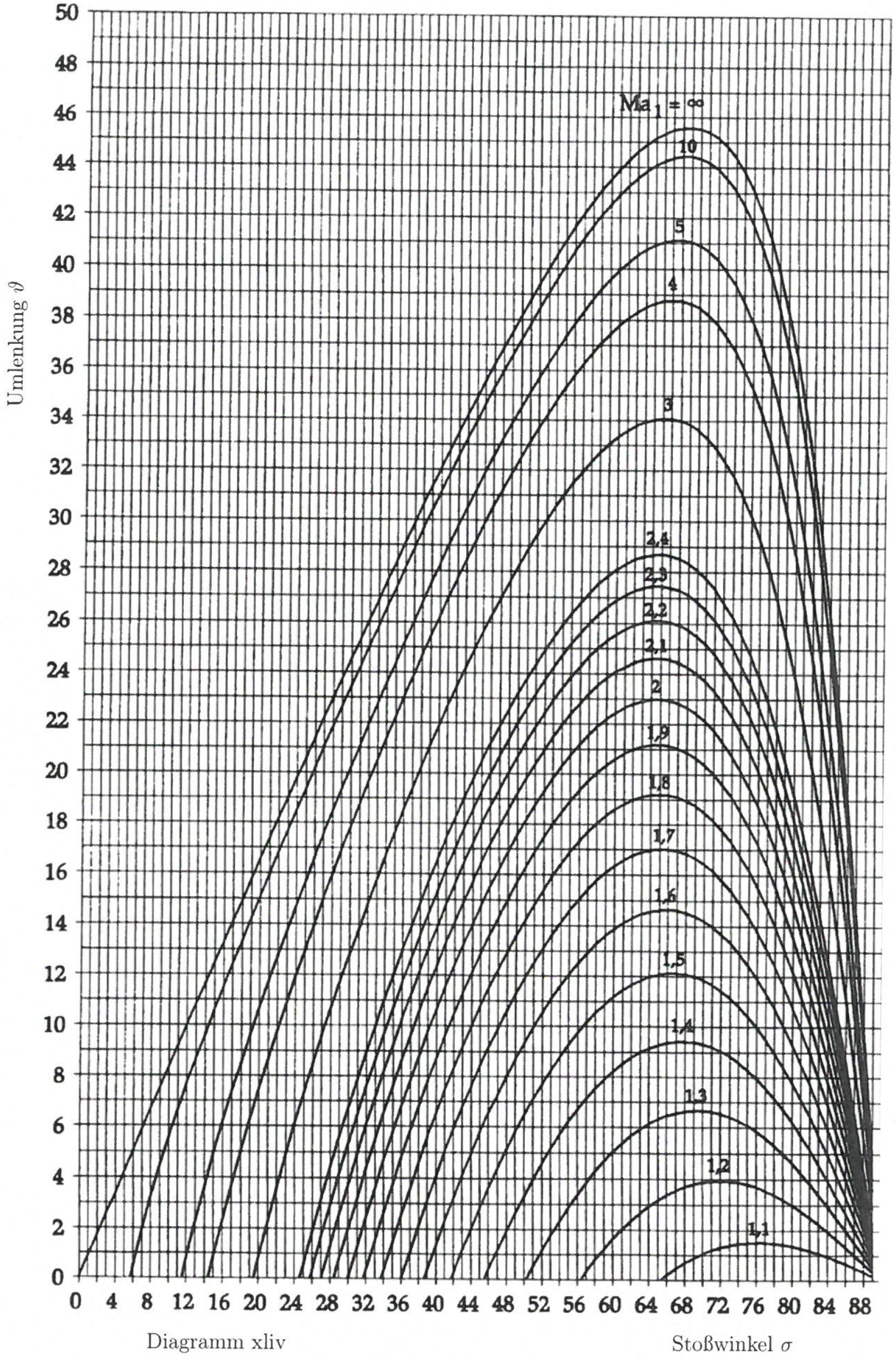


Diagramm xliv

Abgelößter Verdichtungsstoß
Abgelößte Kopfwelle

Anhang

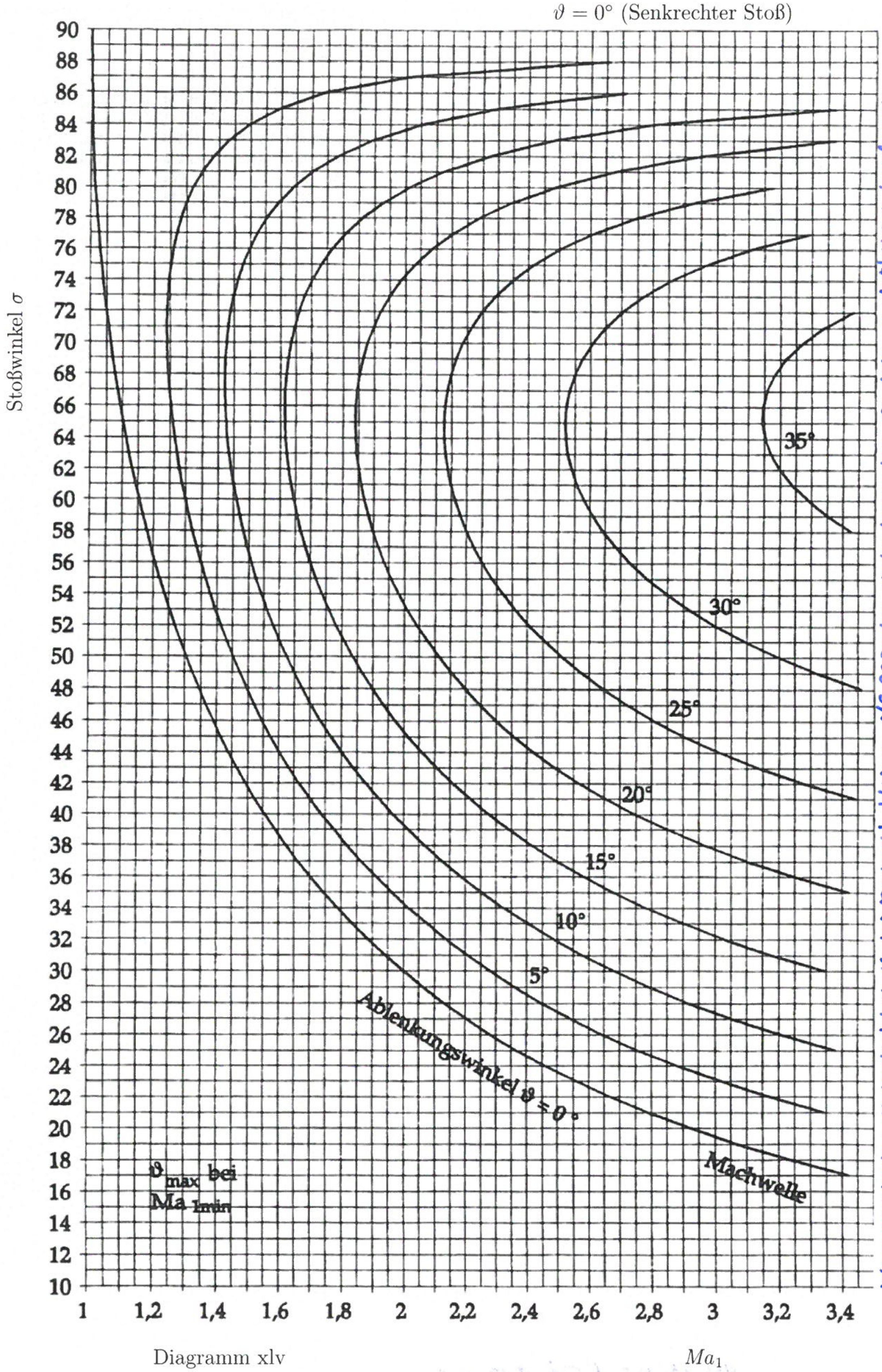


Diagramm xlv

Wenn maximaler Umlenkwinkel ϑ bei Machzahl kleiner als gesamte Umlenkung des Profils -> abgelösene Kopfwelle

Maximaler Umlenkwinkel bei bestimmter Machzahl mit Diagramm xLiv finden

Anhang

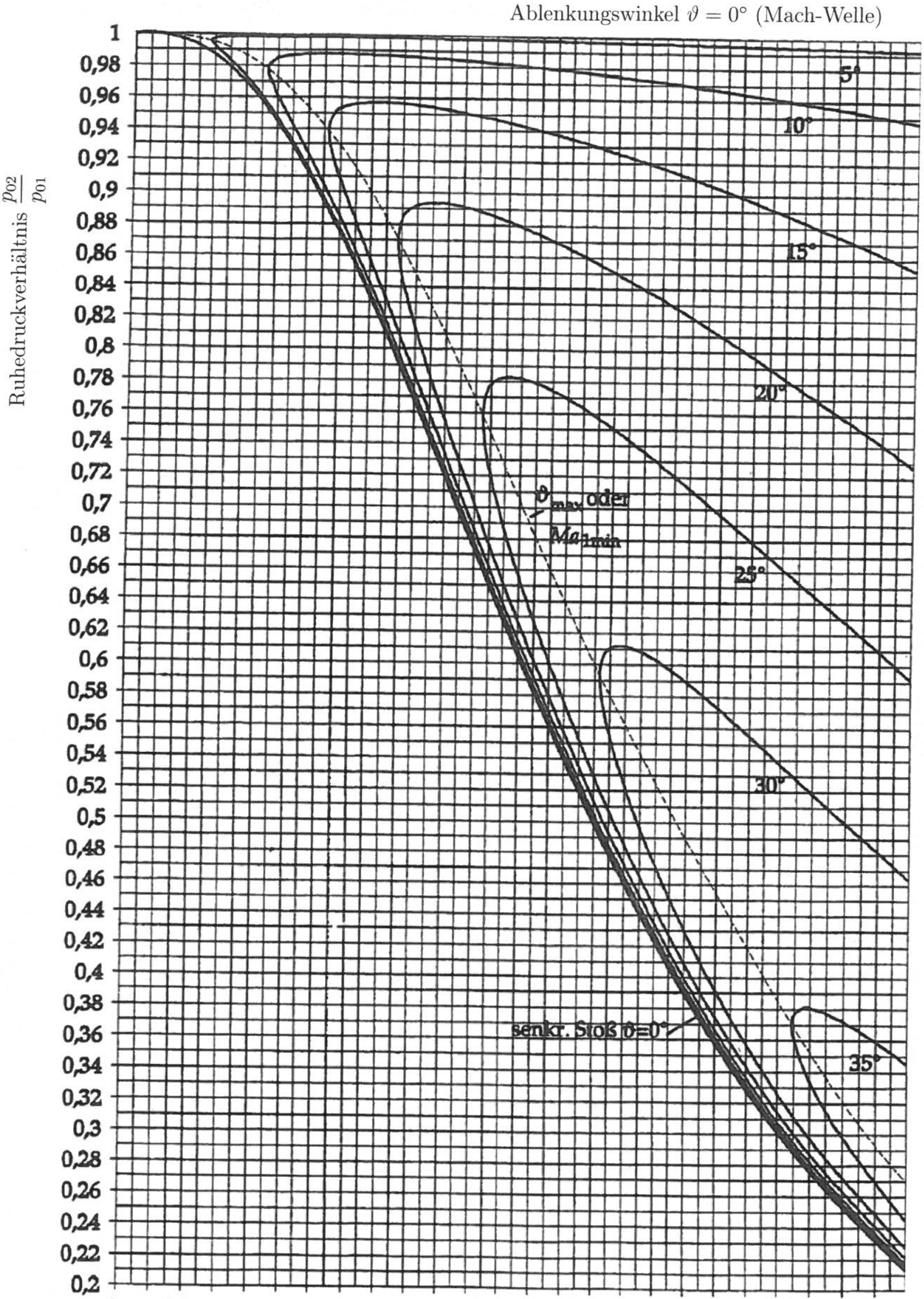


Diagramm xlvi

Anhang

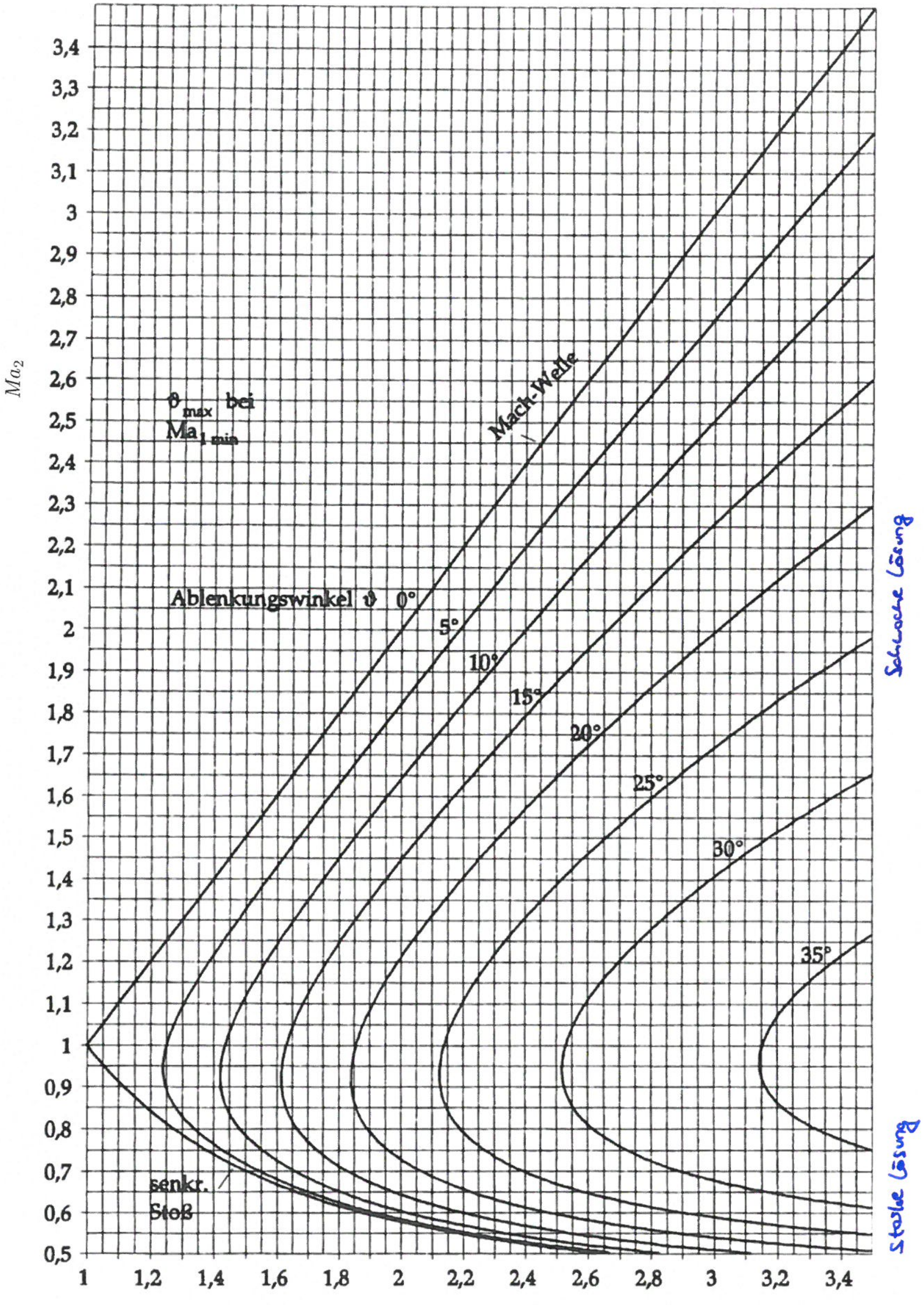


Diagramm xlvii

Anhang

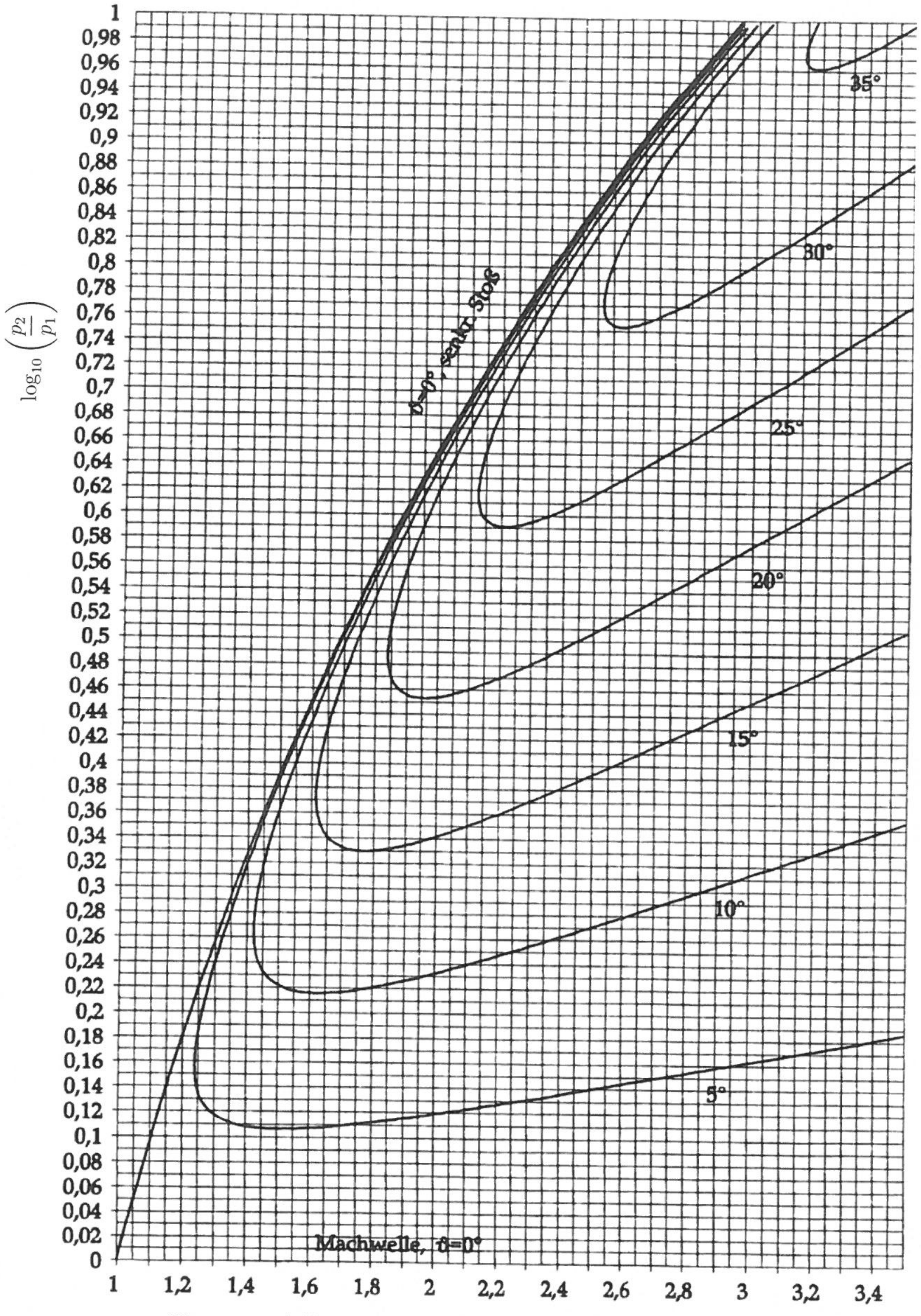


Diagramm xlviii

Ma_1

Strömung Klausur H23

①

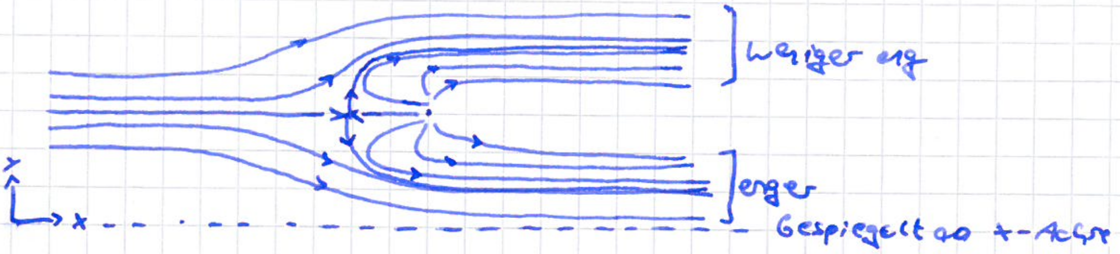
a) $\psi(x,y) = U_\infty y + \frac{E}{2\pi} \arctan\left(\frac{y-a}{x-d}\right) + \frac{E}{2\pi} \arctan\left(\frac{y+b}{x-d}\right)$

Translationsströmung in x Richtung mit Stärke U_∞

Quelle bei (d,a) mit Stärke E

Quelle bei $(d,-b)$ mit Stärke E

b)



c) Staupunkte; $u,v = 0$

$$V_x = 0 = U_\infty + \frac{E}{2\pi} \frac{x-d}{(x-d)^2+(y-a)^2} + \frac{E}{2\pi} \frac{x-d}{(x-d)^2+(y+b)^2}$$

$$V_y = 0 = 0 + \frac{E}{2\pi} \frac{y-a}{(x-d)^2+(y-a)^2} + \frac{E}{2\pi} \frac{y+b}{(x-d)^2+(y+b)^2}$$

$x_1 = 97{,}5\,m \quad y_1 = -79{,}87\,m$

$x_2 = 97{,}5\,m \quad y_2 = 29{,}88\,m$

d) Volumenstrom $\frac{\dot{V}}{t} = |\psi_2 - \psi_1|$

$\psi_1 = \psi(97{,}5, -79{,}87) = -32{,}4\ \frac{m^2}{s}$

$\psi_2 = \psi(97{,}5, 29{,}88) = 52{,}4\ \frac{m^2}{s}$

$\frac{\dot{V}}{t} = 84{,}8\ \frac{m^2}{s}$

e) $V_{Rei} = (U,0)$

$$\psi(x,y) = U_\infty y + \frac{E2}{2\pi} \arctan\left(\frac{y-d}{x-b}\right) - \frac{E1}{2\pi} \arctan\left(\frac{y-d}{x-a}\right) + \frac{\mu}{2\pi} \frac{y-d}{(x-c)^2+(y-d)^2}$$

$U_\infty < 0 \quad E_1 > 0 \quad E_2 > 0 \quad \mu > 0$

f) (1) : Strömung wird eingesogen und wieder ausgesogen -> keine Verdrängung

(2) : Richtig, Kreiszylinderumströmung wird durch Dipol (der Strömung entgegengesetzt) erzeugt.

(3) : Wird nicht durch Dipol dargestellt -> Stromlinien laufen auseinander, keine Kreise um Singularität -> Hier: Quelle.

g) $\Psi(x,y) = U_\infty y + \frac{E}{2\pi} \arctan\left(\frac{y}{x-6}\right)$ $U_\infty = -2 \frac{m}{s}$

Damit Plankton das Maul trifft muss gelten: $\Psi(20,2, \frac{1,5}{2}) \overset{!}{=} \Psi(100, y_1)$

$\Psi(20,2, -\frac{1,5}{2}) = \Psi(100, y_2)$

$y_1 = 0,6464\,m$ $y_2 = -0,1464\,m$

max. Breite des Planktons : 1,293 m

② $\kappa = 1,4$ $R = 287,1 \frac{J}{kgK}$

a) $p_0 = 9$ bar $\rho_0 = 4,467 \frac{kg}{m^3}$

$Ma_1 = 2,65$ $A_e = 0,1257\,m^2$

Aus Tabelle: $\frac{A_e}{A^*} = 3,03588 \rightarrow A^*_e = 0,04139\,m^2$

$\frac{p_1}{p_0} = 0,47170 \cdot 10^{-1} \rightarrow p_1 = 0,424$ bar

Rayleigh-Pitot-Formel : $\frac{p_{Pitot}}{p_1} = 9,519$

$p_{Pitot} = 4,036$ bar Abgelöste Kopfwelle vor Pitotsonde führt zu Totaldruckverlust.

$c_1 = \sqrt{\kappa R T_1}$

$= \sqrt{\kappa \frac{p_1}{\rho_1}}$

$= 345,2 \frac{m}{s} \rightarrow T_1 = 296,4\,K$

~~$\frac{T_1}{T_0} = 0,41589 \rightarrow T_1 = (1 + \frac{\kappa-1}{2} Ma_1^2)^{-1}$~~

b) Formel 9.49 : $\vartheta_{12} = 9,898°$ 1->2

9,59 : $Ma_2 = 2,225$ (mit Diagramm)

2->3 : 9.49 : $\vartheta_{23} = 10° \rightarrow \sigma_{23} = 475°$

$Ma_3 = 1,42$

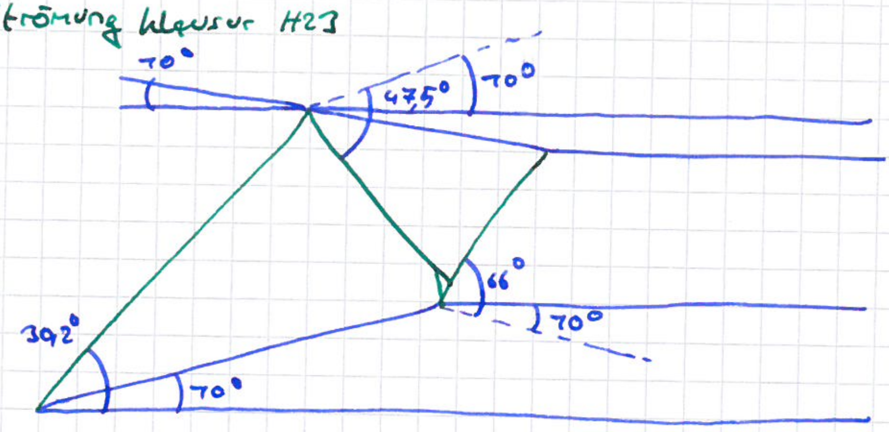
3->4 : $\vartheta_{34} = 10°$ $\sigma_{34} = 66°$

$Ma_4 = 0,95$

(Alles aus Diagramm x(Ma) und x(ν))

Strömung Klausur H23

c)



d) $Ma_1 = 2,8$ $Ma_2 = 2,0$ $Ma_3 = 1,5$

$\frac{p_{02}}{p_{01}} = 0,97$ $\frac{p_{03}}{p_{02}} = 0,89$ $\frac{p_{04}}{p_{03}} = 0,98$ Diagram XLVI

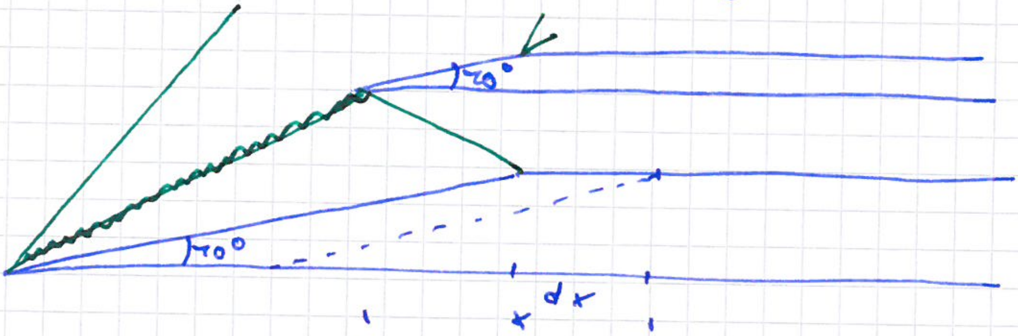
$\frac{p_{0e}}{p_{01}} = 0,846$

e) Anhang B2 → $\frac{p_{02}}{p_{01}} = 0,44156$

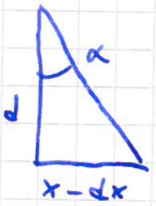
f) Verhältnis $\frac{A_e}{A^*}$ müsste ~~vergrößert werden~~. verkleinert werden

Nur A_e verkleinern oder A^* vergrößern geht auch.

g)



h) $T_0 = 411,2\ K$ $\frac{p_2}{\rho_2} = 84120\ \frac{m^2}{s^2}$ $d = 50\ mm$ $x = 120\ mm$



$\overset{\tan}{\cancel{\sin}}(\alpha) = \frac{x - dx}{d}$

$\frac{p_2}{\rho_2} = RT_2 \rightarrow T_2 = 293\ K \rightarrow \frac{T_2}{T_0} = 0,7125$

Tabelle Anhang B1 : $Ma_2 = 1,42$

$\vartheta_{12} = 10°$ $\vartheta_{23} =$

~~[illegible] = 9,565 [illegible] + [illegible]~~

~~[illegible]~~ $\nu(Ma_6) = \nu(Ma_2) + \vartheta_{23} \rightarrow = \cancel{\ } 10°$

$= 19,565 \rightarrow Ma_6 = 1,76$

~~[illegible]~~

$\alpha = \sigma_2 - 10°$: ~~[illegible]~~ 49.49, Tabelle XLV $\vartheta_2 = \cancel{\ } 10°$ $Ma_2 = 1,42$

$= \cancel{\ } 66°$

$\rightarrow dx = 86,27\ mm$

Strömung Klausur F23

$\boxed{1}$

①

a) Parallelströmung in x-Richtung ; Quelle mit Stärke $2\dot{E}$ bei $(-1,1)$

Zwei Senken mit Stärke $\dot{E}$ bei $(1,1)$ und $(2,1)$

b) $\Psi(x,y) = U_\infty y + \frac{2\dot{E}}{2\pi}\arctan\left(\frac{y-1}{x+1}\right) - \frac{\dot{Q}}{2\pi}\left(\arctan\left(\left(\frac{x-1}{y-1}\right)^{-1}\right) + \arctan\left(\left(\frac{x-2}{y-1}\right)^{-1}\right)\right)$

$u(x,y) = U_\infty + \frac{2\dot{E}}{2\pi}\frac{x+1}{(x+1)^2+(y-1)^2} - \frac{\dot{E}}{2\pi}\left(\frac{x-1}{(x-1)^2+(y-1)^2} + \frac{x-2}{(x-2)^2+(y-1)^2}\right)$

$V(x,y) = 0 + \frac{2\dot{E}}{2\pi}\frac{y-1}{(x+1)^2+(y-1)^2} - \frac{\dot{E}}{2\pi}\left(\frac{y-1}{(x-1)^2+(y-1)^2} + \frac{y-1}{(x-2)^2+(y-1)^2}\right)$

c) Aufgrund der Symmetrie sind alle Staupunkte bei $\underline{\underline{y_{sp} = 1}}$

x Koordinaten über $v(x,1) \overset{!}{=} 0$

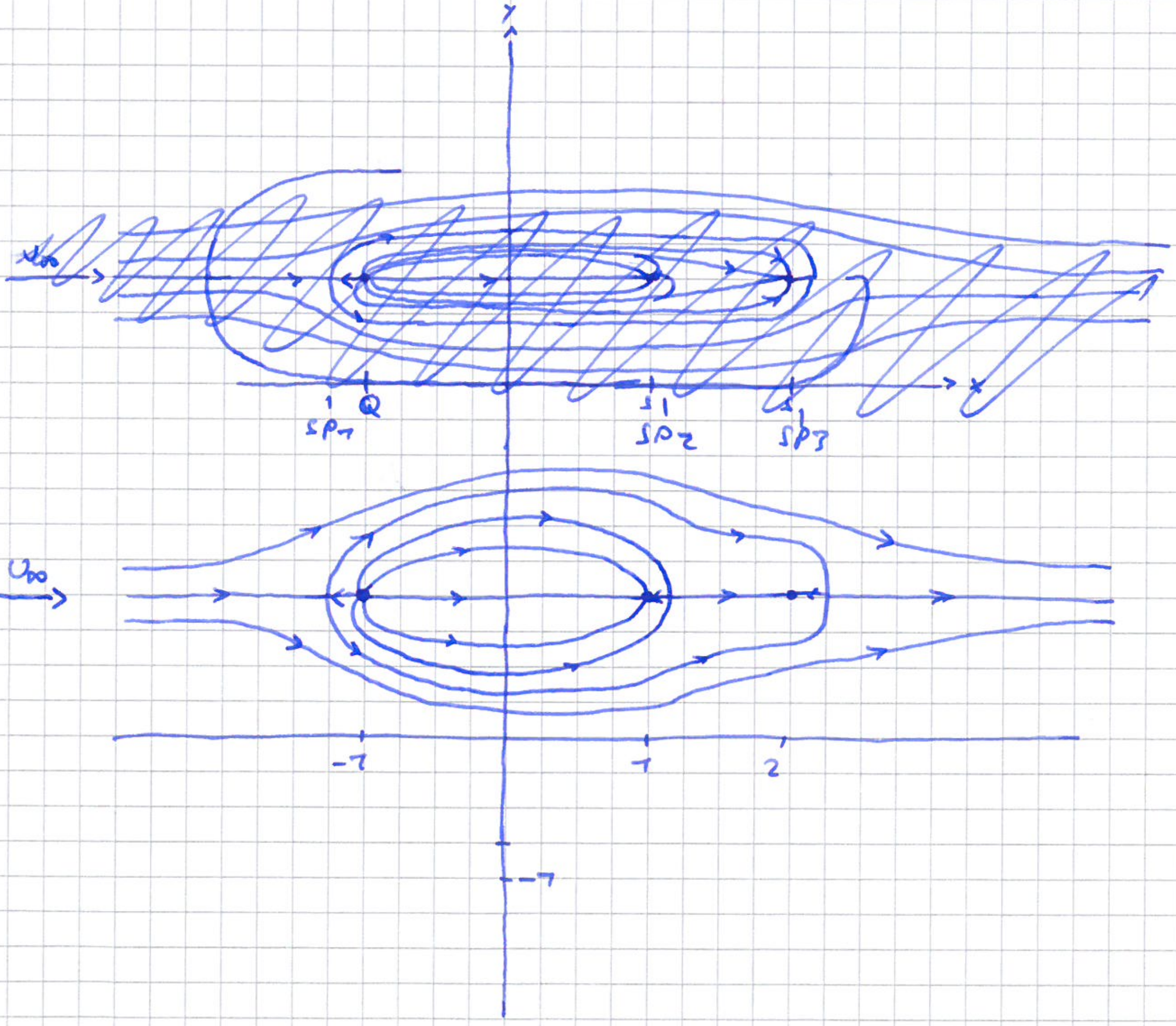
$v(x,1) \overset{!}{=} 0$

Drei Staupunkte: für $U_\infty = \dot{E}$ gilt:

$x_{sp1} = -1{,}285 \qquad x_{sp2} = 1{,}12 \qquad x_{sp3} = 2{,}165$

Länge $l = x_{sp1} + |x_{sp1}| = 3{,}42$

d)



e) Wirbel ~~gegen den~~ Im Urzeigersinn

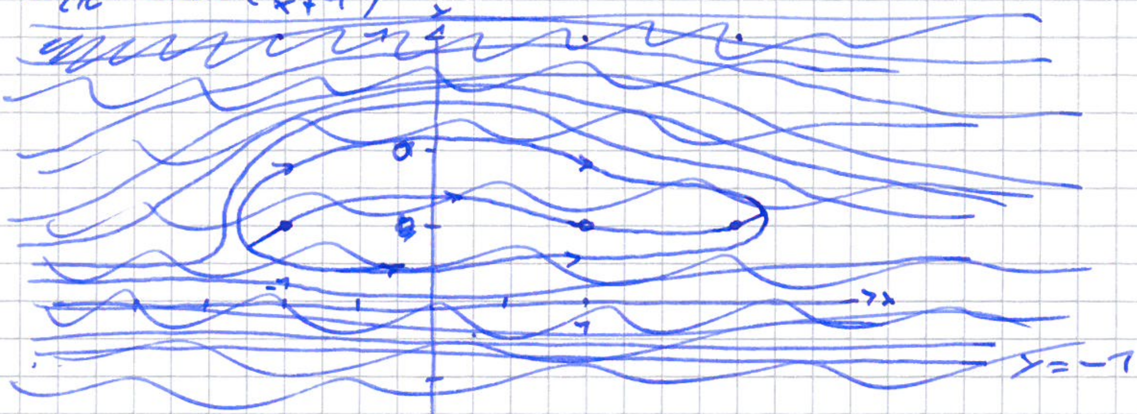
$$\Psi(x,y) = U_\infty y + \frac{2E}{2\pi}\arctan\left(\frac{y-1}{x+1}\right) - \frac{E}{2\pi}\left(\arctan\left(\frac{y-1}{x-1}\right) + \arctan\left(\frac{y-1}{x-2}\right)\right) +$$
$$+ \frac{\Gamma}{2\pi}\ln\left(\sqrt{(x+1)^2+(y-1)^2}\right)$$

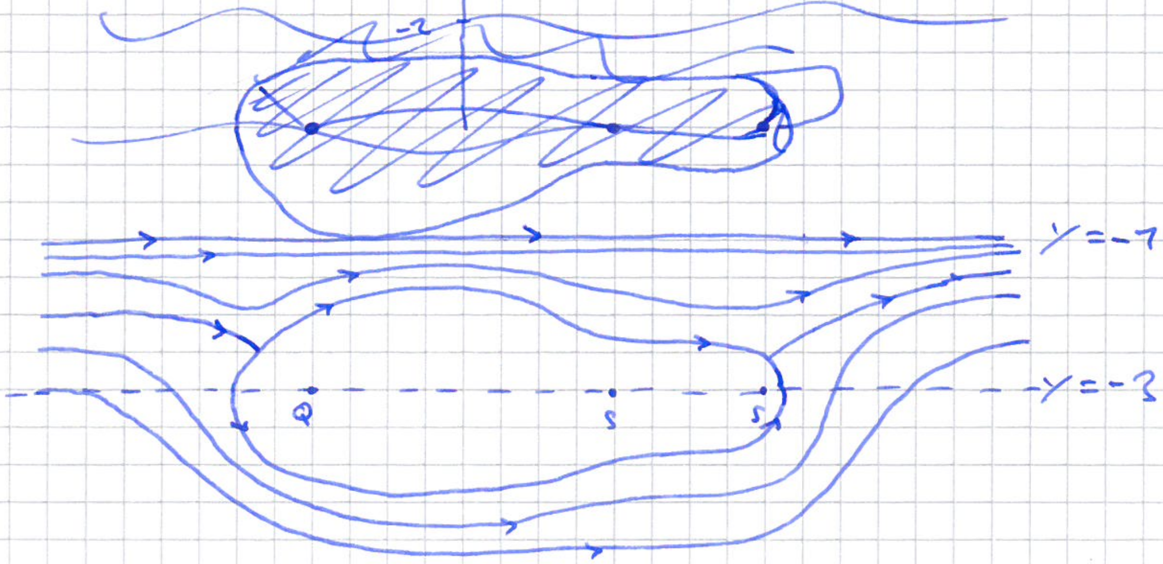
f) Bodenfläche bei $y = -1$

Spiegelung der Singularitäten + Umkehrung des Wirbels :

$$\psi(x,y) = U_\infty y + \frac{2E}{2\pi}\arctan\left(\frac{y-1}{x+1}\right) - \frac{E}{2\pi}\left(\arctan\left(\frac{y-1}{x-1}\right) + \arctan\left(\frac{y-1}{x-2}\right)\right) +$$
$$+ \frac{\Gamma}{2\pi}\ln\left(\sqrt{(x+1)^2+(y-1)^2}\right) \cancel{+ \frac{2E}{2\pi}\arctan\left(\frac{y+3}{x+1}\right) + \frac{E}{2\pi}\arctan\left(\frac{y+3}{x-2}\right)} + -$$
$$\cancel{+\frac{\Gamma}{2\pi}\ln}$$
$$+ \frac{2E}{2\pi}\arctan\left(\frac{y+3}{x+1}\right) - \frac{E}{2\pi}\left(\arctan\left(\frac{y+3}{x-1}\right) + \arctan\left(\frac{y+3}{x-2}\right)\right) - \frac{\Gamma}{2\pi}\ln\left(\sqrt{(x+1)^2+(y+3)^2}\right)$$

$$\phi(x,y) = U_\infty x + \frac{2E}{2\pi}\ln\left(\sqrt{(x+1)^2+(y-1)^2}\right) - \frac{E}{2\pi}\left(\ln\left(\sqrt{(x-1)^2+(y-1)^2}\right) + \ln\left(\sqrt{(x-2)^2+(y-1)^2}\right)\right)$$
$$\mp \frac{\Gamma}{2\pi}\arctan\left(\frac{y-1}{x+1}\right)$$
$$+ \frac{2E}{2\pi}\ln\left(\sqrt{(x+1)^2+(y+3)^2}\right) - \frac{E}{2\pi}\left(\ln\left(\sqrt{(x-1)^2+(y+3)^2}\right) + \ln\left(\sqrt{(x-2)^2+(y+3)^2}\right)\right)$$
$$+ \frac{\Gamma}{2\pi}\arctan\left(\frac{y+3}{x+1}\right)$$





Strömung Klausur F23 $\boxed{3}$1

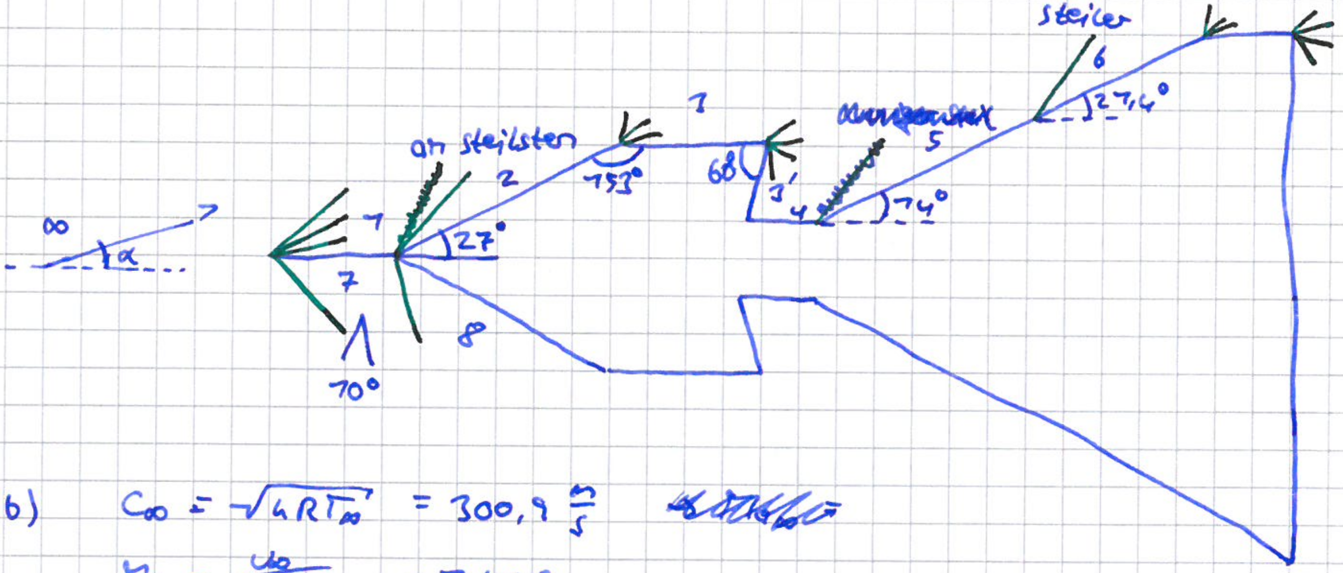
g) In Reibungsbehafteten Fall würde sich an der Profiloberfläche und an der Wand eine Grenzschicht ausbilden die den Wiederstand erhöht und die Geschwindigkeiten in Profilnähe verringert.

$V(2,-1) = 0$, weil Wand nur Geschw. in x-Richtung

$v(2,-1) = \frac{17}{65\pi} E - \frac{2}{13\pi} \Gamma + v_\infty$? leicht anders wie Lösung

② $u_\infty = 436{,}0 \frac{m}{s}$ $T_\infty = -48°C = 225{,}2\,K$ $p_\infty = 25 \frac{kN}{m^2} = 0{,}25\,bar$ $\alpha = 10°$

a)



b) $c_\infty = \sqrt{\kappa R T_\infty} = 300{,}9 \frac{m}{s}$

$Ma_\infty = \frac{u_\infty}{c_\infty} = 1{,}449$

$\sigma_{01} = 61{,}16°$ (9.49)

$Ma_1 = 1{,}03$ (9.59)

$p_{0,\infty} = 0{,}8528\,bar$ (9.20a)

c) $Ma_\infty = 2{,}5$ $p_\infty = 0{,}25\,bar$

9.74 : $\nu(Ma_\infty) = 39{,}12°$

$\nu(Ma_1) = 49{,}12°$ (9.75)

$Ma_1 = 2{,}967$ (9.74)

$\sigma_{12} = 47{,}35°$ (9.49)

$Ma_2 = 1{,}58$ (9.59)

Öffnungswinkel des Fächers: $\delta = \mu_2 - \mu_1 + \Delta\vartheta_{\infty,1}$

$\mu = \arcsin\left(\frac{1}{Ma}\right) \rightarrow \mu_1 = 23{,}58$; $\mu_2 = 19{,}7$

$\delta = + 13{,}88°$

d) $M_7 = 2,1$

ϑ_{max} bei M_7: $\approx 25° < 27°$

-> Es bildet sich ein abgelößter verdichtungsstoß aus

e) $M_3 = 2,1$

Expansionsfächer von 3 zu 3'

9.74 : $\nu(Ma_1) = 29,1°$

9.75 : $\nu(Ma_3') =$ ~~[illegible]~~ $149,1°$ Unmöglich weil > 130,454

9.74 : $Ma_3' =$ ~~[illegible]~~ ∞

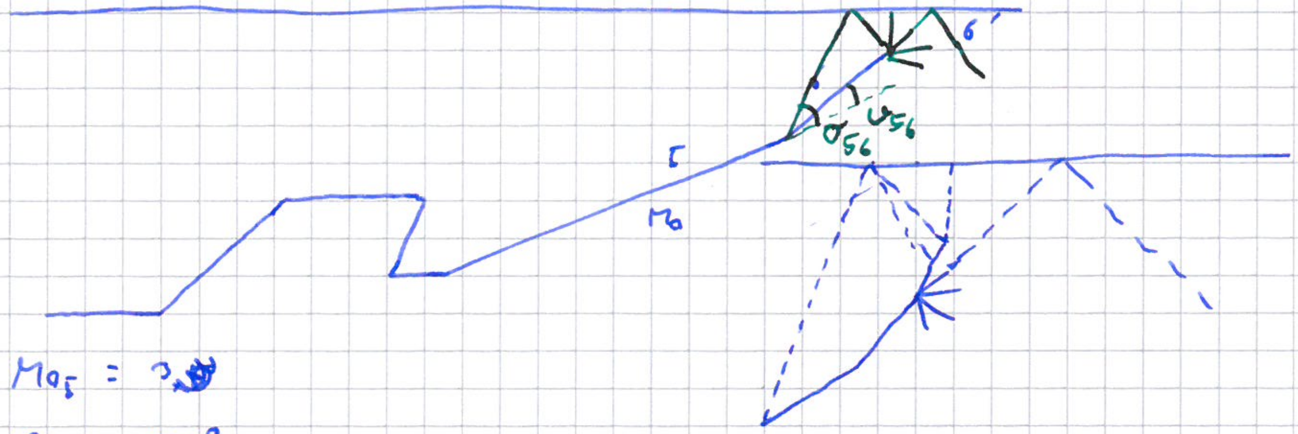
$\mu_3' = 0°$

$\Delta\vartheta_{3,3'} = \nu_3 - \nu_3' = 101,4°$ $\mu_3 = \arcsin\left(\frac{1}{Ma_3}\right) = 28,44°$

$\delta = \mu_3' - \mu_3 + \Delta\vartheta_{3,3'} = 129,8°$

Nahe der Wand entsteht ~~[illegible]~~ "vor" dem Expansionsfächer ein Vakuum. In Wirklichkeit würde durch Reibungseffekte ein wesentlich geringerer Öffnungskegel entstehen.

f)



$Ma_5 = 3,$[illegible]

$\vartheta_{56} = 7,4°$

9.49 : ~~Ma~~ $\sigma_{56} = 25,1°$ -> 9.59 : $Ma_6 = 2,632$

9.74 : $\nu(Ma_6) = 42,73°$ -> 9.75 : $\nu(Ma_6') = 3,834$

$\delta = \mu_{6'} - \mu_6 + \Delta\nu_{66'}$

↳ $= 21,4°$?

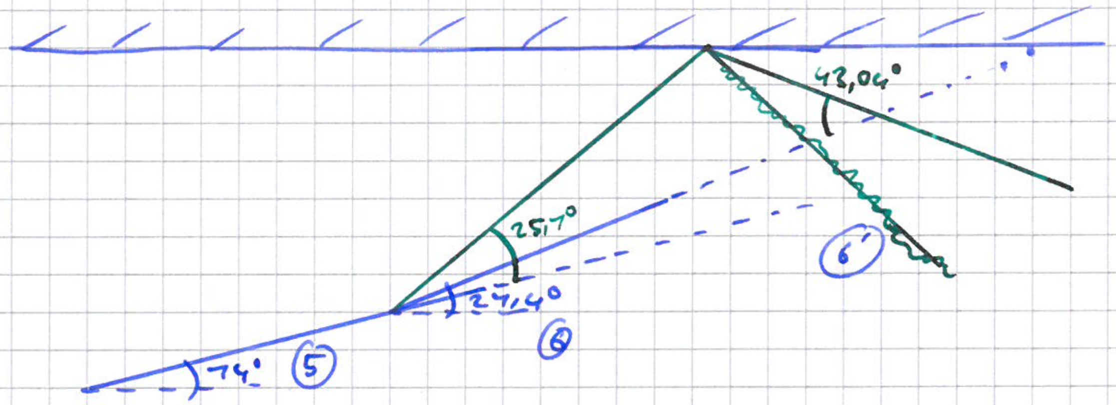
$\mu_6 = 20,8°$ $\mu_{6'} = 14,62°$

$\delta = 15,21$ ↯

Nochmal neuer Versuch auf nächster Seite!

Strömung klausur F23

c)



$Ma_5 = 3$

$\vartheta_{56} = 7{,}4°$

9.49 : $\sigma_{56} = 25{,}7°$

9.59 : $Ma_6 = 2{,}632$

$\vartheta_{66'} = 27{,}4°$

9.49 : $\sigma_{66'} = 43{,}04°$

9.59 : $Ma_{6'} = 1{,}674$

Strömung Klausur H22

①

a) $-U_\infty x + \frac{M}{2\pi}\frac{x-a}{(x-a)^2+y^2} = \psi(x,y)$

Staupunkt soll bei $(a, -\frac{d}{2})$ liegen

$v_y(x,y) = U_\infty + \frac{M}{2\pi}\frac{2y(x-a)}{((x-a)^2+y^2)^2}$

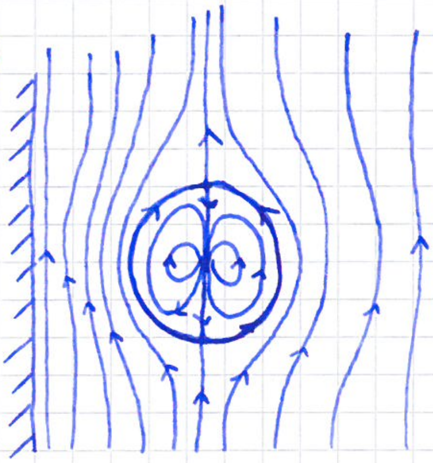
$M = \frac{d^2\pi U_\infty}{2}$

b) Spiegelungsprinzip: Gleicher Dipol qua bei $(-a, 0)$

$F(z) = -iU_\infty z - i\frac{M}{2\pi}\cdot\frac{1}{z-a} - i\frac{M}{2\pi}\cdot\frac{1}{z+a}$

c) $\omega'(z) = \frac{dF(z)}{dz} = \frac{iM}{2\pi}\left(-\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right) = 0$

d)



Gespiegelt auf anderer Seite des Ufers

e) $y=0 \quad \psi(x,y) = -U_\infty x + \frac{M}{2\pi}\frac{x-a}{(x-a)^2+y^2} + \frac{M}{2\pi}\frac{x+a}{(x+a)^2+y^2}$

$\frac{\dot{V}}{t} = |\psi_2(0,0) - \psi_1(a-\frac{d}{2},0)|$

$\psi_2(0,0) = 0 \qquad \psi_1(a-\frac{d}{2},0) = -7933$

$\dot{V} = t\cdot|\psi_2-\psi_1| = 6\cdot t\cdot \langle u\rangle$

$\langle u\rangle = \frac{|\psi_2-\psi_1|}{6} = 1{,}133\ \frac{m}{s}$

f) $\psi(x,y) = U_\infty y + \frac{E_1}{2\pi}\arctan\left(\frac{y}{x+b/2}\right) + \frac{E_2}{2\pi}\arctan\left(\frac{y}{x-b/2}\right)$

$U_\infty y$: Translationsströmung in x-Richtung mit Geschw. $U_\infty > 0$

Quelle bei $(-\frac{b}{2}, 0)$ mit Stärke $E_1 > 0$

Quelle bei $(\frac{b}{2}, 0)$ mit Stärke $E_2 < 0$ (Senke)

g) $E_1 = -E_2$ (Damit sich eine geschlossene Körperkontur bildet.

h) $\psi(x',y') = \psi_h(x,y)$ $\qquad x' \to \infty$

$u_\infty y' + \frac{E}{2\pi}\overset{\to=0}{\cancel{\arctan}(0)} + \frac{E}{2\pi}\overset{\to=0}{\cancel{\arctan}(0)} = \psi_h(x,y)$

$y' = \frac{\psi_h}{u_\infty}$

i) rot (u) = 0 → Winkel ändert sich nicht → $\alpha_h = \alpha_\infty$

② $A_2 = 400\,cm^2$ $\quad A_{min} = 237\,cm^2$ $\quad \kappa = 1{,}4$ $\quad R = 287{,}1\,\frac{J}{kgK}$

$T_1 = 0°C$ $\quad p_1 = 1\,bar$ $\quad M_1 = 0{,}5$

a) Lavaldüse ; Lavalzahl, Strömungsgeschwindigkeit bezogen auf Lavalgeschwindigkeit, identisch zu Ma bei Ma = 0, Ma = 1

b) 9.26 → $Ma_2 = 2$

9.20a → $p_0 = 1{,}186$ bar

↳ $T_0 =$ 286,8K

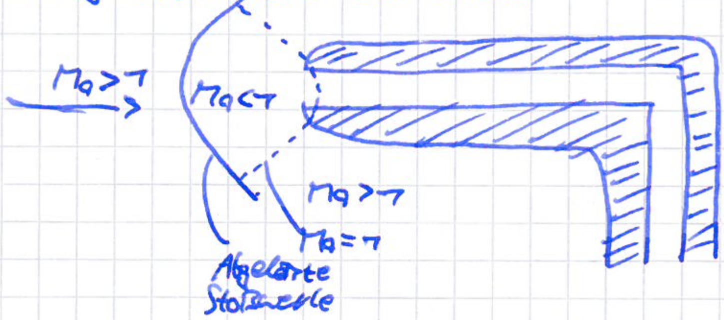
9.24 → $\dot{m} = 6{,}707\,\frac{kg}{s}$ ↯ $7{,}77\,\frac{kg}{s}$ in Lösung trotz gleicher Zwischenergebnisse

c) $p_{Pit,2} = 10\,bar$

9.41 → $p_2 = 1{,}773$ bar

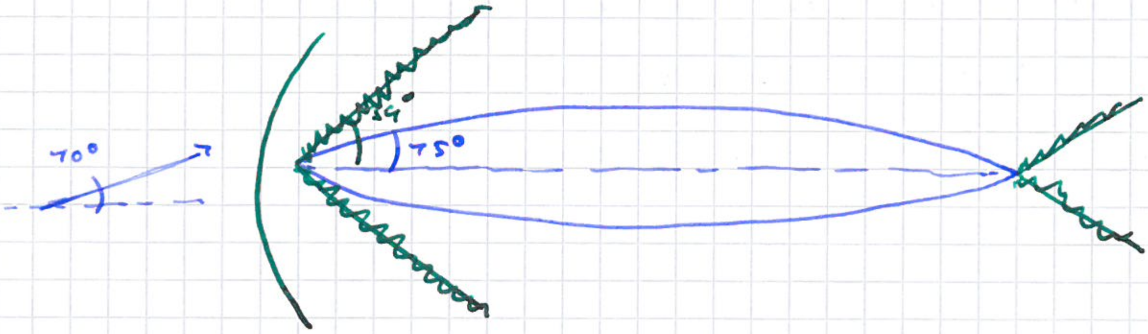
9,20a → $p_{02} = 13{,}87$ bar

Totaldruckverlust durch abgelößten Stoß vor der Sonde



d) $\vartheta = 15°$ $\quad \alpha = 10°$

tlix



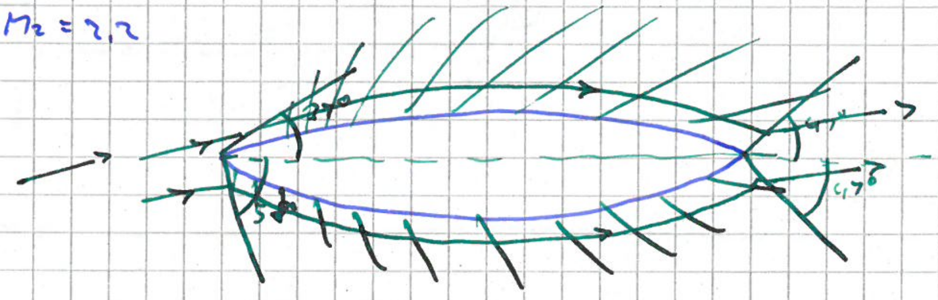
d) Maximaler Druckverlust bei $\sigma = 90°$

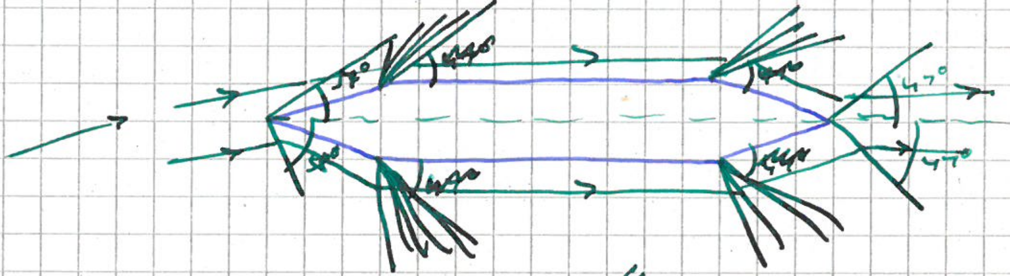
9.58 -> $p_{02} = 10$ bar -> $\frac{p_{02}}{p_{01}} = 0,721$

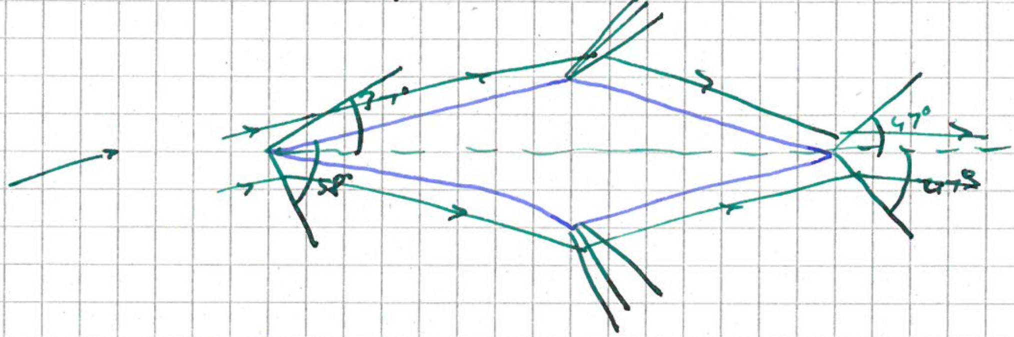
Anstellwinkel um 2° verringern

Machzahl erhöhen auf $Ma_2 = 2,125$ -> $\vartheta_{max} = 25°$

e) $M_2 = 2,2$







In Wirklichkeit "haftet" die Strömung an der Profilwand und wird dort so auf subsonische Geschwindigkeiten gebremst.

f) $M_2 = 2,2$ $P_2 = 1,773$ bar

$M_{3,oben}$: $\sigma = 37°$; $\vartheta = 15°$; 9.59 -> $Ma_{3,oben}$ = 2

-> $Ma_{3,unten}$ = 1,1

: $P_{3,oben}$ = 2,36 bar

9.470 : $P_{3,unten}$ = 6,905 bar $\vartheta_{14} = 30°$

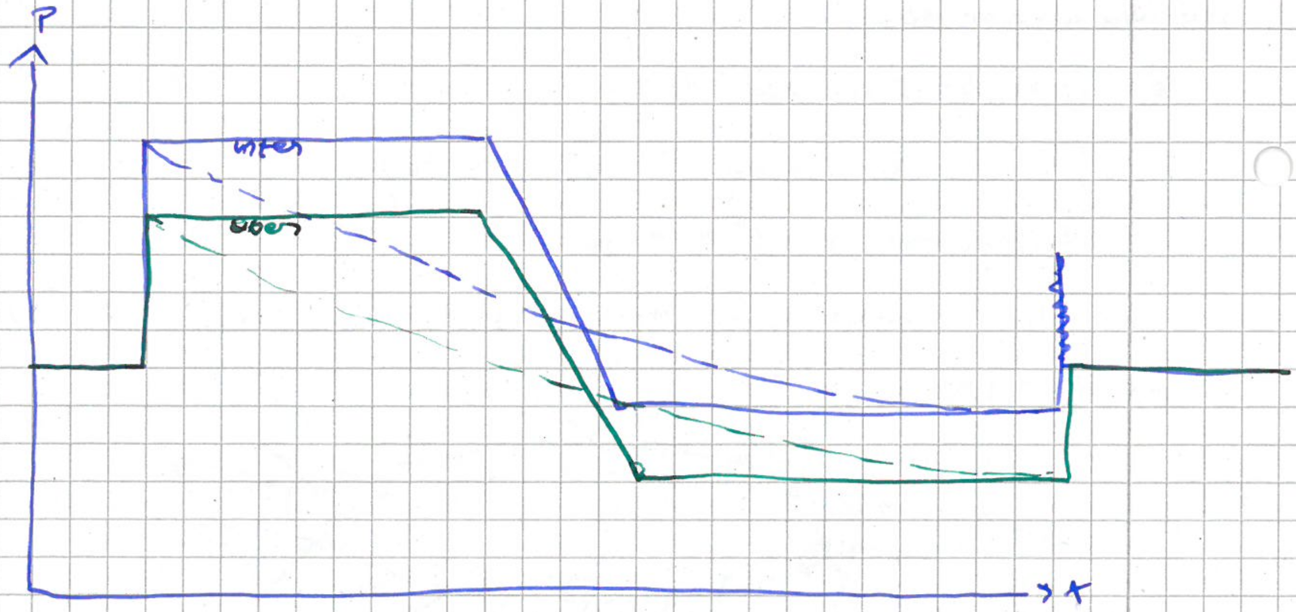
$Ma_{4,oben}$ = (mit 9.75) = 3,268

$Ma_{4,unten}$ = 2,185

9.776 : $P_{4,oben} = 0,2925$ bar

$P_{4,unten} = 1,472$ bar

g)



Strömung Klausur F22

⑦

a) $F(z) = i \ln(z) \overset{\text{neg}}{\frac{\Gamma}{2\pi}} - i \overset{\text{Pos}}{\frac{\Gamma}{2\pi}} \ln(z-2Hi) + \frac{M}{2\pi} \frac{1}{z-(D+Hi)}$

$\Gamma_{1,2} > 0 , M > 0$

b) $w^*(z) = \frac{dF(z)}{dz} = \frac{i\Gamma}{2\pi z} - \frac{1 \cdot i\Gamma}{(z-2Hi)2\pi} \cancel{+} - \frac{M}{2\pi} \frac{1}{(z-(D+Hi))^2}$

Wand : ~~[illegible]~~ warum "+"

$w^*(z = x+Hi) = \frac{H\Gamma}{\pi(x^2+H^2)} - \frac{M}{2\pi(x-D)^2}$

Keine Komponente in y-Richtung?

$\mathrm{Im}(w^*(z=x+Hi)) = 0$ ✓ → Fiktive Wand zwischen Wirbeln

c) $x = 0 \quad y \in [H-k, H+k] ; |k| < H$

Bei $|k| > H$ würden die Bereiche auf der anderen Seite der Dipole den inneren Volumenstrom wieder ausgleichen.

$\Psi(z) = \mathrm{Im}(F(z)) = v_y(x,y) =$

$= \frac{-\Gamma_1}{2\pi} \frac{x}{x^2+(y-2H)^2} + \frac{\Gamma_2}{2\pi} \frac{x}{x^2+y^2} + \frac{M}{2\pi} \frac{2(x-d)(y-H)}{((x-D)^2+(y-H)^2)^2}$

$\frac{\dot{V}}{t} = |\Psi_2(x,y) - \Psi_1(t,y)|$

~~$\Psi_2 = \Psi(0,2H) = -\frac{DHM}{(D^2+H^2)^2\pi}$~~

~~$\Psi_1 = \Psi(0,0) = \frac{DHM}{(D^2+H^2)^2\pi}$~~

$\Psi_2 = \Psi(0, H-k) =$

$\Psi_2 = \Psi(0, H+k) =$

Nicht nötig für Aufgabe c)

$\Psi(x,y) = \frac{-\Gamma}{2\pi} \ln(-\sqrt{x^2+(y-2H)^2}) + \frac{\Gamma}{2\pi} \ln(-\sqrt{x^2+y^2}) - \frac{M}{2\pi} \frac{y-H}{(x-D)^2+(y-H)^2}$

$\frac{\dot{V}}{t} = \frac{\Gamma}{2\pi} \ln\left(\left|\frac{H-k}{H+k}\right|\right) + \frac{kM}{2(D^2+k^2)\cdot\pi}$

d) kein Dipol , $v_y(x,y) = 0$ auf $y = H$

~~[illegible]~~

$v_x(x=x, y=H) = -\frac{\Gamma}{2\pi} \frac{-H}{x^2+(-H)^2} + \frac{\Gamma}{2\pi} \frac{H}{x^2+H^2}$

$= \frac{H\Gamma}{\pi(x^2+H^2)} \quad ; x=0$

$\Rightarrow \frac{\Gamma}{\pi H}$

$U_\infty = \frac{\Gamma}{\pi H}$; Dipol bei $(D,0)$

e) $F(z) = U_\infty \cdot z + \frac{M}{2\pi} \frac{1}{z-D}$

f) Staupunkt : $w' = 0 = \frac{dF(z)}{dz} = U_\infty - \frac{M}{2\pi(z-D)^2}$

Staupunkt bei $z = D \pm \frac{\sqrt{2M}}{2\sqrt{\pi \cdot U_\infty}}$

Symmetrie: $y_{sp} = 0 \rightarrow x_{sp_{1,2}} = D \pm \sqrt{\frac{M}{2\pi U_\infty}}$

Länge des Windkanalmodells: $x_{sp,2} - x_{sp,1} = 2\sqrt{\frac{M}{2\pi U_\infty}}$

g) $x_s = D$

y Pos. der Sonde ist auf Oberfläche der Dipol-Kontur

$\rightarrow y_s = \sqrt{\frac{M}{2\pi U_\infty}}$

$v_y(\text{Sonde}) = 0$

$v_x(x,y) = U_\infty - \frac{M}{2\pi} \frac{(x-D)^2 - y^2}{((x-D)^2+y^2)^2}$

$v_x(0, y_s) = 2U_\infty$

Ungebungsdruck :

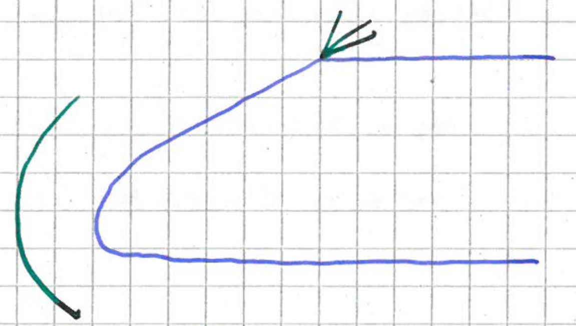
$p_\infty + \frac{\rho}{2} U_\infty^2 = p_s + 2\rho U_\infty^2$

$p_\infty = s + \frac{3}{2} \rho U_\infty^2$

Strömung Klausur F22

② $T = T_{0m} - 6{,}5\,K\,\frac{\text{Höhe}}{1000\,m}$

a) $Ma_1 = 2{,}0$ $H_1 = 20\,km$ $T_{0m} = 288{,}75\,K$



b) $T_{0,20km} = T_1 = 158{,}2\,K$

BZ Tabelle $T_2 = 267\,K$

9.19 : $T_0 = 284{,}8\,K = T_{max}$

maximaler relatives Totaldruckverlust auf Staustromlinie

9.396 $\frac{p_{02}}{p_{01}} = 0{,}72087$

c) Dreieck $(0,0) - (N,0) - (0, \tan(\sigma) \cdot N)$

$\sigma = 45^\circ$

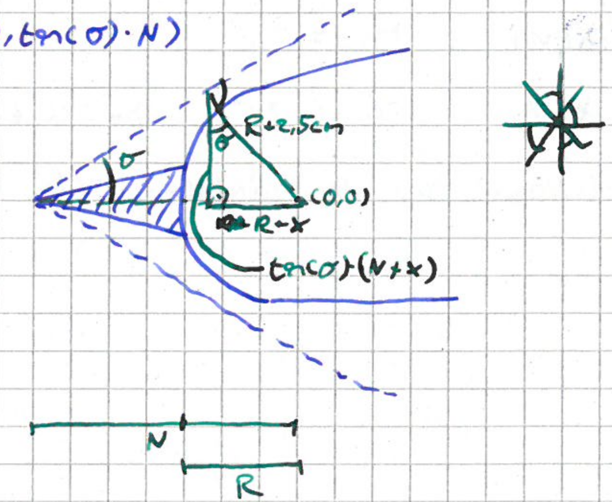
$\sin(\sigma) = \frac{R - x}{R + 2{,}5\,cm}$

$\cos(\sigma) = \frac{\tan(\sigma)(N + x)}{R + 2{,}5\,cm}$

$x = 12{,}88\,cm$

$N = 24{,}24\,cm$ ✓

~~$\sin(\sigma) = \frac{R+2{,}5}{N+R}$~~



$R = 0{,}5\,m$

Nasenspitze bei $(74{,}25, 0)$ cm

d) XLVI : $\frac{p_{02}}{p_{01}} = 0{,}95$

XLVII : $Ma_2 = 1{,}45$

Dann senkrechter Stoß → BZ

$Ma_3 = 0{,}71956$!

$\frac{p_{03}}{p_{02}} = 0{,}94484$

$\frac{p_{04}}{p_{01}} = \frac{p_{03}}{p_{02}} \cdot \frac{p_{02}}{p_{01}} = 0{,}8976$

Insgesamt : $\Delta = 1 - \frac{1 - \frac{p_{04}}{p_{00}}}{1 - \frac{p_{01}}{p_{00}}} = 0{,}63371 \rightarrow 63\,\%$

e) Totaldruckverlust sinkt mit spitzerem Keilwinkel

Nachstoßmachzahl steigt " " "

(Kopfwelle hebt ab einer Umlenkung von größer 23° ab.)

↳ wodurch der Totaldruckverlust über den senkrechten Stoß steigt.

Qualitative Aussage über gesamten Totaldruckverlust kann nicht getroffen werden, da sich bei einem bestimmten Winkel ein Optimum einstellt.

f) $Ma_3 = 1.2$ $T_3 = 350\,K$ $\vartheta = 35°$ $Ma_1 = 3,5$

9.79 : $\nu(Ma_3) = 3,558°$

$\nu(Ma_4) = \nu(Ma_3) + \vartheta$

$\nu(Ma_4) = 38,558° \rightarrow Ma_4 = 2,476$

9.77a : $T_4 = 202,5\,K$

g) x(vii : $Ma_2 = 2,6$

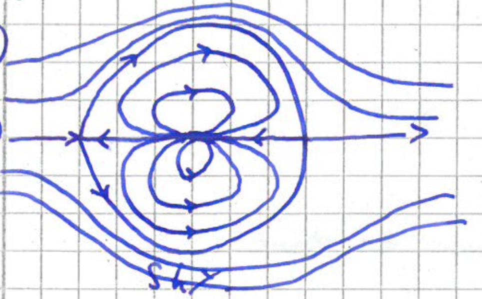
x(vii : $Ma_1 \overset{!}{=} 1,5$, $Ma_2 = 2,6 \rightarrow \vartheta = 24°$

Winkel zur Horizontalen : $\vartheta_{23} = 39°$

Strömung Klausur H21

①

a)



Skylla und Charybdis werden durch gleich aufgebaute Dipole dargestellt.

b) $\Psi(x,y) = U_\infty y - \frac{M_{Sk}}{2\pi}\frac{y-a}{x^2+(y-a)^2} - \frac{M_{Ch}}{2\pi}\frac{y+a}{x^2+(y+a)^2} -$
$- \frac{\Gamma}{2\pi}\ln\left(\sqrt{x^2+y^2}\right) - \frac{Q}{2\pi}\arctan\left(\frac{y}{x}\right)$

$M, \Gamma, Q > 0$

c) $U(x,y) = U_\infty - \frac{M}{2\pi}\left(\frac{x^2-(y-a)^2}{(x^2+(y-a)^2)^2} + \frac{x^2-(y+a)^2}{(x^2+(y+a)^2)^2}\right) - \frac{Q}{2\pi}\frac{x}{x^2+y^2} - \frac{\Gamma}{2\pi}\frac{y}{x^2+y^2}$

$V(x,y) = -\frac{M}{2\pi}\left(\frac{2x(y-a)}{(x^2+(y-a)^2)^2} + \frac{2x(y+a)}{(x^2+(y+a)^2)^2}\right) - \frac{Q}{2\pi}\frac{y}{x^2+y^2} + \frac{\Gamma}{2\pi}\frac{x}{x^2+y^2}$

Bedingung: Er muss sich in einem Staupunkt befinden: $U(x,y) = V(x,y) = 0$

d) Es ist leichter, nahe an Charybdis vorbei zu fahren, weil der Wirbel im Ursprung die Strömung auf dieser Seite beschleunigt.

e) Breite der Meerenge: a

$\frac{\dot{V}}{t} = \left|\Psi(x, a\cdot\frac{1}{2}) - \Psi(x, -a\cdot\frac{1}{2})\right| = 6\cdot\underbrace{\langle U\rangle}_{\text{Mittlere Geschw.}}$; $x = 0$

$\Psi(x,y)$ ist entlang einer Stromlinie konstant. Strom an Uferwänden der Meerenge entspricht daher den gegebenen Staupunkten der beiden Dipole

$|1{,}26\,U_\infty\cdot a + 1{,}12\,U_\infty\cdot a| = \langle U\rangle\cdot a$

$\langle U\rangle = 2{,}38\,U_\infty$

f) $\Delta y = \frac{3}{4}a$

$|E| = \frac{a}{2}\pi U_\infty$; $|\Gamma| = \frac{a}{4}\pi U_\infty$; $|M| = \frac{225}{544}\pi a^2 U_\infty$

$E > 0$ (in obiger Formel ist "-" schon enthalten).

$\Gamma > 0$

$M > 0$

Potentialtheoretisch geht die Geschw. hier gegen unendlich, in Wirklichkeit natürlich nicht.

g) $C_p = -\frac{u^2+v^2}{U_\infty^2} + 1$

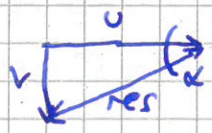
Vorbeifahrt bei $(0, -\frac{1}{4}a)$

$u(0, \frac{1}{4}a) = U_\infty \cdot 2$

$v(0, \frac{1}{4}a) = +U_\infty$ ~~[illegible]~~

$C_p =$ ~~[illegible]~~ -4 → Es besteht Unterdruck

h) $\tan(\alpha) = \frac{v}{u}$



$\alpha = 26{,}57°$ zur Transl.str.

$\Omega = 0$, weil entlang einer Stromlinie die Wirbelstärke immer null ist. (Potentialtheorie)

i) $S_1 \overset{!}{=} S_2$, um Ellipsenform zu erreichen

Translationsströmung U_∞ in x-Richtung

Quelle S_1 bei $(0,0)$; Senke S_2 bei $(b,0)$

j) $F(z) = U_\infty z + \frac{S_1}{2\pi}\ln(z) - \frac{S_2}{2\pi}\ln(z-b)$

$w^*(z) = \frac{dF(z)}{dz} = U_\infty + \frac{S_1}{2\pi z} - \frac{S_2}{2\pi(z-b)}$

Staupunkte bei $w^*(z) = 0$

Symmetrie: $y_{sp} = 0$ → $\mathrm{Im}(w^*(z)) = 0$

$$x_{sp} = \frac{1}{2\sqrt{\pi}U}\left(\sqrt{b(2S+b\pi U_\infty)U_\infty} + b\sqrt{\pi}U\right)$$

$$= \frac{\sqrt{b(2S+b\pi U_\infty)U_\infty}}{2\sqrt{\pi}U_\infty} \pm \frac{b}{2}$$

~~[illegible]~~

$$= \frac{b}{2} \pm \sqrt{\frac{2b(2S+b\pi U_\infty)}{\pi U_\infty}}$$

② $H = 150\,mm$ $b = 750\,mm$ $M_1 = 2{,}5$ $p_0 = 15\,bar$ $T_0 = 450\,K$

a) 9.19 : $T_1 = 200\,K$

9.20a : $p_1 = 0{,}8779\,bar$

$c_1 = \sqrt{\kappa R T_1} = 283{,}5\,\frac{m}{s}$ → $V_1 = 708{,}8\,\frac{m}{s}$

$c_1 = \sqrt{\kappa \frac{p_1}{\rho_1}}$ ⇒ $\rho_1 = 1{,}529\,\frac{kg}{m^3}$

$A_1 =$ ~~[illegible]~~ $H \cdot b = 0{,}1125\,m^2$

9.26 : $A^* = \left(\frac{2{,}63672}{A_1}\right)^{-1} =$ ~~[illegible]~~ $0{,}04267\,m^2$

Strömung Klausur H21

b) $\frac{A_1}{A^*} = 2,63672$ (Aus Anhang B1)

$\frac{A_A}{A_*} \overset{!}{=} 2,63672 \rightarrow Ma_A = 0,22$ (Aus Anhang B1)

$\frac{p_A}{p_0} = 0,96685 \rightarrow p_A = 14,5$ bar

$\frac{T_A}{T_0} = 0,99041 \rightarrow T_A = 445,7$ K

c) $\alpha = 20^\circ$ $\beta = 12,5^\circ$

II: Senkrechter Verdichtungsstoß Ma, p, T

$Ma_1 = 2,5$ $p_1 = 0,8779$ bar $T_1 = 200$ K

9.38c : $p_2 = 6,255$ bar

9.35c : $Ma_2 = 0,513$

9.38c : $T_2 = 427,5$ K

III Schräger Stoß mit $\vartheta = 20^\circ$

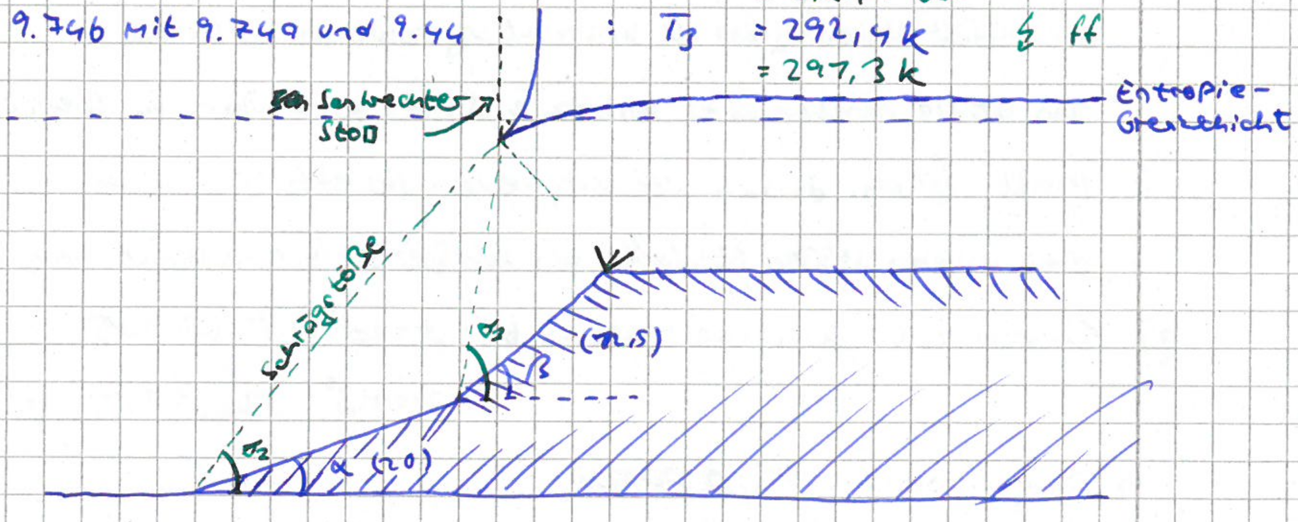
Diagramm xlv : $\sigma_2 = 43^\circ$

Diagramm xlvii : $Ma_3 = 1,5$ =1,65 in Lösung -> schlecht abgelesen?

~~Diagramm xlvii~~ 9.47a : $p_3 = 2,413$ bar ↯ ff
= 2,841 bar

9.74b mit 9.74a und 9.44 : $T_3 = 292,4$ K ↯ ff
= 291,3 K

d)



$Ma_4 = 0,95$ (Diagramm xlvii)

$\sigma_3 = 53,25^\circ$ (9.49) ; Expansionsfächer: Umlenkung um $-12,5^\circ$

Senkrechter Stoß: $Ma_1 = 2,5$ zu groß für $\vartheta_{ges} = 12,5^\circ$

1 -> 3 : Umlenkung um 20° bei Ma_1, 3 -> 4 Umlenkung um $12,5^\circ$ bei Ma_3

Entropietrennschicht: Zwischen 2 und 6 herrscht unterschiedl. ~~[illegible]~~ T und v

4 -> 6 : Schrägstoß stellt gleichen p und Richtung zwischen $\overset{2}{\cancel{4}}$ und 6 her

e) Diagramm xlvii : $Ma_4 = 1,175$

9.79 : $\nu(Ma_4) = 2,956^\circ$ $-32,5^\circ$

9.75 : $\nu(Ma_5) = \nu(Ma_4) + \vartheta_{45}$

$\nu(Ma_5) = 35,46^\circ$

$Ma_5 = 2,347$; $\sigma_5 = 155,7^\circ$

$c_4 = \sqrt{\kappa R T_4}$; $c_5 = \sqrt{\kappa R T_5}$

$T_2 = 427,5\,K$; $T_1 = 200\,K$

9.74b mit 9.47a und 9.44 : $T_3 = 291,3$

$T_4 = 351\,K$

9.77a : $T_5 = 213,7\,K$

$c_4 = 375,6 \frac{m}{s} \rightarrow v_4 = 441,3 \frac{m}{s}$

$c_5 = 292,7 \frac{m}{s} \rightarrow v_5 = 686,9 \frac{m}{s}$ Leichte Abweichung bei den Diagrammen, sonst richtig.

f) Für Zustand 6 müssen Druck und Strömungswinkel identisch zu der Strömung sein, die direkt über den Kreuzungspunkt den Stoß darüber durchquert. Der Strömungswinkel stellt damit gleichzeitig die effekt. Umlenkung des als senkrecht angenommenen Stoßes dar.
Also ist die Annahme eines senkrechten Stoßes genau über dem Kreuzungspunkt falsch, da sich hier entsprechen der effektiven Umlenkung der drei unteren Stößen ebenfalls ein schräger Stoß einstellen kann.

g) Diagramm xliv: kritische Machzahl bei $\vartheta = 20^\circ$: $Ma_{S1} \geq 1,85$

$\vartheta = 12,5^\circ$: $Ma_{S3} \geq 1,57$ ↯

→ Diagramm xlvii :

$Ma_1 \overset{!}{\geq} 2,3$

Strömung Klausur F 21

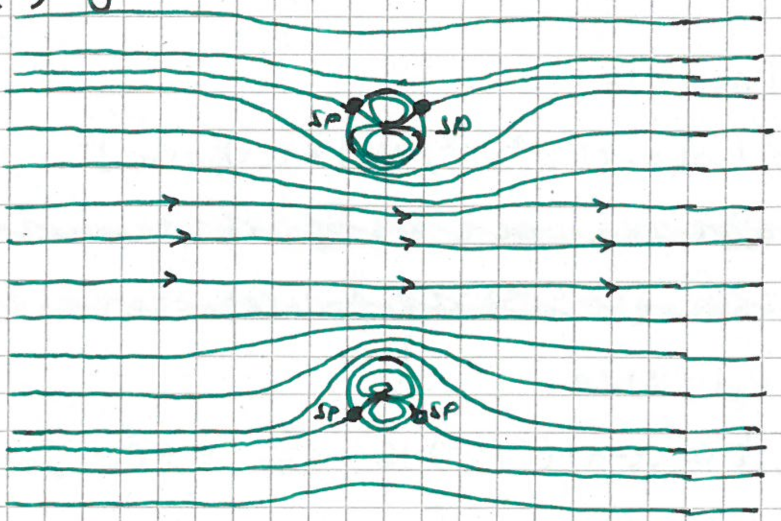
①

Lösung hat hier ein "+"

a) $F(z) = U_\infty \cdot z - i\frac{\Gamma}{2\pi}\ln(z-ai) + i\frac{\Gamma}{2\pi}\ln(z+ai) - \frac{\mu}{2\pi}\left(\frac{1}{z+ai} + \frac{1}{z-ai}\right)$

$\Gamma, \mu > 0$

b)



Hier am Ende müssen die Stromlinien eigentlich etwas auseinanderlaufen!

c) An der Wand gilt $v(x,y) = 0 \rightarrow \mathrm{Im}\left(\frac{dF(z)}{dz}\right) \overset{!}{=} 0$ für $y=0$

$$w^*(z) = \frac{dF(z)}{dz} = U_\infty + \frac{a\Gamma}{\pi(z^2+a^2)} - \frac{\mu(z^2-a^2)}{\pi(z^2+a^2)^2}$$

Alle "i" entfallen auf $y=0$

$\rightarrow \underbrace{\mathrm{Im}(w^*(z))}_{=v(x,y)} = 0$ für $y=0$ $\rightarrow$ Fiktive Wand

d)

$$\frac{\dot{V}}{t} = |\Psi(0,R-a) - \Psi(0,a-R)|$$

$$\Psi(x=0, |y|<a) = \mathrm{Im}(F(z))$$

$$F(0+(R-a)i) = -3{,}142 - 5{,}352i$$

$\Psi(0,R-a) = -5{,}352$ $\rightarrow$ Symmetrie ergibt:

$$\frac{\dot{V}}{t} = 2\cdot\Psi(0,R-a) = 10{,}7\ \frac{m^3}{sm}$$

e) $\phi = U_\infty \left(r\cos(\varphi-\alpha) + \frac{R^2}{r}\cos(\alpha-\varphi)\right) + \frac{\Gamma}{2\pi}\varphi$

$V_r(r, \overset{\varphi}{\cancel{\phi}}) = \frac{\partial \phi}{\partial r} = U_\infty \cos(\varphi-\alpha) \cancel{+} - \frac{1}{r^2}\left(\cos(\alpha-\varphi)\cdot R^2 \cdot U_\infty\right)$

$V_\varphi(r,\varphi) = \frac{1}{r}\frac{\partial\phi}{\partial\varphi} = \frac{\Gamma}{2\pi r} + \frac{1}{r^2}\left(\sin(\alpha-\varphi)\cdot R^2\cdot U_\infty\right) + \sin(\alpha-\varphi)\cdot U_\infty$

$\Psi(r,\varphi) = \int V_r(r,\varphi)\,dr$

$= \left[\frac{1}{r}\left(\cos(\alpha-\varphi)\cdot(r^2+R^2)\cdot U_\infty\right) + C(\varphi)\} + C_1\right]$

~~[illegible]~~

~~$\Psi(r,\varphi) = \frac{1}{r}\cos(\alpha-\varphi)(r^2+R^2)U_\infty - \frac{\Gamma}{2\pi}\ln(R) - 2\cos(\alpha-\varphi)R\,U_\infty$~~

$\Psi(r,\varphi) = \int V_\varphi(r,\varphi)\,d\varphi$

$= \left[\frac{1}{r^2}\cos(\varphi-\alpha)\cdot(r^2+R^2)\cdot U_\infty + \frac{\Gamma}{2\pi r}\cdot\varphi + C(r)\} + C_2\right]$

$C(\varphi) = 0 \quad ; \quad C(r) = -\frac{\Gamma}{2\pi}\ln(r) \qquad C_1 = C_2 = C$

$\Psi(r,\varphi) = \frac{1}{r}\left(\cos(\alpha-\varphi)\cdot(r^2+R^2)\cdot U_\infty\right) + C$

$\Psi(R,\varphi) = -\frac{\Gamma}{2\pi}\ln(R) \quad \rightarrow$ ↯ ?

$\Psi(r,\varphi) = U_\infty r \sin(\varphi-\alpha)\left(1-\frac{R^2}{r^2}\right) - \ln(r)\frac{\Gamma}{2\pi}$

Neuer Versuch unten...

e) $\phi(r,\varphi) = U_\infty\left(r\cos(\varphi-\alpha) + \frac{R^2}{r}\cos(\alpha-\varphi)\right) + \frac{\Gamma}{2\pi}\varphi$

$V_r(r,\varphi) = \cdot\frac{\partial\phi}{\partial r} = -\frac{1}{r^2}\left(\cos(\alpha-\varphi)(r^2-R^2)\cdot U_\infty\right)$

$V_\varphi(r,\varphi) = \frac{1}{r}\frac{\partial\phi}{\partial\varphi} = \frac{1}{r^2}\left(\sin(\alpha-\varphi)\cdot R^2 U_\infty\right) + \sin(\alpha-\varphi)U_\infty + \frac{\Gamma}{2\pi r}$

$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi} \rightarrow \psi_1 = \frac{1}{r}\left(\sin(\varphi-\alpha)(r^2-R^2)U_\infty\right) + C_1(\varphi)$

$V_\varphi = -\frac{\partial\psi}{\partial r} \rightarrow \psi_2 = \frac{1}{r}\left(\sin(\alpha-\varphi)R^2U_\infty\right) - \sin(\alpha-\varphi) - \frac{\Gamma}{2\pi}\ln(r) + C_2(r)$

~~$\frac{\partial\psi}{\partial r} = \frac{1}{r^2}\left(\sin(\alpha-\varphi)(r^2+R^2)U_\infty\right) + C_1'(\varphi)$~~

$\rightarrow$ ~~$C_1'(\varphi)$~~ $\Rightarrow \psi_1 = \frac{-\sin(\alpha-\varphi)(r^2-R^2)U_\infty}{r} + C(\varphi)$

$\psi_2 = \frac{-\sin(\alpha-\varphi)(r^2-R^2)U_\infty}{r} - \frac{\ln(r)\Gamma}{2\pi} + C(r)$

$C(r) = C$

$C(\varphi) = -\frac{\ln()\Gamma}{2\pi}$

$\psi(r,\varphi) = \frac{-\sin(\alpha-\varphi)(r^2-R^2)U_\infty}{r} - \frac{\ln(r)\Gamma}{2\pi} + 0$ ✓

Strömung Klausur F21

f) Staupunkt bei $r \overset{!}{=} R$ $\quad v_\varphi(r,\varphi) \overset{!}{=} 0$

$$v_\varphi(R,\varphi) = \frac{1}{R^2}\left(\sin(\alpha-\varphi)R^2 U_\infty\right) + \sin(\alpha-\varphi)U_\infty + \frac{\Gamma}{2\pi R} = 0 \; ; \; \alpha = 0°$$

$$\varphi_{sp} = \arcsin\left(\frac{\Gamma}{4\pi R v_\infty}\right) \qquad r_{sp} = R$$

~~[illegible]~~

~~[illegible]~~

Keine Lösung für $\left|\frac{\Gamma}{4\pi v_\infty}\right| > 1$; Nur eine für $\left|\frac{\Gamma}{4\pi v_\infty}\right| = 1$

↳ weil Staupunkte dann nicht mehr bei $r = R$ liegen...

g) $\Gamma = 2\sqrt{2}\pi R U_\infty$

$$c_p = 1 - \left(\frac{-\sqrt{U^2+V^2}}{U_\infty}\right)^2 \; ; \; r = R$$

$c_p =$ ~~[illegible]~~

$$c_p = -4\sin^2(\alpha-\varphi) - 4\sin(\alpha-\varphi)\sqrt{2} - 1$$

An der Körperkante ist $v_r = 0$!

~~[illegible]~~

h) $\alpha = 0$; $c_{p,max} = 1$; $R = 0{,}5\,m$

$\rightarrow \varphi_1 = \frac{\pi}{4} \qquad \varphi_2 = \frac{3\pi}{4}$

$r_1 = R \qquad r_2 = R$

Druckbeiwert am größten in den Staupunkten

i) Kraft wirkt in neg. y-Richtung, da ~~über~~ unter des Runde ein Druckminimum – und ~~unter~~ über des Runde ein Maximum herrscht. (Magnus-Effekt)

②

a) H = 18,3 km p_1 = 0,072 bar T_1 = 220 K ϑ_2 = 10° σ_2 = 28,94°

9.49 : Ma_1 = 2,8

9.79 : T_{01} = 565 K

9.20a : p_{01} = 1,954 bar

9.59 : Ma_2 = 2,34 9.47a : $\frac{p_2}{p_1}$ = 1,975 → p_2 = 0,1422 bar

~~Anhang B1 : $\frac{p_2}{p_{02}}$~~

b) ϑ_3 = 10°

9.49 : σ_3 = 33,79° → δ = 23,79°

9.59 : Ma_3 = 1,947

9.47a : p_3 = 0,2573 bar

c) A_4 = 0,8 m²

9.35 : Ma_4 = 0,5867

~~T~~ 9.74b mit 9.47a und 9.44 : T_2 = 354,5 K

T_3 = 538,2 K

9.38a : T_4 = 825,3 K

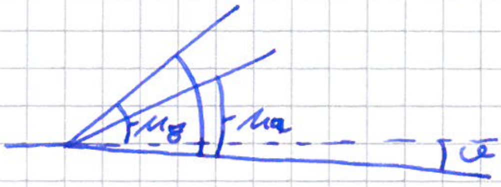
$T_{04} \overset{!}{=} T_{01}$ → T_{04} = 565 K

~~9.20a : p_{04} =~~ 9.38c p_4 = 1,095 bar

Tabelle B1 : p_{04} = 1,386 bar

9.24 : $\dot{m}$ = 156,5 $\frac{kg}{s}$

d) Ma_9 = 2,49 ε = 22,08°



9.74 : $\nu(Ma_9)$ = 38,89°

$\nu(Ma_9) = \nu(Ma_8) + \vartheta_{89}$; ϑ_{89}

Anhang B1 : μ_9 = 23,679° → ϑ = 1,599°

9.75 : Ma_8 = 2,423

d) $\delta = \mu_8 + \theta - \mu_1$

$\mu_8 = \varepsilon + \Phi = \arcsin\left(\frac{1}{Ma_8}\right) = 24{,}38°$

$\mu_1 = 23{,}679°$

$\delta = \Phi = 2{,}295°$

~~[illegible]~~ $P_a = P_1 = 0{,}072\ bar$

~~9.88 : [illegible]~~

9.776 : $\frac{P_a}{P_8} = 0{,}9009\ bar \rightarrow P_8 = 0{,}07992\ bar$

9.20a : $P_{08} = 1{,}2176\ bar$

e) $\frac{c_6}{c_5} = 2{,}04$ $V_6 = 193{,}9\ \frac{m}{s}$ $\dot m_6 = 165\ \frac{kg}{s}$

$Ma_6 \cdot c_6 = V_6$

$c_6^* = \sqrt{\kappa R T_6}$

$\frac{c_6^*}{c_5^*} = \sqrt{\frac{T_6}{T_5}}$ $T_{05} = T_{01} = 565\ K$

$c_5^* = \sqrt{2 \frac{\kappa}{\kappa+1} R T_{05}} = 435\ \frac{m}{s}$

$c_6^* = 887{,}5\ \frac{m}{s}$

$Ma_6 = \frac{V_6}{c_6} = 0{,}2185$

$c_6^* = \sqrt{2 \frac{\kappa}{\kappa+1} R T_{06}} \rightarrow T_{06} = 2352\ K$

~~$\dot m_6 = \rho_6 \cdot A_6 \cdot V_6$~~ $P_{06} = P_{08} = 1{,}2176\ bar$

~~$\dot m_5 = \rho_5 \cdot A_5 \cdot V_5$~~

9.24 : $A_6 = 4{,}336\ m^2$

f) $F_B = \dot m_8 V_8 + A_8 (P_a - P_1)$

$T_{06} = T_{08} = 2352\ K$; $P_{08} = 1{,}217\ bar$; $P_8 = 0{,}9004\ bar$; $Ma_8 = 2{,}423$

$\dot m_8 = 41{,}11 \cdot A_8\ \frac{kg}{s}$ ~~$c_8 = \sqrt{\kappa R T_8}$~~ $P_1 = 0{,}072\ bar$

9.19 : $T_8 = 1082\ K \rightarrow c_8 = \sqrt{\kappa R T_8} = 659{,}5\ \frac{m}{s} \rightarrow V_8 = 1598\ \frac{m}{s}$

$\dot m_6 = \dot m_8 = 165\ \frac{kg}{s} \rightarrow A_8 = 4{,}014\ m^2$

$F_8 = 2{,}668 \cdot 10^5\ N$

Strömung Klausur H20

①

a) $F(z) = U_\infty z + \frac{M}{2\pi}\frac{1}{z-1} + \frac{M}{2\pi}\frac{1}{z+1}$

$= U_\infty z + \frac{M}{\pi}\frac{z}{z^2-1}$

b) $w^*(z) = \frac{dF(z)}{dz} = U_\infty - \frac{M}{\pi}\frac{(z^2+1)}{(z^2-1)^2}$

Breite ist Distanz zwischen vorderem und hinterem Staupunkt

$Im(w^*) = 0$ (keine Geschwindigkeit in y-Richtung)

$Re(w^*) = 0 \rightarrow x_{sp1} = -\sqrt{5}$

$x_{sp2} = +\sqrt{5}$

Breite $B = 2\sqrt{5}$

c) Konturlinie ist Stromlinie Durch Staupunkt

$\Psi(x,y) = U_\infty y - \frac{3U_\infty}{4}\frac{y}{(x-1)^2+y^2} - \frac{3U_\infty}{4}\frac{y}{(x+1)^2+y^2}$

$\Psi(-\sqrt{5},0) = 0$

Konturgleichung: $0 = U_\infty y - \left(\frac{3}{4}\right)^{-1} U_\infty \frac{y}{(x-1)^2+y^2} - \left(\frac{3}{4}\right)^{-1} U_\infty \frac{y}{(x+1)^2+y^2}$

$\Psi_u(0, H_{VT}) = 0 \rightarrow H_{VT} = \frac{\sqrt{15}}{3}$

d) $v(0,H_{VT}) = 0$

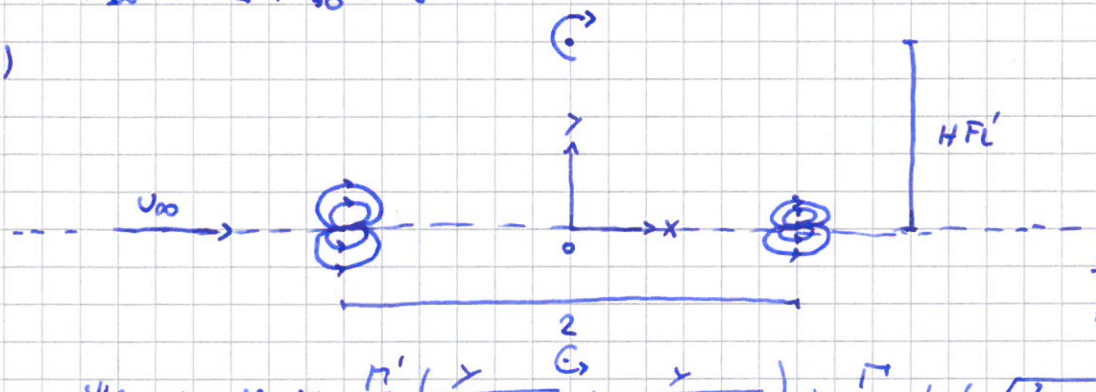
$u(x,y) = U_\infty - \frac{M}{2\pi}\frac{(x-1)^2-y^2}{((x-1)^2+y^2)^2} - \frac{M}{2\pi}\frac{(x+1)^2-y^2}{((x+1)^2+y^2)^2}$

$u_1(0,H_{VT}) = \frac{5}{4}U_\infty \rightarrow U_\infty = \frac{4}{5}U_s$

Bernoulli: $p_\infty + \frac{\rho}{2}U_\infty^2 = p_s + \frac{\rho}{2}U_s^2$

$p_\infty = p_s + \frac{9}{50}\rho U_s^2$

e)



Spiegelachse, Spiegelwirbel auf anderer Seite quasi vorhanden.

$\Psi(x,y) = U_\infty y - \frac{M'}{2\pi}\left(\frac{y}{(x-1)^2+y^2} + \frac{y}{(x+1)^2+y^2}\right) + \frac{\Gamma}{2\pi}\ln\left(\sqrt{x^2+(y-H_{FL}')^2}\right)$

$- \frac{\Gamma}{2\pi}\ln\left(\sqrt{x^2+(y+H_{FL}')^2}\right)$; $U_\infty, M', \Gamma > 0$

f) $M' = k \cdot M$; $H_{vr}' = H_{vr}$

$M = \frac{8\pi}{3} U_\infty$; $H_{vr} = \frac{5}{3}$ $\Gamma = -\frac{5}{3} \cdot 2\pi U_\infty$; $H_{\bar{F}c}' = 9 H_{vr}'$

Wandkontur soll gleich bleiben: $\Psi'(x,y) \overset{!}{=} \Psi_k(x,y) = 0$

$$\Psi(x,y) = U_\infty y - \frac{M'}{2\pi} \frac{y}{(x+1)^2 + y^2} - \frac{M'}{2\pi} \frac{y}{(x-1)^2 + y^2} -$$

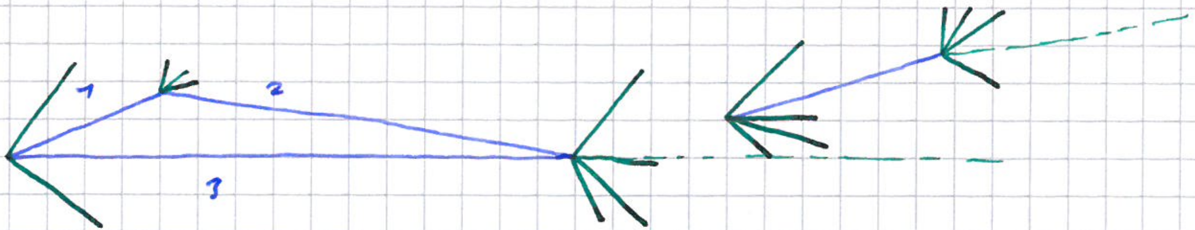
$$- \frac{\Gamma}{2\pi} \ln\left(\sqrt{x^2 + (y + H_{\bar{F}c}')^2}\right) + \frac{\Gamma}{2\pi} \ln\left(\sqrt{x^2 + (y \pm H_{\bar{F}c}')^2}\right) \overset{!}{=} 0$$

$y = H_{vr}'$; $x = 0$ $\lightning$

② $U_0 = 743,4 \frac{m}{s}$ $p_0 = 0,24\,bar$ $\rho_0 = 0,38 \frac{kg}{m^3}$

$\gamma_1 = 15°$ $\gamma_2 = 5°$ $\gamma_3 = 15°$ $\alpha = 5°$ $l_1 = 7m$ $l_2 = 3m$ $l_3 = 7m$ $\beta = 2°$

a)



b)

~~9.38 : P_{01} =~~ $c_0 = \sqrt{k \frac{p_0}{\rho_0}} = 297,4 \frac{m}{s}$; $T_0 = 220K$

$Ma_0 = 2,5$ ~~[illegible]~~

~~9.58 : P_{01} = 0,2234 bar~~

~~9.49 : σ_{12} =~~

~~9.59 : Ma_2 = 1,874~~

~~9.49 : σ_{12} = 52,66°~~

~~9.58 : P_{02} = 0,1482 bar~~

9.49 : $\sigma_{01} = 37,85$; $\vartheta_{01} = 10°$

9.47a : $P_1 = 0,4473\,bar$

(Mit 9.59 : $Ma_1 = 2,086$) ~~[illegible]~~

$\vartheta_{12} = 20°$ -> 9.49 : $\sigma_{12} = 50,79°$

~~9.47a : P_2 = 0,1287 bar~~

~~9.776~~ 9.74 : $\nu(Ma_1) = 28,72°$

9.75 : $\nu(Ma_2) = 48,72°$ → $Ma_2 = 2,947$

9.776 : $P_2 = 0,1178\,bar$ ↳ $Ma_3 = 2,292$

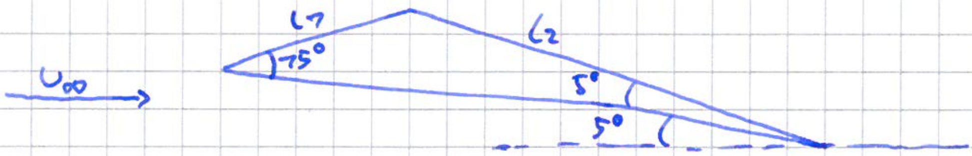
9.47a : $P_3 = 0,3377\,bar$ Mit $\sigma_{03} = 27,42°$ $\vartheta_{03} = 5°$

~~Totaldruck P_{01} ≠ P_{03}, weil Ma_0 ≠ Ma_3~~

Totaldruckverlust von 0->1 ist größer weil der Stoß mit größerem ~~[illegible]~~ Umlenkung auch stärker ist.

c) $A = -p_1 l_1 \cos(10°) - p_2 l_2 \cos(10°) + p_3(l_1 \cos(15°) + l_2 \cos(5°)) \cos(5°) = 57{,}69 \frac{kN}{m}$

$W = p_1 l_1 \sin(10°) - p_2 l_2 \sin(10°) + p_3(l_1 \overset{\cos}{\cancel{\sin}}(15°) + l_2 \overset{\cos}{\cancel{\sin}}(5°)) \sin(5°) = 13{,}07 \frac{kN}{m}$



Es ist bekannt, dass ein Profil in reibungsfreier Unterschallströmung keinen Wiederstand erzeugt (d'Alembertsches Paradoxon). Im Überschall jedoch existiert ein Wiederstand auch bei reibungsfreier Strömung. Er wird Wellenwiederstand genannt und hängt unmittelbar mit dem Totaldruckverlust und dem Entropieanstieg über die durch das Profil erzeugten schrägen Stößen zusammen. In dem obigen Beispiel entsteht der d) Wiederstand durch die Verdrängungswirkung und die Anstellung des Profils. Er setzt sich daher aus dem volumenbedingten und auftriebbedingten Wellenwiederstand zusammen.

∇ 0

d) · $Ma_2 = 2{,}9$ $p_2 = 0{,}13 bar$ $M_\infty = 2{,}5$ ETS : 2°

$\mu = 30°$

Tabelle B1 : $Ma_1 = 2$ ~~$\vartheta_{12} = 20°$ [illegible] $= 51{,}42$~~

$\vartheta_{01} = 15° - x \leftarrow$ Anstellwinkel

~~[illegible]~~

9.49 : $\vartheta_{01} = 12°$ (x(vii)

Anstellwinkel von 3°

e) Ma_4 gesucht:

$\vartheta_{24} = \gamma_2 + \beta + \alpha = 10°$

9.49 : $\sigma_{24} = 28,13°$

9.59 : $Ma_{24} = 2,423$

f) Totaldruck in Gebiet 5 höher. Verlust ist an den steileren Stößen bei 0->1 und 2->4 größer als unter dem Flügelprofil

g) $A = -P_6 \cdot L_6 \cdot \cos(13°) + P_7 L_6 \cos(13°)$ $\qquad L_3 = 1m$

$W = P_6 L_6 \sin(13°) - P_7 L_6 \sin(13°)$

$P_2 = 0,13\,bar \quad Ma_2 = 2,9$

$Ma_4 = 2,423 \qquad \sigma_{24} = 28,13° \qquad \vartheta_{46} = \vartheta_{47} = 13°$

$\sigma_{24} = 28,13° \qquad \vartheta_{24} = 10°$

9.47a : $P_4 = 0,2679\,bar$

9.49 : $\sigma_{46} = 35,75°$

9.47a : $P_6 = 0,5687\,bar$

9.74 : $\nu(Ma_4) = 37,3°$

$\nu(Ma_7) = 13 + \nu(Ma_4) = 50,3°$

$Ma_7 = 3,028$

9.776 : $P_7 = 0,1036\,bar$

$A = -4,532 \cdot 10^4\,N/m$

$W = 1,046 \cdot 10^4\,N/m$

Bei einer angestellten Platte stellt sich nur Auftriebsbedingter Wellenwiderstand ein.

h) Anströmmachzahl: $Ma_4 = 2,423$

$\vartheta_{max} = 29°$ -> Bei einem Anstellwinkel von 31° oder mehr würde sich eine ~~Abgelöste~~ Abgehobene Kopfwelle bilden. (Abgelesen aus xliv)