FLURUS

FACHSCHAFT LUFT- UND RAUMFAHRTTECHNIK UNIVERSITÄT STUTTGART

Raumfahrt

Formelsammlung

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Konstanten

Gravitationskonstate γ	$= 6,674 \cdot 10^{-11}$	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$	Erdgravitation (bei R_0) g_0	$= 9,81$	$\mathrm{m\,s^{-2}}$
Universelle Gaskonstante $\mathfrak{R}$	$= 8,31434$	$\mathrm{J\,mol^{-1}\,K^{-1}}$	Solarkonstante (bei 1AE) S	$= 1,371$	$\mathrm{W\,m^{-2}}$
Lichgeschwindigkeit c	$= 2,997925 \cdot 10^{8}$	$\mathrm{m\,s^{-1}}$	Astronomische Einheit 1AE	$= 1,49598 \cdot 10^{11}$	m
Avogadrokonstante N_A	$= 6,02214 \cdot 10^{26}$	$\mathrm{kmol^{-1}}$	Boltzmann-Konstante k	$= 1,38065 \cdot 10^{-23}$	$\mathrm{J\,K^{-1}}$
Molvolumen (Ideales Gas) V_0	$= 22,414$	$\mathrm{m^3\,kmol^{-1}}$	Elementarladung e	$= 1,60218 \cdot 10^{-19}$	C
Elektrische Feldkonstante ε_0	$= 8,85419 \cdot 10^{-12}$	$\mathrm{A\,s\,V^{-1}\,m^{-1}}$	Protonenmasse m_p	$= 1,67262 \cdot 10^{-27}$	kg
Magnetische Feldkonstante μ_0	$= 4\pi \cdot 10^{-7}$	$\mathrm{V\,s\,A^{-1}\,m^{-1}}$	Elektronenmasse m_e	$= 9,10938 \cdot 10^{-31}$	kg
Stefan-Boltzmann-Konstante σ	$= 5,6704 \cdot 10^{-8}$	$\mathrm{W\,m^{-2}\,K^{-4}}$	Wirkungsquantum h	$= 6,62607 \cdot 10^{-34}$	J s
			Elektronenvolt 1eV	$= 1,602 \cdot 10^{-19}$	J

Erde

$\mu_E = 3,986 \cdot 10^{14}\ \mathrm{m^3\,s^{-2}}$

$M_E = 5,974 \cdot 10^{24}$ kg

$R_0 = 6,378 \cdot 10^{6}$ m

Sonne

$\mu_S = 1,327 \cdot 10^{20}\ \mathrm{m^3\,s^{-2}}$

$M_S = 1,989 \cdot 10^{30}$ kg

$R_0 = 6,957 \cdot 10^{8}$ m

Kosmische Geschwindigkeiten

$v_1 = \sqrt{\gamma \frac{M}{r}}$ (Erde: 7,9km/s)

$v_2 = \sqrt{2\gamma \frac{M}{r}}$ (Erde: 11,2km/s)

$v_3 = \sqrt{2\gamma \frac{M_S}{r}}$ (Erde: 42,4km/s)

astronomischer Tag (für geostationäre Orbits): 23h 56min 4s = 86164 s

$\varepsilon_T = \frac{c_{e,ideal}^2}{2}$

$c_{e,ideal} = \sqrt{2\varepsilon_T}$

Trägerraketen

Spezifischer Impuls	Schub	Austrittsgeschwindigkeit		Äußerer Wirkungsgrad
$I_s = \frac{F}{g_0 \dot{m}_T} = \frac{c_e}{g_0}$	$F = \dot{m} c_e$	$c_e = \sqrt{2\eta_I \varepsilon_T}$	$\eta_I = \left(\frac{c_e}{c_{e,\text{ideal}}}\right)^2$	$\bar{\eta}_A = \frac{v_b^2}{c_e^2\left(e^{\frac{v_b}{c_e}} - 1\right)} = \frac{\left(\ln\left(\frac{m_0}{m_b^*}\right)\right)^2}{\frac{m_0}{m_b^*} - 1}$

Einstufig

Gesamtmasse	Antriebsbedarf	Nutzlastverhältnis	Strukturmassenverhältnis
$m_0 = m_M + m_S + m_T + m_L$	$\Delta v_{ch} = c_e \ln\left(\frac{m_0}{m_b^*}\right) = c_e \ln\left(\frac{1}{\sigma + \mu_L}\right)$	$\mu_L = \frac{m_L}{m_0}$	$\sigma = \frac{m_M + m_S}{m_0}$

$m_{SM,i} = \frac{\sigma_i}{1-\sigma_i} m_{T,i}$

$v_R(t) = c_e \ln\left(\frac{m_0}{m_R(t)}\right)$

Tandemstufung

Strukturmassenverhältnis

$$\sigma_i = \frac{m_{M,i} + m_{S,i}}{m_{0,i}} = \frac{m_{0,i} - m_{T,i} - m_{0,i+1}}{m_{0,i}}$$

Relativmasse

$$\left[\mu_i = \frac{m_{0,i}}{m_0}\right] \quad m_{0,i} = \mu_i \frac{m_L}{\mu_L}$$

Startmasse

$$m_0 = \sum_1^n m_i + m_L$$

m_i = Masse der i-ten Raketenstufe
$m_{0,i}$ = Masse der i-ten Unterrakete
$m_{b,i}$ = Leermasse der i-ten Raketenstufe
$m_{b,i}^*$ = Leermasse der i-ten Unterrakete
$m_{T,i}$ = Treibstoffmasse der i-ten Raketenstufe
$m_{M,i}$ = Motorenmasse der i-ten Raketenstufe
$m_{S,i}$ = Strukturmasse der i-ten Raketenstufe
$m_{L,i}$ = Nutzlastmasse

Massenauslegung

$$m_{0,i} = \mu_i m_0 = m_L + \sum_1^n m_j$$
$$m_{T,i} = m_{0,i}(1 - \sigma_i) - m_{0,i+1}$$
$$m_i = \sigma_i m_{0,i} + m_{T,i} = m_{S,i} + m_{M,i} + m_{T,i}$$
$$m_{b,i} = \sigma_i m_{0,i} = m_i - m_{T,i}$$
$$m_{b,i}^* = m_{b,i} + m_{0,i+1}$$
$$m_{0,i+1} = m_{0,i} - m_i$$

Antriebsvermögen

$$\Delta v_{ch} = \sum_1^n \Delta v_i$$
$$\Delta v_{ch} = \sum_1^n c_{e,i} \ln\left(\frac{1}{\sigma_i + \frac{\mu_{i+1}}{\mu_i}}\right)$$

Optimierung

$$\mu_{i,opt} = A_i + \sqrt{A_i^2 + B_i}$$
$$A_i = \frac{\mu_{i+1}}{\sigma_i} \frac{c_{e,i} - c_{e,i-1}}{2c_{e,i-1}}$$
$$B_i = \frac{\mu_{i+1}}{\sigma_i} \frac{c_{e,i}}{c_{e,i-1}} \sigma_{i-1}\mu_{i-1}$$

Parallelstufung

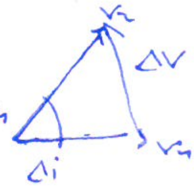
Antriebsvermögen	Effektive Austrittsgeschwindigkeit	Schub
$\Delta v_{ch} = \bar{c}_e \ln\left(\frac{m_0}{m_b^*}\right)$	$\bar{c}_e = \frac{\sum_1^n \dot{m}_i c_{e,i}}{\sum_1^n \dot{m}_i}$	$F = \sum_1^n F_i = \sum_1^n \dot{m}_i c_{e,i}$

Oberste Stufe: $m_{b,i} = \frac{\sigma_i \mu_i m_L}{\mu_L} + m_L$

Effektive Startbeschleunigung: $a_{eff} = \frac{F_{ges}}{m_0} - g_0$

$v_0 = \sqrt{v_\infty^2 + \frac{2\mu}{r_{Orbit}}}$

Inklinationsänderung: $\Delta v = \Delta i \cdot v_1$ ↑ [rad]



Bahnmechanik

Gravitationsfeld

Gravitationspotenzial	Gravitationskraft	Erdbeschleunigung
$U(r) = -\gamma \frac{mM}{r}$	$F = \gamma \frac{mM}{r^2}$	$g(r) = g_0 \frac{R_0^2}{r^2}$

e = Exzentrität
m, M = Massen
r = Abstand der Massenschwerpunkte
a = Große Halbachse
r_{Peri} = Radius des kleinsten Abstandes
r_{Apo} = Radius des größten Abstandes
ε = Spezifische Bahnenergie
v_∞ = Hyperbolische Exzessgeschwindigkeit

Vis-Viva-Gleichung

Allgemein	Bahngeschw.	Satellit (Kreisbahn)
$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = \frac{v_\infty^2}{2} = \text{konst.}$	$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$	$v_K^2 = \frac{\mu}{r_K}$

$v_{Erde}^2 = \mu_{Sonne}\left(\frac{1}{r_{Erde}}\right)$ (Bei Kreisbahn)

Umlaufbahnen

$\Delta v_{ch,Flucht} = (\sqrt{2}-1) \cdot v_{Kreisbahn}$

Geometrie

das heißt RED

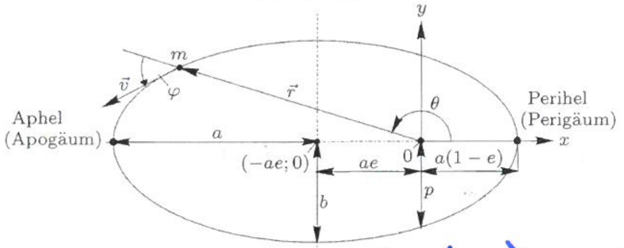
$$a = \frac{(r_{Peri} + r_{Apo})}{2}$$

$e = 1 - \frac{r_{peri}}{a}$

$$b = a\sqrt{1-e^2}$$

$$p = a\left(1-e^2\right) = \frac{h^2}{\mu}$$

Notation



Kegelschnittgleichungen

Ellipse/Hyperbel

$$r = \frac{p}{1 + e\cos(\theta)}$$

$$1 = \frac{(x+ae)^2}{a^2} + \frac{y^2}{a^2\left(1-e^2\right)}$$

Parabel

$$y^2 = -2p\left(x - \frac{p}{2}\right)$$

Gravitationsparameter

$$\left[\mu = \gamma M\right]$$

Größe	Kreis	Ellipse	Parabel	Hyperbel
e	0	$0 < e < 1$	$e = 1$	$1 < e$
a	r	$0 < a < \infty$	$\pm\infty$	$-\infty < a < 0$
b	r	$a\sqrt{1-e^2}$	-	$\lvert a \rvert \sqrt{e^2-1}$
p	a	$a\left(1-e^2\right)$	$\frac{h^2}{\mu}$	$a\left(1-e^2\right)$
r_{Peri}	a	$a(1-e)$	$\frac{h^2}{\mu}$	$a(1-e)$
r_{Apo}	a	$a(1+e)$	∞	∞
$[\frac{J}{kg}]$ ε	$-\frac{\mu}{2a}$	$-\frac{\mu}{2a} < 0$	$-\frac{\mu}{2a} = 0$	$-\frac{\mu}{2a} > 0$
v_∞	$\mathbb{C}$	$\mathbb{C}$	0	$\mathbb{R}$
$v_{Peri/Apo}$	$\sqrt{\frac{\mu}{a}}$	$\sqrt{\mu\left(\frac{2}{r_{Peri/Apo}} - \frac{1}{a}\right)}$	$\sqrt{\frac{2\mu}{r_{Peri}}}$	$\sqrt{\mu\left(\frac{2}{r_{Peri}} - \frac{1}{a}\right)}$

für v Flucht $v_{F,F} = \sqrt{\frac{2\mu}{r}}$

Massenspezifischer Drehimpuls

$$h = \left|\frac{\vec{H}}{m}\right| = rv\sin(\varphi) = rv\cos(\gamma) = r_{Peri}v_{Peri} = r_{Apo}v_{Apo} = \text{konst.}$$

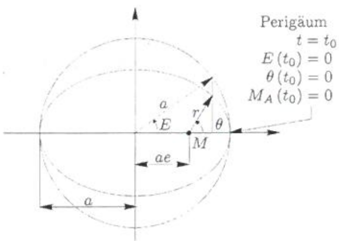
Keplerbahnen

Umlaufzeit (Einkörper)	Zusammenhang	Bahngleichung	Geometrie	
$P = 2\pi\sqrt{\frac{a^3}{\mu}} = \pi\mu\sqrt{-\frac{1}{2\varepsilon^3}}$	$\cos(\theta) = \frac{\cos(E) - e}{1 - e\cos(E)}$	$r = \frac{a\left(1-e^2\right)}{1+e\cos(\theta)}$	$r\cos(\theta) = a\left(\cos(E) - e\right)$	$\frac{r\sin(\theta)}{a\sin(E)} = \sqrt{1-e^2}$

Keplergleichung

$= \frac{1}{31536000}$ Jahr [a]
→ Sek

$$M_A = E - e\sin(E) = \sqrt{\frac{\mu}{a^3}}\,(t - t_0)$$



θ = Wahre Anomalie
E = Exzentrische Anomalie
M_A = Mittlere Anomalie

t = Zeit (Seit Perigäum)
M = Zentralmasse
ε = Spezifische Bahnenergie

~~Hohman~~

Hohman-Transfer mit gleichzeitiger Bahnebenendrehung:

$\Delta v_1 = \sqrt{v_{k,2}^2 + v_{A,E}^2 - 2 v_{k,2} \cdot v_{AE} \cdot \cos(i_2 - i_1)}$

$\Delta t = \frac{P}{2} = \frac{\pi}{\sqrt{\mu}} \left(\frac{a_{Hoh}}{2}\right)^{\frac{3}{2}}$

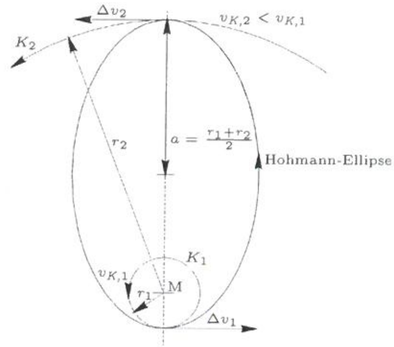
Antriebsbedarf

Hohmann-Transfer

Antriebsbedarf

$$\Delta v_{ch} = \Delta v_1 + \Delta v_2 = v_{K,1}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1 + \sqrt{\frac{r_1}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)\right) \approx \underbrace{v_{K,1}\frac{r_2-r_1}{2r_1}}_{r_2-r_1 \ll r_1}$$

$\Delta v_1 = \sqrt{\frac{\mu}{r_1}} \cdot \left(\sqrt{\frac{2r_1}{r_1+r_2}} - 1\right)$



Bahnenergiedifferenz

$$\varepsilon_{K,2} - \varepsilon_{K,1} = \frac{\mu}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Geschwindigkeiten

$$v_{K,i} = \sqrt{\frac{\mu}{r_i}}$$

Dreiimpulsübergang

$\frac{v_1^2}{2} - \frac{\mu_s}{r_E} = \frac{v_2^2}{2} - \frac{\mu_s}{r_P}$

$$\Delta v_{ch} = v_{K,1}\left(\sqrt{2} - 1\right)\left(1 + \sqrt{\frac{r_1}{r_2}}\right)$$

Aufspiralen

Aufspiralen $r_0 \to r$

$$\Delta v_{ch} = v_{K,0} - v_K = \sqrt{\mu}\left(\sqrt{\frac{1}{r_0}} - \sqrt{\frac{1}{r}}\right)$$

Bahnkurve

$$r = \frac{r_0}{\left(1 + \sqrt{\frac{r_0}{\mu}}\frac{F}{\dot{m}}\ln\left(1 - \frac{\dot{m}}{m_0}(t - t_0)\right)\right)^2}$$

Gravity-Assist-Manöver

$\cos(\gamma_0) = \frac{r_P v_2}{r_E v_1} \cos(\gamma_1)$

Kosinussätze

$$v_2^2 = v_3^2 + v_p^2 - 2v_3v_p\cos(\alpha)$$
$$v_2^2 = v_3^2 + v_p^2 + 2v_3v_p\cos(\beta_1)$$
$$v_5^2 = v_3^2 + v_p^2 + 2v_3v_p\cos(\beta_2)$$
$$v_3^2 = v_2^2 + v_p^2 - 2v_2v_p\cos(\gamma_1)$$
$$v_4^2 = v_5^2 + v_p^2 - 2v_pv_5\cos(\gamma_2)$$

v_5

$$v_5^2 = v_2^2 - 2\frac{k_1}{k_2^2} + \frac{4v_3v_p}{k_2}\sqrt{1 - \frac{1}{k_2^2}}\sqrt{1 - \left(\frac{k_1}{2v_pv_3}\right)^2}$$

$$k_1 = v_2^2 - v_p^2 - v_3^2 \qquad k_2 = 1 + \frac{r_{Peri}}{\mu_p}v_3^2 = 1 + x^2$$

Energieänderung (Heliozentrisch)

$$\Delta\varepsilon = 2v_p\sqrt{\frac{\mu_p}{r_{Peri}}}\left(\frac{x^2\sqrt{2+x^2}}{(1+x^2)^2}\sin(\beta_1) - \frac{x}{(1+x^2)^2}\cos(\beta_1)\right)$$

$$x = v_3\sqrt{\frac{r_{Peri}}{\mu_p}}$$

Maximale Energieänderung

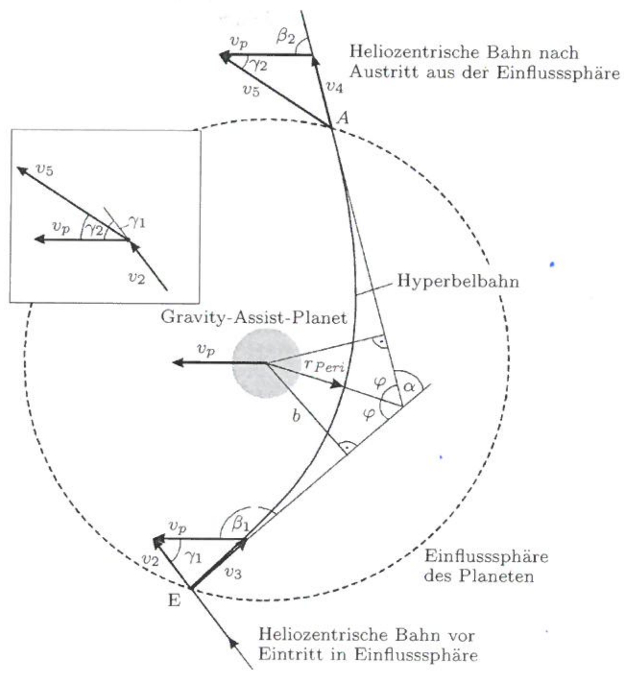
$x = 1$

$\beta_1 = 120°$

$$\Delta\varepsilon_{max} = v_pv_{3,max} = v_p\sqrt{\frac{\mu_p}{r_{Peri}}}$$

Zusammenhänge

$$\beta_2 = \beta_1 - \alpha = \beta_1 + 2\varphi - 180°$$
$$b^2v_3^2 = r_{Peri}^2v_3^2 + 2r_{Peri}\mu_p$$
$$\cos(\varphi) = \frac{1}{1 + \frac{r_{Peri}}{\mu_P}v_3^2}$$



HZ = Heliozentrisches Koordinatensystem

PZ = Planetenfestes Koordinatensystem

$\vec{v}_2$ = Flugkörpergeschwindigkeit vor dem Manöver (HZ)

$\vec{v}_3$ = Flugkörpergeschwindigkeit vor dem Manöver (PZ)

$\vec{v}_4$ = Flugkörpergeschwindigkeit nach dem Manöver (PZ)

$\vec{v}_5$ = Flugkörpergeschwindigkeit nach dem Manöver (HZ)

$\vec{v}_p$ = Planetengeschwindigkeit (HZ)

$$|v_3| = |v_4|$$

$\Delta\gamma = \gamma_2 - \gamma_1$

$PV = nRT = \frac{m}{M} RT$

$p = \rho \frac{R_m}{M} T$

$R = \frac{R_m}{M} \left[\frac{J}{kgK}\right]$

$\rightarrow c_F = \frac{F}{p_0 A_t} = \Gamma \sqrt{\frac{2\kappa}{\kappa-1}} \sqrt{1-\left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}} + \varepsilon\left(\frac{p_e-p_a}{p_0}\right)$

Thermische Raketen

Indizes

$0, c$ = Brennkammer (Ruhezustand)

a = Umgebungszustand (Ambient)

$\sim$ = Mittlewert

t = Düsenhals

e = Düsenende

Raketenschub

$$F = \dot{m}\tilde{w}_e + (\tilde{p}_e - p_a) A_e$$

Schubkoeffizient

$$c_F = \frac{F}{p_0 A_t}$$

Charakt. Brennkammerlänge

$$L^* = \frac{V_0}{A_t}$$

V_0 = Brennkammervolumen

A_t = Düsenhalsquerschnittsfläche

Feststoffrakete

Gesetz Von Robert & Vieille

$$\dot{r} = a\left(\frac{p_0}{p_{ref}}\right)^n$$

$\dot{r}$ = Regressionsgeschwindigkeit Der Treibstoffoberfläche

a = Empirische Treibstoffabhängige Konstante

p_0 = Brennkammerdruck

p_{ref} = Referenzdruck

n = Verbrennungsindex $(0{,}006 < n < 0{,}12)$

Idealisierte Rakete

$w_{e,\alpha=0} = \sqrt{2(h_0 - c_p T_e)}$

effekt. Austrittsgeschw. $\rightarrow c_e = \frac{F}{\dot{m}}$ $\left[\frac{m}{s}\right]$

A = Querschnitt

c_p = Spezifische Wärmekapazität

F = Schub

h = Spezifische Enthalpie

$\dot{m}$ = Massenstrom

p = Druck

R = Spezifische Gaskonstante

T = Temperatur

w = Strömungsgeschwindigkeit

Γ = Konstante

ε = Flächenverhältnis

κ = Adiabatenexponent

ρ = Dichte

Massenstrom

$$\dot{m} = \rho w A = \frac{p_0 A_t \Gamma}{\sqrt{RT_0}}$$

Funktion

$$\Gamma = \sqrt{\kappa\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}$$

Zusammenhänge

$$R = \frac{\Re}{M}$$

$\left[\frac{J}{kgK}\right]$

$$c_p = R\frac{\kappa}{\kappa-1}$$

$$h_0 = \eta_V \varepsilon_T$$

$\left[\frac{kJ}{kg}\right]$

Strömungsgeschwindigkeit

bei angepasster Düse $w = c_e$

$$w = \sqrt{2h_0}\sqrt{1-\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}} = \sqrt{\frac{2\kappa}{\kappa-1}}\sqrt{RT_0}\sqrt{1-\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}}$$

Divergenzverlust

$w_{e,real} = \lambda w_{e,\alpha=0}$

$$\lambda = \frac{1}{2}\left(1+\cos^2(\alpha)\right) = \frac{F_{real}}{F_{id}} = \frac{w_{e,real}}{w_{e,\alpha=0}}$$

Innerer Wirkungsgrad

$= \frac{c_e^2}{2\varepsilon_T}$

$$\eta_I = \left(\frac{F}{F_{max}}\right)^2 = \left(\sqrt{1-\left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}} + \frac{1}{\Gamma}\sqrt{\frac{\kappa-1}{2\kappa}}\frac{A_e}{A_t}\left(\frac{p_e-p_a}{p_0}\right)\right)^2$$

Maximalschub

$$F_{max} = \dot{m}w_{e,max} = \dot{m}\sqrt{2\varepsilon_T}$$

Flächenverhältnis

$$\varepsilon = \frac{A_e}{A_t} = \Gamma\left(\frac{p_e}{p_0}\right)^{-\frac{1}{\kappa}}\left(\frac{2\kappa}{\kappa-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right)\right)^{-\frac{1}{2}}$$

Schub

$$F = p_0 A_t\left(\Gamma\sqrt{\frac{2\kappa}{\kappa-1}}\sqrt{1-\left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}} + \frac{A_e}{A_t}\left(\frac{p_e-p_a}{p_0}\right)\right)$$

Brennkammerschub

$$F_{konv.} = p_0 A_t\left(2\left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}} - \frac{p_a}{p_0}\right)$$

Schubgewinn

$$\frac{F}{F_{konv.}} = \frac{\Gamma\sqrt{\frac{2\kappa}{\kappa-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right)} + \frac{A_e}{A_t}\left(\frac{p_e-p_a}{p_0}\right)}{2\left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}} - \frac{p_a}{p_0}}$$

Adiabate Strömungsvorgänge

Ruhezustand

$$\frac{T_0}{T} = 1 + \frac{\kappa-1}{2}Ma^2$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\kappa}{\kappa-1}} = \left(1+\frac{\kappa-1}{2}Ma^2\right)^{\frac{\kappa}{\kappa-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\kappa-1}} = \left(1+\frac{\kappa-1}{2}Ma^2\right)^{\frac{1}{\kappa-1}}$$

auch für nicht ideal zu benutzen

$$h_0 = h + \frac{w^2}{2} \Leftrightarrow T_0 = T + \frac{w^2}{2c_p}$$

Kritischer Zustand

$$\frac{T_t}{T_0} = \frac{2}{\kappa+1}$$

$$\frac{p_t}{p_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$$

$$\frac{\rho_t}{\rho_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}$$

$$\frac{A}{A_t} = \frac{1}{Ma}\left[\frac{2}{\kappa+1}\left(1+\frac{\kappa-1}{2}Ma^2\right)\right]^{\frac{\kappa+1}{2(\kappa-1)}}$$

$w_t = \sqrt{\kappa R T_t}$ für $Ma_t = 1$

Inklinationsänderung im Knoten der Bahnebenen, ΔV senkrecht zur Bahnebene
$\Delta V = 2 \cdot V_k \cdot \sin\left(\frac{\Delta i}{2}\right)$

$m_{T,i} = \frac{I_i}{I_{sp} g_0}$

Lage- & Bahnregelung

$\Theta_x = \Theta_{x0} \cdot \frac{M_0 - M_{T,i}}{M_0}$

Drallstabilisierung

Allgemeine Beziehung

$$\omega_q = \sqrt{\omega_x^2 + \omega_y^2}$$

$$\tan(\vartheta_N) = \frac{\theta_x \omega_q}{\theta_z \omega_z} = \frac{\omega_q}{\omega_N}$$

$$\omega_N = \frac{\theta_z \omega_z}{\theta_x} = 2\pi f_N$$

Passive Nutationsdämpfung

$$\vartheta_N(t) = \vartheta_N(t_0)\, e^{-\frac{t}{T_D}}$$

Bed.: $\theta_z > \theta_x, \theta_y$

$\vartheta_N(t)$ = Halber Öffnungswinkel (Nutationskegel)
$\vartheta_N(t_0)$ = Anfangswert von ϑ_N
t = Zeit
T_D = Dämpfungskonstante

Aktive Nutationsdämpfung

$$\Delta I = \frac{\theta_x \omega_q}{L} \frac{\alpha}{\sin(\alpha)}$$

$\omega_q = \omega_N \tan(\vartheta_N)$

$$\Omega = \frac{\theta_x - \theta_z}{\theta_x} \omega_z$$

$= \frac{\Theta_z \omega_z}{\Theta_x} \tan(\vartheta_N)$

$$\omega_x = \omega_q \sin(\Omega t)$$

$$\omega_y = \omega_q \cos(\Omega t)$$

Bed.: ω_z = konst.

ΔI = Impulsbedarf
L = Hebelarm der Schubdüsen
θ = Trägheitsmoment um die betrachtete Achse
θ_z = Trägheitsmoment um die Spinachse
ω_z = Winkelgeschwindigkeit um die Spinachse
ω_N = Kreisfrequenz der Nutationsbewegung
Ω = Kreisfrequenz der Stellmomente

Präzessionsbewegung & Zielausrichtung

!

$$\Delta I = S \Delta t = \sum_i \Delta I_i = \frac{\delta \theta_z \omega_z}{L} \frac{\alpha}{\sin(\alpha)}$$

[Ns] [rad] [rad]

$$\alpha = \frac{\omega_z \tau_{Puls}}{2}$$ [s]

$$D_z = \theta_z \omega_z$$ [rad]

S = Gesamtschub
Δt = Gesamte Blasdauer der Düse(n)
δ = Winkel zwischen alter und neuer Drallrichtung
τ_{Puls} = Dauer eines Schubimpulses
D_z = Drall um die Spinachse
α = Halber Zündwinkel

Dreiachsenstabilisierung

Geschwindigkeitsdämpfung

$$\Delta I = S \Delta t = \frac{\theta \Delta \omega}{L}$$

Brenndauer ≠ Manöver-Zeit

ΔI = Impuls
S = Gesamtschub
Δt = Gesamte Zündzeit der Düse(n)
L = Hebelarm der Schubdüsen
$\Delta\omega$ = Winkelgeschwindigkeitsdifferenz

$t = \Delta t \cdot \frac{360}{2\alpha}$

$\dot{\varphi}_{max}$

$$\dot{\varphi}_{max} = \dot{\varphi}\left(\frac{\Delta\varphi}{2}\right) = \frac{M}{\theta}\frac{\Delta t}{2}$$

Zielausrichtung

$\omega_0 = 2\pi f_0$ $\left[\frac{rad}{s}\right]$

$$\dot{\varphi}^2 - \dot{\varphi}_0^2 = \pm 2 \frac{M}{\theta}(\varphi - \varphi_0)$$

(Für $\dot{\varphi}_{max}$ nötig)

$$\varphi = \frac{M}{2\theta} t^2 + \dot{\varphi}_0 t + \varphi_0$$

$\left[\frac{1}{s^2}\right]$

$$\pm\ddot{\varphi} = m_\theta = \frac{M}{\theta} = \frac{SL}{\theta} \quad \Big| \quad \Delta I = \frac{2\theta\omega_{max}}{L}$$

φ = Lagewinkel
$\dot{\varphi}$ = Winkelgeschwindigkeit
$\ddot{\varphi}$ = Winkelbeschleunigung
$\varphi_0, \dot{\varphi}_0$ = Anfangswerte von $\varphi, \dot{\varphi}$
M = Stellmoment $= F \cdot L$
m_θ = Normiertes Stellmoment
ω_{max} = Maximale Winkelgeschwindigkeit

Lagestabilisierung

$$\Delta I = n \Delta I_{min} = m_T g_0 I_s = n F \tau$$

$$n = T \frac{\Delta I_{min} L}{4\theta\varphi_G}$$

$$\Delta I_{min} = F_{min}\tau_{min} = \frac{m_T g_0 I_s}{n}$$

ΔI_{min} = Realisierbarer Minimalimpuls
n = Anzahl der Stellimpulse
φ_G = Lagewinkel der geforderten Ausrichtgenauigkeit
T = Gesamte Missionszeit
m_T = Treibstoffbedarf
θ = Trägheitsmoment

Geostationäre Satelliten

Positionierung

Kontinuierlicher Anteil

$$\Delta v = \frac{4}{3} R_s \frac{\Delta\lambda}{\delta t}$$

Gepulstes System

$$\Delta v = \frac{2}{3} R_s \frac{\Delta\lambda}{\Delta t}$$

Streckenänderung

$$\Delta s = R_s \Delta\lambda$$

Winkelkorrektur

$$\Delta v_{NS} = v_{GEO} \Delta i \frac{\tau\pi}{2\sin\left(\frac{\tau\pi}{2}\right)}$$

$v_{GEO} = 3074\,\mathrm{m\,s^{-1}}$
Δi = Inklinationsabweichung
τ = Normierte Antriebszeit

Translationskorrektur

$$\Delta v_{OW} = b_\lambda T$$

$$b_\lambda = -5{,}568 \cdot 10^{-8} \sin(2(\lambda - \lambda_0))$$

T = Missionszeit
b_λ = Störbeschleunigung
λ = Geographische Länge $(-180° < \lambda \leq +180°)$
$\lambda_0 = +74{,}6° / -105.4°$ (Näheres Wählen)

Exzentrizitätskorrektur

Solare Störbeschleunigung

$$b_s = p_s G$$

$$G = (1 + \sigma)\frac{A}{m}$$

$p_s = 4{,}5 \cdot 10^{-6}\,\mathrm{N\,m^{-2}}$
G = Satellitenparameter
σ = Mittlerer Reflexionskoeffizient des Satelliten
A = Bezugsfläche des Satelliten
m = Satellitenmasse

Exzentrität Aufgrund b_s

$$e(t) = e_A \left|\sin\left(\frac{\dot{\Lambda}_E t}{2}\right)\right|$$

$$e_A = \frac{3 p_s G}{v_{GEO} \dot{\Lambda}_E} = 0{,}022 G$$

e_A = Amplitude
$\dot{\Lambda}_E = 1{,}9914 \cdot 10^{-7}\mathrm{s}^{-1}$
= Mittlere Winkelgeschwindigkeit der Erde umd die Sonne

Kompensation Durch Gegenblasen Pro Jahr

$$\Delta v_s = p_s G \frac{2\pi}{\dot{\Lambda}_E}$$

1.1

1. Phase: Booster $\Delta V_1 = \bar{c}_e \ln\left(\frac{m_0}{m_b^*}\right)$

$$\cancel{\bar{c}_e = \frac{\sum_1^n \dot{m}_i c_{e,i}}{\sum_1^n \dot{m}_i} = \frac{(500 \cdot 4300) \cdot 3}{3 \cdot 500}}$$

$$I_s = \frac{c_e}{g_0} \rightarrow c_{e,SRB} = 2943 \frac{m}{s}$$

$$\bar{c}_e = \frac{\sum \dot{m}_i c_{e,i}}{\sum \dot{m}_i} \quad \cancel{\frac{2(4767 \cdot 2943)}{2 \cdot 4767}}$$

$$\dot{m}_{SRB} = \frac{500\,000}{120} = 4167 \frac{kg}{s}$$

$$\bar{c}_e = \frac{2(4167 \cdot 2943) + 3 \cdot (500 \cdot 4300)}{2 \cdot 4167 + 3 \cdot 500} = 3150 \frac{m}{s}$$

$$\Delta V_1 = 3150 \cdot \ln\left(\frac{2047000}{837000}\right)$$

$$\Delta V_1 = 2777 \frac{m}{s}$$

2. Phase: Verbrauchter Treibstoff: 180 000 kg

$$\Delta V_2 = \bar{c}_e \ln\left(\frac{m_1}{m_{b,1}^*}\right)$$

$$\bar{c}_e = \frac{3 \cdot (500 \cdot 4300)}{3 \cdot 500} = 4300$$

$$m_1 = 2047\,000 - 2 \cdot m_{SRB} - 180000$$

$$m_{b,1}^* = 2047000 - 2 \cdot m_{SRB} - m_{ET,tr})$$

$$\Delta V_2 = 6545 \frac{m}{s}$$

3. Phase $\Delta V_3 = c_{e,oms} \ln\left(\frac{m_{0,3}}{m_{3,b}^*}\right)$

$$m_{0,3} = 2047000 - 2 \cdot m_{SRB} - m_{ET}$$

$$m_{b,3}^* = 2047000 - 2 \cdot m_{SRB} - m_{ET} - 11000$$

$$\Delta V_3 = 313{,}7 \frac{m}{s}$$

$$\Delta V_{ges} = \Delta V_1 + \Delta V_2 + \Delta V_3 = 9629 \frac{m}{s}$$

1.2

a) $\Delta t_1 = 123\,s$

$\Delta t_2 = 590 - 123 = 467\,s$

$\Delta t_3 = 1390 - 590 = 800\,s$

b) $\Delta V_0 = c_e \ln\left(\frac{M_0}{M_b^*}\right) \overset{!}{=} 795\,\frac{m}{s}$

$c_e = \frac{F}{\dot m}$

$\dot m = \frac{m}{\Delta t_3}$

$M^*_{b,0} = 24030\,kg \rightarrow$ ~~$M^*_{b,0}$~~

c) $M_{SM,U} = 15000\,kg$

$I_{SU} = 390\,s = \frac{c_e}{g_0} \rightarrow c_{e,U} = 3826\,\frac{m}{s}$

$\Delta V_U = c_{e,U} \ln\left(\frac{M_{0,U}}{M^*_{b,U}}\right)$

~~$M^*_{b,0}$~~

$\dot m_U = \frac{M_{T,U}}{510\,s} = 262{,}7\,\frac{kg}{s}$

~~$= 122{,}4\,t$~~

$M_{T,U2} = M_{T,U} - \Delta t_1 \cdot \dot m_U = 122{,}7\,t$

$M_U = 122{,}7 + 15 = 137{,}7\,t$

$M_{0,U} = M_U + M_{b,0} + M_{T,0} = 168{,}9\,t$

$M^*_{b,U} = M_{SM,U} + M^*_{b,0} + M_{T,0} = 46{,}23\,t$

$\Delta V_U = 4957\,\frac{m}{s}$

d) $F = \dot m \cdot c_e$

$\Delta V = c_e \ln\left(\frac{1}{\sigma + \mu_L}\right) = c_e \ln\left(\frac{M_{0,B}}{M^*_{b,B}}\right)$

$\mu_L = \frac{M_L}{M_0} =$

$\Delta V_1 = \Delta V_{ch} - \Delta V_0 - \Delta V_U = 3747{,}1\,\frac{m}{s}$

$M^*_{b,1B} = \frac{\sigma_1}{1-\sigma_1} M_{T,1} = 34{,}98\,t$

$M_{0,1} = 731182\,kg$

$M_{b,1} = 238869\,kg$

$\overline{F} = 6{,}201\,MN$

1.3

a) $\varepsilon_T = 5\cdot 10^6 \ J/kg$, 80% Energie zur Verfügung

$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$

$c_{e,ideal} = \sqrt{2\varepsilon_T} = 3162 \ \frac{m}{s}$

$\eta_1 = \left(\frac{c_e}{c_{e,ideal}}\right)^2$

$c_e = 2828 \ \frac{m}{s}$

$\Delta v \overset{!}{=} 3400 \ \frac{m}{s} = c_e \ln\left(\frac{m_0}{m_b^*}\right)$ $\quad m_b^* = 50 \ kg$

$m_0 = 766,35 \ kg$ ~~[illegible]~~ 766,4 kg

b) $m_T = m_0 - m_b^* = 716,35 \ kg$

$\cancel{v_R(t) = c_e \ln\left(\frac{m_0}{m_R(t)}\right)}$

$\cancel{m_R(t) = m_0 - \dot{m} \cdot t}$

$\cancel{v_R(t = t_{end}) \overset{!}{=} 3400 \ \frac{m}{s}}$

$\cancel{\Delta s \overset{!}{=} 1000 \ m = \int_0^{t_{end}} v_R(t)\, dt = \int_0^{t_{end}} c_e \ln\left(\frac{m_0}{m_0 - \dot{m} \cdot t}\right) dt}$

$\cancel{= \frac{1}{\dot{m}}\left(c_e\left((\dot{m} \cdot t_{end} - m_0)\ln\left(\frac{m_0}{m_0 - \dot{m} t}\right) \ldots\right)\right)}$

$v_R = \frac{ds}{dt}$

$v_R(t)\, dt = ds \overset{!}{=} 1000 \ m$

$v_R(t) = c_e \ln\left(\frac{m_0}{m_0 - \dot{m}_T \cdot t}\right)$

$v_R(t)\, dt = ds$

$\int_{t_0}^{t} v_R(t)\, dt = \int_0^{1000} ds \quad ; \quad t = \frac{m_T}{\dot{m}_T} \qquad (1)$

$1000 \ m = \frac{1}{\dot{m}_T}\left(c_e\left((\dot{m}_T \cdot t - m_0) \cdot \ln\left(\frac{-m_0}{\dot{m}_T \cdot t - m_0}\right) + \dot{m}_T \cdot t\right)\right) \qquad (2)$

$\dot{m}_T = 754,7 \ \frac{kg}{s} \quad ; \quad t = 0,7375 \ s$

c) $\dot{m}_{T,new} = 378,2 \ \frac{kg}{s}$

$t = 0,3658 \ s \quad , \quad v_1 = 3400 \ \frac{m}{s}$

(2) auflösen nach Δs:

$\Delta s = 500,4 \ m$

Raumfahrt Übung 2

2.1

a) $\mu_3 = \frac{m_3}{m_{0,3}}$; $m_L = 1{,}5\,t$; $\Delta V_3 \overset{!}{=} 1{,}5 \cdot c_{e3}$

$\Delta V_3 = c_{e3} \ln\left(\frac{m_{0,3}}{m^*_{b,3}}\right)$ ges: $m_{0,3}$ und $m^*_{b,3}$

$\mu_{L,3} = \frac{m_L}{m_{0,3}} \rightarrow m_{0,3} = \frac{m_L}{\mu_{L,3}}$ $\sigma_3 = \frac{m^*_{b,3}}{m_{0,3}}$

$\frac{m^*_{b,3}}{\sigma_3} = \frac{m_L}{\mu_{L3}}$ $m_L = \frac{\mu_L}{} \cdot m_0 \rightarrow m_0 = 207{,}7\,t$

$\mu_{L3} \cdot m_{0,3} = \mu_L \cdot m_0 = 1{,}5 \cdot 10^3$

$m_{0,3} = \frac{\mu_3 \sigma_3 m_L}{\mu_L} + m_L$ $m_{0,3} = \mu_3 \cdot \frac{m_L}{\mu_L}$

$\Delta V_3 = c_{e,3} \ln\left(\frac{\mu_3 \cdot \frac{m_L}{\mu_L}}{\frac{\mu_3 \sigma_3 m_L}{\mu_L} + m_L}\right) \rightarrow \mu_3 = 0{,}07234$

b) $\Delta V_{ges} \overset{!}{=} f(\mu_2) = \Delta V_1 + \Delta V_2 + \Delta V_3$

$\hookrightarrow = 3750\,\frac{m}{s}$

$\mu_{i,opt} = A_i + \sqrt{A_i^2 + B_i}$; $A_i = \frac{\mu_{i+1}}{\sigma_i} \frac{c_{e,i} - c_{e,i-1}}{2 c_{e,i-1}} \Rightarrow A_2 = -0{,}1005$

$B = \cdot$ ——— — $\Rightarrow B_2 = 0{,}0422$

$\mu_{2,opt} = 0{,}1282$ (Leicht anders wie Lösung)

$\mu_{2,opt} = 0{,}7041 \qquad \mu_3 = 0{,}07233$

$\Delta V_{ges} = \sum_1^n c_{e,i} \ln\left(\frac{1}{\sigma_i + \frac{\mu_{i+1}}{\mu_i}}\right) = 9505 \frac{m}{s} \qquad ; \mu_4 = \mu_L \; ; \mu_1 = 1$

c) $m_{T,1}$: $m_{0,2} = \frac{m_0 \cdot \mu_3}{\cancel{\mu_{2,3}}} = \cancel{\ldots} = 14{,}55\,t$

$m_{0,2} = \cancel{\ldots} = m_0 \cdot \mu_2 = 20{,}13\,t$

$m_{T,1} = m_0 - m_{0,2} - m_{S,1}$

$m_{S,1} = \sigma_1 \cdot m_0$

$m_{T,1} = 166{,}1\,t$

d) $\frac{166{,}1\,t}{1000\,kg/s} = 166{,}1$ sekunden Brenndauer der 1. Stufe $= \Delta t_1$

$m_{T,2} = \cancel{\ldots}\; m_{0,2} - m_{0,3} \cancel{\ldots} - m_{S,2}$

$m_1 = m_{S,1} + m_{T,1} = 180{,}2\,t$

$m_{S,2} = \sigma_2 \cdot m_{0,2} = 1{,}674\,t$

$m_{T,2} = \cancel{\ldots}\; 4{,}106\,t$

$\frac{m_{T,2}}{\dot m_2} = 18{,}82\,s = \Delta t_2$

$\Delta t_1 + \Delta t_2 = 184{,}9\,s \longrightarrow$ zweite Stufe brennt

$V_R(t) = \Delta V_1 + c_{e2} \ln\left(\frac{m_{0,2}}{m_R(t)}\right)$

$m_R(t) = m_{0,2} - \dot m_2 \cdot t \qquad ; \quad t = 166 - 180 = 13{,}9\,s$

~~$V_R(t=13{,}9s) =$~~

$\Delta V_1 = c_{e,1} \cdot \ln\left(\frac{m_0}{m_{B,1}^{*}}\right) = 5245 \frac{m}{s}$

$\Delta V_R(t=13{,}9s) = 5608 \frac{m}{s}$

$m(t=13{,}9s) = m_{0,2} - \dot m \cdot \underset{13{,}9s}{t} = 17{,}46\,t$

$4{,}789 \cdot 10^7\, kg \cdot \frac{m}{s} = I_R$

Raumfahrt Übung 2 $\boxed{C1}$

a) m_L gesucht

$m_{S,1}+m_{T,1}$

$m_{0,2} = m_2 + m_{0,3} = 135\,t$

$\sigma_1 = \frac{m_{S,1}}{m_{0,1}}$; $m_{0,1} = m_{S,1} + m_{T,1} + m_{0,2}$

$m_{S,1} = 41,47\,t \rightarrow m_{0,1} = 276,5\,t$

$\rightarrow m_L = \mu_L \cdot m_0 = 4,748\,t$

Benötigtes ΔV : $7,9 \frac{km}{s}$

$\Delta V_1 + \Delta V_2 + \Delta V_3 \overset{!}{=} 7900 \frac{m}{s}$

$\Delta V_1 = c_{e,1} \cdot \ln\left(\frac{m_0}{m^*_{b,0}}\right)$; $m^*_{b,0} = m_0 - m_{T,1} = 176,5\,t$

$\Delta V_1 = 1347 \frac{m}{s}$

$\Delta V_2 = c_{e,2} \cdot \ln\left(\frac{m_{0,2}}{m^*_{b,2}}\right)$; $m^*_{b,2} = m_{S,2} + \overbrace{m_{0,3}}^{55\,t}$; $c_{e,2} = I_{s,2} \cdot g_0 = 4120 \frac{m}{s}$

$\Delta V_3 = c_{e,3} \cdot \ln\left(\frac{m_{0,3}}{m^*_{b,3}}\right)$; $m_3 = m_{0,3} - m_L = 50,85\,t$

$\Delta V_3 = 4127 \frac{m}{s}$; $m_{T,3} = 0,6 \cdot m_3 = 30,51\,t$

$7900 = \Delta V_1 + \Delta V_2 + \Delta V_3$; $m^*_{b,3} = m_{0,3} - m_{T,3} = 24,49\,t$

$\rightarrow m_{S,2} = 19,92\,t$; $c_{e,3} = I_{s,3} \cdot g_0 = 5101 \frac{m}{s}$

$\sigma = \frac{m_{S,2}}{m_{0,2}} = 0,1476$

b) $\Delta V_{ges} \overset{!}{\geq} 11,2 \frac{km}{s}$

$\Delta V_2, \Delta V_3$ bleiben gleich:

$\Delta V_1 \overset{!}{=} 4647 \frac{m}{s} = \bar{c}_e \ln\left(\frac{m_0^{+2\cdot 60\,t}}{m^*_b}\right)$

$\bar{c}_e = \frac{\dot{m}_{T,B} c_{e,B} + \dot{m}_{T,B} c_{e,B} + \dot{m}_{T,1} c_{e,1}}{\dot{m}_{T,B} + \dot{m}_{T,B} + \dot{m}_{T,1}}$ $\quad m_{0,1,neu} = 396,5\,t$

$\dot{m}_{T,B}$:

$m_{S,B} = 3\,t \rightarrow m_{T,B} = 57\,t \rightarrow \dot{m}_{T,B} = 570 \frac{kg}{s}$

$\bar{c}_e = 3959 \frac{m}{s}$; $\dot{m}_{T,1} = 1000 \frac{kg}{s}$

$m^*_b = m_{0,1} - 2 \cdot m_{T,B} - m_{T,1} = 182,5\,t$

$\Delta V_1 = 3072 \frac{m}{s} < 4647 \frac{m}{s}$

Zwei Booster sind nicht ausreichend

c) $t = 300\,s$

$t_1 = 100\,s$ $\quad c_{e,2} = 4120\,\frac{m}{s}$

$\dot m_2 = \frac{F_2}{c_{e,2}} = 233\,\frac{kg}{s}$

$t_2 = \frac{m_{T,2}}{\dot m_2} = \frac{60{,}08\,t}{233\,\frac{kg}{s}} = 257{,}9\,s$

→ Es brennt die zweite Stufe bei $t = 300\,s$

$t_{2,rest} = 57{,}9\,s$

$m_{T,verbraucht} = \dot m_{T,2}\cdot(t_2 - t_{2,rest}) = 46{,}6\,t$

$\Delta v_2' = c_{e,2}\cdot\ln\left(\frac{m_{0,2}}{m_{0,2}-m_{T,verbr.}}\right) = 1744\,\frac{m}{s}$

$\Delta v_{ges}' = \Delta v_1 + \Delta v_2' + \Delta v_3 = 8943\,\frac{m}{s}$

$a_{eff} = \frac{F_2}{m_{0,2}-m_{T,verbr.}} - g_0 = 1{,}05\,\frac{m}{s^2}$

2.3

a) $\cancel{g_0 = 9{,}81\,\frac{m}{s^2} \text{ und } a_{eff} = \frac{F}{M_0} - g_0}$

Gewichtskraft $M_0\, g_0 = 7{,}378\cdot10^6\,N$

Antriebskraft $F_{ges} = F_1 + 2\cdot F_B$

$\dot m_1 = 268\,\frac{kg}{s}$ $\quad c_{e,1} = I_s\cdot g_0 = 3041\,\frac{m}{s}$

$\dot m_B = 1939\,\frac{kg}{s}$ $\quad c_{e,B} = 2845\,\frac{m}{s}$

$F_1 = \dot m_1\cdot c_{e,1} = 814{,}9\,kN$

$F_B = \dot m_B\cdot c_{e,B} = 5{,}515\cdot10^6\,N$

$F_{ges} = 1{,}185\cdot10^7\,N$

$a_{eff} = \frac{F_{ges}}{M_0} - g_0 = 6{,}069\,\frac{m}{s^2}$

$F_{Bes} = F_{ges} - F_g = 4{,}528\cdot10^6\,N$

Raumfahrt Übung 2

b) $\Delta V_1 = \bar{c}_e \ln\left(\frac{M_{0,1}}{M^*_{b,1}}\right)$

$\bar{c}_e = \frac{2 \cdot \dot{m}_B c_{e,B} + \dot{m}_1 c_{e,1}}{2 \cdot \dot{m}_B + \dot{m}_1} = 28\cancel{xx}58 \frac{m}{s}$

$M^*_{b,1} = M_{0,1} - 2 \cdot M_{T,B} - M'_{T,1} = \cancel{xxx} 232\,t$

$\cancel{xxx} = M'_{T,1} = \dot{m}_1 \cdot t_1 = \cancel{xxx} 31{,}232\,t$

$\Delta V_1 = 3338 \frac{m}{s}$

$\Delta V_2 = c_{e,2} \ln\left(\frac{M_{0,2}}{M^*_{b,2}}\right) \qquad c_{e,2} = 4228 \frac{m}{s}$

$M_{0,2} = M_0 - M'_{T,1} - 2 \cdot M_{S,B} = 162\,t$

$M^*_{b,2} = M_{0,2} - (M_{T,2} - \dot{m}_1 \cdot t_1) = 37{,}11\,t$

$\Delta V_2 = \cancel{xxx} 6237 \frac{m}{s}$

$\Delta V_3 = c_{e,3} \ln\left(\frac{M_{0,3}}{M^*_{b,3}}\right) = 1964 \frac{m}{s}$

$M_{0,3} = M_3 + M_L + M_{Fairing}$

$M_{0,1} = \underbrace{M_1 + M_2}_{M_B + M_1} + M_3 + M_L + M_{Fairing} \qquad c_{e,3} = 3749 \frac{m}{s}$

$M_{B+1} = \cancel{xxx} 725{,}1\,t$

$M_3 = 10{,}9\,t$

$M_L = 6\,t \quad ; \quad M_{0,3} = 20{,}9\,t$

$M^*_{b,3} = 11{,}2\,t$

$\Delta V_3 = 1964 \frac{m}{s}$

$\Delta V_{ges} = 11{,}533 \frac{km}{s}$

c) $\Delta V_1 = 3338 \frac{m}{s}$

$\Delta V_2 = c_{e,2} \cdot \ln\left(\frac{M_{0,2}}{M^*_{b,2}}\right)$

$M^*_{b,2} = M_{0,2} - \dot{m}_{T,2} \cdot t_B \qquad t_B = 500\,s - 124\,s$

$= \cancel{xxx} 67{,}23\,t$

$\Delta V_2 = 4914 \frac{m}{s}$

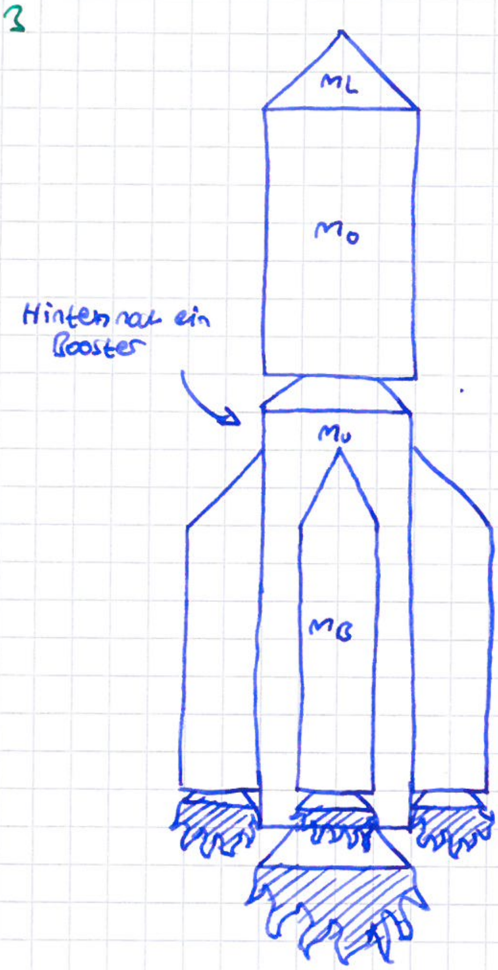
$\Delta V_3 = 1964$ ||

$\Delta V_{ges} \overset{!}{=} 7{,}9 \frac{km}{s} \leq \Delta V_1 + \Delta V_2 + \Delta V_3 = 9476$

Sonstige Flugverluste dürfen $1576 \frac{m}{s}$ nicht überschreiten

Raumfahrt Übung 3

3.1. a)



Phase 1: 128 s, Booster + Unterstufe

Phase 2: 128-766 s (38 s), Unterstufe

Phase 3: 766-Ende, Oberstufe

Startmasse:

$M_B \cdot 4 + M_{T,U} + M_{S,U} + M_\sigma + M_L = 460{,}9\,t$
$= M_0$

Nutzlastverhältnis:

$\mu_L = \frac{M_L}{M_0} = 0{,}01823$

Treibstoff: $M_T = (M_B - M_{S,B}) \cdot 4 + M_{T,U} +$
$+ M_\sigma - m_\sigma \sigma_\sigma - M_L \sigma_\sigma = 424{,}76$

b) $F_1 = \sum_1^n F_i = 4 \cdot (\dot m_B \cdot c_{e,B}) + \dot m_U c_{e,U}$

$\dot m_B = \frac{M_{T,B}}{t_{b,B}} = 295{,}3 \frac{kg}{s}$ $\quad \dot m_U = \frac{M_{T,U}}{t_{b,U}} = 1130 \frac{kg}{s}$

$c_{e,B} = I_{s,B} \cdot g_0 = 2855 \frac{m}{s}$ $\quad c_{e,U} = I_{s,U} \cdot g_0 = 2835 \frac{m}{s}$

$F_1 = 6{,}576 \cdot 10^6\,N$

$F_B = 3{,}372 \cdot 10^6\,N$

$a_{eff} = \frac{F_{ges}}{M_0} - g_0 = 4{,}458 \frac{m}{s^2}$

c) $\Delta V_{ges} = \bar c_e \ln\left(\frac{M_0}{M_{b,0}}\right) + \Delta V_2 + \Delta V_3$

$\bar c_e = \frac{(\dot m_B c_{e,B}) \cdot 4 + \dot m_U c_{e,U}}{4 \cdot \dot m_B + \dot m_U} = 2845 \frac{m}{s}$

$M_{b,0} = 4 \cdot m_{s,B} + m_{s,U} + (M_{T,U} - \dot m_U \cdot 128\,s) + M_{0,\sigma} = 165{,}06\,t$

$\Delta V_2 = c_{e,U} \cdot \ln\left(\frac{M_0 - 4 \cdot M_B - \dot m_U \cdot 128}{M_0 - 4 \cdot M_B - M_{T,U}}\right) = 937{,}2 \frac{m}{s}$

$\Delta V_3 = c_{e,\sigma} \cdot \ln\left(\frac{M_\sigma + M_L}{(M_\sigma + M_L) \cdot \sigma_L + M_L}\right) = 8244 \frac{m}{s}$

$\Delta V_{ges} = 12100 \frac{m}{s}$

$V_2 = 11{,}2 \frac{km}{s} < \Delta V_{ges}$

→ Rakete kann Fluchtgeschwindigkeit der Erde übertreffen!

d) $\Delta V = 12082{,}97\ \frac{m}{s} \overset{!}{=} C_e \cdot \ln\left(\frac{M_0}{M_{B,0}^*}\right)$; $M_0 = 460{,}9\,t$, $M_T = 424{,}7\,t$

$C_e = 4749\ \frac{m}{s}$

3.2

a)

$M_{S,OS} = 0{,}09 \cdot M_{T,OS} = 12{,}6\,t$

$M_{OS} = M_{S,OS} + M_{Triebw,OS} + M_{T,OS} = 155\,t$

$M_0 = \cancel{\dots}\ 2 \cdot M_B + M_{ZS} + M_{OS}$

$M_B = 735\,t$

$M_{ZS} = 0{,}05 \cdot M_0 + M_{T,ZS}$

$I_{s,ZS} = \frac{F_{ZS}}{g_0\, \dot{m}_{T,ZS}} \rightarrow \dot{m}_{T,ZS} = 876{,}6\ \frac{kg}{s}$

$M_{T,ZS}^* = 261{,}3\,t$ (Für ein Triebwerk)

$M_{T,ZS} = 1307\,t$

$M_0 = 3749\,t$

$M_{ZS} = 1464\,t$

$F_1 = 2 \cdot F_B + 5 \cdot F_{ZS}$

$F_{ZS} = \dot{m}_{ZS} \cdot C_{e,ZS} = 2900\,kN$

$F_1 = 42{,}5\,MN$

$a_{eff} = \frac{F_1}{M_{0,1}} - g_0 = 3{,}686\ \frac{m}{s^2}$

$\Delta V_1 = \bar{C}_e \ln\left(\frac{M_{0,1}}{M_{B,1}^*}\right)$

$\bar{C}_e = \frac{2 \cdot \dot{m}_B C_{e,B} + \dot{m}_{ZS} \cdot C_{e,ZS}}{2 \cdot \dot{m}_B + \dot{m}_{ZS}} = 2756\ \frac{m}{s}$

$F_B = \dot{m}_B \cdot C_{e,B} \rightarrow \dot{m}_B = 5668\ \frac{kg}{s}$

$\dot{m}_{ZS,ges} = \frac{M_{T,ZS}}{t_{ZS}} = 4084\ \frac{kg}{s}$

$C_{e,ZS} = 3557\ \frac{m}{s}$

$M_{B,1}^* = M_0 - M_{T,B} \cdot 2 - \dot{m}_{ZS,ges} \cdot t_1 = 1422\,t$; $t_1 = 112\,s$

$\Delta V_1 = 2792\ \frac{m}{s}$; $t_2 = 208\,s$

$\Delta V_2 = C_{e2} \ln\left(\frac{M_{0,2}}{M_{B,2}^*}\right)$; $C_{e2} = I_{s,vakuum} \cdot g_0 = 4022\ \frac{m}{s}$

$M_{0,2} = \cancel{\dots}\ M_{B,1}^* - M_{S,B} \cdot 2 = 1222\,t$; $M_{B,2}^* = M_{0,2} - \dot{m}_{ZS,ges} \cdot t_2 = 372{,}5\,t$

$\Delta V_2 = 4178\ \frac{m}{s}$

$V_2 = \Delta V_1 + \Delta V_2 = 6970\ \frac{m}{s}$

Raumfahrt Übung 3

c) $\Delta V_1 = 2792 \frac{m}{s}$ $\Delta V_2 = 4778 \frac{m}{s}$ $\Delta V_3 \overset{!}{=} 2030 \frac{m}{s}$

$\cancel{\Delta V_{soll} = 8808 \frac{m}{s}}$

$\Delta V_3 = c_{e,0s} \cdot \ln\left(\frac{M_{0,3}}{M^*_{b,3}}\right)$

$M_{0,1} = 275 t$ $c_{e,3} = 4267 \frac{m}{s}$

$M^*_{b,3} = M_{0,3} - t \cdot \dot{M}_{T,3}$ $\dot{M}_{T,3} = \frac{M_{T,3}}{t_3} = 376,4 \frac{kg}{s}$

$t = 257,3\ s$

d) $M_{T,3,rest} = 5.859 \cdot 10^4\ kg$

$\Delta V_{3,rest} = c_{e,0s} \cdot \ln\left(\frac{M^*_{b,3}}{M^*_{b,3,ende}}\right)$

$\Delta V_{3,rest} = 2463 \frac{m}{s}$

~~[illegible]~~

$\rightarrow \Delta V_{Flucht} = \sqrt{\frac{2\mu_E}{r}} = 10,85 \frac{km}{s}$

~~[illegible]~~

~~[illegible]~~

$V_{k,400km} = \sqrt{\frac{\mu_E}{r}} = 7,669 \frac{km}{s}$

$\Delta V_{Flucht} = 3,187 \frac{km}{s} > \Delta V_{3,rest}$

Die Nutzlast kann nicht auf Fluchtgeschwindigkeit beschleunigt werden.

4.1

a) $V_{Erde}^2 = \mu_s\left(\frac{2}{r} - \frac{1}{r}\right) \to r_{Erde} = 1{,}496\cdot10^{11}\,m$

Bahnenergie $\mathcal{E} = -\frac{\mu_s}{2r_E} = -4{,}435\cdot10^{7}\,\frac{m^2}{s^2} < 0 \to$ Elliptisch

$P = \pi\cdot\mu_s\cdot\sqrt{-\frac{1}{2\mathcal{E}^3}} = 9{,}888\cdot10^8\,s$

$= 31{,}36$ Jahre

$P = 2\pi\sqrt{\frac{a^3}{\mu}}$

$a = 1{,}487\cdot10^{12}\,m$

b) $r_{peri} = 1{,}07\cdot10^{11}\,m$

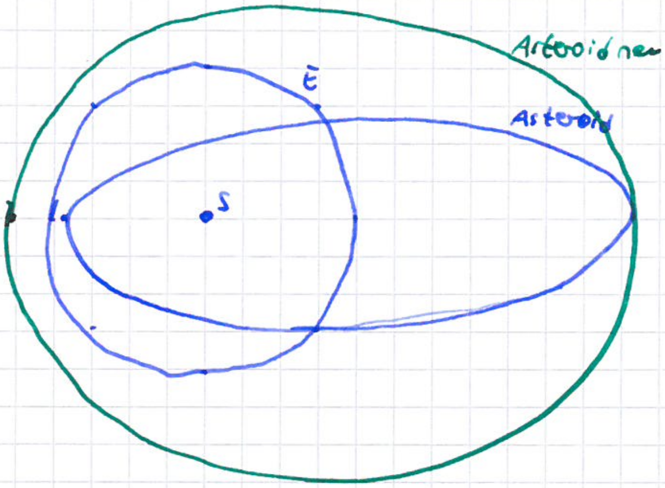
$a = \frac{(r_{peri} + r_{apo})}{2}$

$r_{apo} = 2{,}873\cdot10^{12}\,m$

$v_{apo}^2 = \mu_s\left(\frac{2}{r_{Apo}} - \frac{1}{a}\right)$

$v_{apo} = 1{,}711\,\frac{km}{s}$

c) $r_{peri,neu} = 2{,}244\cdot10^{11}\,m$



$v_{neu}^2 = \mu_s\left(\frac{2}{r_{apo}} - \frac{1}{a_{neu}}\right)$; $a_{neu} = 1{,}549\cdot10^{12}\,m$

$v_{neu} = 2585\,\frac{m}{s}$

$\Delta v = v_{neu} - v_{Apo} = 874\,\frac{m}{s}$

$\Delta v = c_e\cdot\ln\left(\frac{M_0}{M_B}\right) = c_e\cdot\ln\left(\frac{m_A+m_T}{m_A}\right)$

$m_T = m_A\left(e^{874/4000} - 1\right)$

$= 0{,}2257\,m_A$

d) $\Delta V_J = 3784 \frac{m}{s}$

$\Delta V_{,neu} = \Delta V_J + \Delta V_{r_J}$

Bahnradius Jupiter:

$V_J^2 = \mu_s \left(\frac{2}{r_J} - \frac{1}{r_J}\right)$

$r_J = 7,768 \cdot 10^{11}$ m

Geschw. A auf Jupiterbahn:

$V_{A,J}^2 = \mu_s \left(\frac{2}{r_J} - \frac{1}{a_{neu}}\right)$

$V_{A,J} = 76\,000 \frac{m}{s}$

$\cancel{\Delta} V_{,neu} = 79784 \frac{m}{s}$

$V_{parabel,J} = \sqrt{2 \frac{\mu}{r_J}} = 78480 \frac{m}{s} < V_{,neu}$

Fluchtgeschwindigkeit wird erreicht.

4.2

a) $a = 4 \cdot 10^{11}$ m $\quad r_{peri} = 1 \cdot 10^{11}$ m

$a = \frac{r_{peri} + r_{apo}}{2} \rightarrow r_{apo} = 7 \cdot 10^{11}$ m

$e = 1 - \frac{r_{peri}}{a} = \frac{3}{4} = 0,75$

$b = a\sqrt{1-e^2} = 2,646 \cdot 10^{11}$ m

$V_{apo}^2 = \mu_s \left(\frac{2}{r_{apo}} - \frac{1}{a}\right) \quad V_{peri}^2 = \mu_s \left(\frac{2}{r_{peri}} - \frac{1}{a}\right)$

$V_{apo} = 6884 \frac{m}{s} \quad V_{peri} = 48790 \frac{m}{s}$

$P = 2\pi \sqrt{\frac{a^3}{\mu_s}} = 1,38 \cdot 10^8$ s

$= 4,376$ a

b) $\Delta V = 5000 \frac{m}{s}$ im Perihel

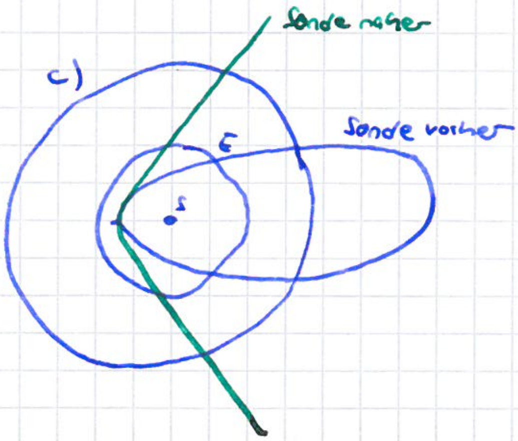
$V_{peri,neu} = V_{peri} + \Delta V = 53790 \frac{m}{s}$

$V_{peri,neu}^2 = \mu_s \left(\frac{2}{r_{peri}} - \frac{1}{a_{,neu}}\right)$

$a_{,neu} = -7,575 \cdot 10^{11}$ m

$a < 0 \rightarrow$ Hyperbel

c)



4.3

a) $V_{k,350} = \sqrt{\frac{\mu_E}{H_1 + R_E}} = 7697 \frac{m}{s}$

$V_{k,800} = \text{" " } = 7452 \frac{m}{s}$

$V_{k,20.000} = \text{" " } = 3887 \frac{m}{s}$

Näherung der kleinere Körper kreist um den größeren, sie kreisen also nicht um ihren gemeinsamen Massenschwerpunkt.

b) Umlaufzeit $P = \frac{365,25}{365,25+1} = 86164\,s$

Höhe H : $\cancel{P = \frac{a^3}{\mu}} \; P = 2\pi \sqrt{\frac{a^3}{\mu}}$

$a = 4216 \cdot 10^7\, m \rightarrow H = 35780\, km$

$V_k^2 = \mu_E \left(\frac{1}{a}\right)$

$V_k = 3075 \frac{m}{s}$

4.4 $V_{9000} = \sqrt{\mu_V \cdot \frac{1}{r}} = 6009 \frac{m}{s}$

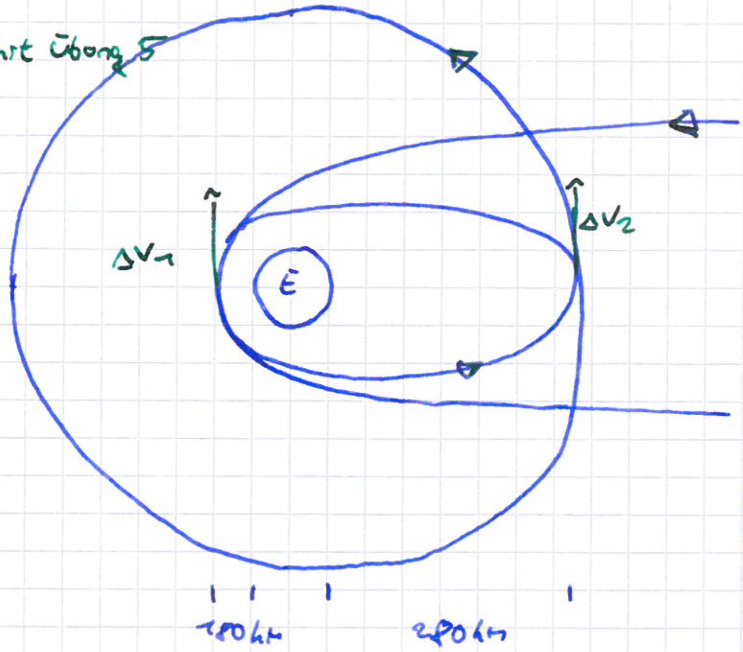
$V_0 = \sqrt{V_\infty^2 + \frac{2\mu_V}{r_4}} = 8737 \frac{m}{s}$

$\Delta V_{ce} = V_0 - V_{9000} = 2727 \frac{m}{s}$

Raumfahrt Übung 5 [E1]

5.1

a)



$r_{peri} = R_E + H_1 = 6{,}558 \cdot 10^6\,m$

$r_{apo} = R_E + H_2 = 6{,}658 \cdot 10^6\,m$

b) Parabel : $V_{peri} = \sqrt{\frac{2\mu_E}{r_{peri}}} = \cancel{\dots} = 11030\,\frac{m}{s}$

Ellipse : $V_{peri,neu} = \sqrt{\mu_E \left(\frac{2}{r_{peri}} - \frac{1}{a}\right)}$

$a = \frac{r_{peri} + r_{apo}}{2} = \cancel{\dots} = 6{,}608 \cdot 10^6\,m$

$V_{peri,neu} = \cancel{\dots} = \cancel{\dots}\ 7{,}826\,\frac{m}{s}$

$\Delta V_1 = V_{peri} - V_{peri,neu} = \cancel{\dots} = 3204\,\frac{m}{s}$

c) Kreis : $V_{kreis} = \sqrt{\frac{\mu}{a}} = 7737\,\frac{m}{s}$

~~[illegible]~~

Ellipse : $V_{apo} = \sqrt{\mu_E \left(\frac{2}{r_{apo}} - \frac{1}{a}\right)} = 7708\,\frac{m}{s}$

$\Delta V_2 = 29\,\frac{m}{s}$

Entgegen dem Geschwindigkeitsvektor in b), in Richtung des Vektors in c)

d) $\Delta V_{ges} = \Delta V_1 + \Delta V_2 = 3233\,\frac{m}{s}$

$\Delta V_{ges} = c_e \cdot \ln\left(\frac{m_S + m_T}{m_S}\right)$; $m_S + m_T = 30\,000\,kg$; $I_{sp} = 380\,s \rightarrow c_e = 3728\,\frac{m}{s}$

$m_S = 12600\,kg$

$m_T = 17400\,kg$

e) $\Delta V = 29\,m/s \longrightarrow m_T = 232{,}5\,kg$

98,66 % Einsparung der ursprünglichen Menge

f) $V_{parabel} = \sqrt{\frac{2\mu}{r_{peri}}} = 10{,}94\,\frac{km}{s}$

$\Delta V = V_{parabel} - V_{kreis} = 3205\,\frac{m}{s}$

$m_T = 17300\,kg$ (wie oben berechnet)

5.2

a) $r_{peri} = H_1 + R_E \qquad r_{apo} = H_2 + R_E$

$= 6{,}478 \cdot 10^6\,m \qquad = 7{,}618 \cdot 10^7\,m$

$a = \frac{r_{peri} + r_{apo}}{2} = 7{,}143 \cdot 10^7\,m$

$\mathcal{E}_{Ellipse} = -\frac{\mu_E}{2a} = -1{,}744 \cdot 10^7\ \frac{J}{kg}$

$v_{peri} = \sqrt{\mu_E\left(\frac{2}{r_{peri}} - \frac{1}{a}\right)} = 9397\ \frac{m}{s}$

$v_{apo} = \quad " \quad " \quad = 3774\ \frac{m}{s}$

b) Perigäum (höhere Ausgangsgeschwindigkeit)

Neue Parabelbahn zum Marsorbit im apogäum:

Nicht nötig für b)

$r_{Erde} = 1{,}496 \cdot 10^{11}\,m$ (Vis-Viva-Gleichung)

$r_{Mars} = 2{,}280 \cdot 10^{11}\,m$ (" " ")

Hohmann-Ellipse: $a = \frac{r_E + r_M}{2} = 1{,}888 \cdot 10^{11}\,m$

$r_{peri} = 6{,}478 \cdot 10^6\,m$; $R_{po} = R_E + (r_E - r_M) =$

$v_{peri,neu} = \sqrt{\frac{2\mu_E}{r_{peri}}} = 11090\ \frac{m}{s} \quad \rightarrow \Delta v = 1699\ \frac{m}{s}$

$v_{apo,neu} = \quad " \quad " \quad " \quad = 6976\ \frac{m}{s} \quad \rightarrow \Delta v = 3265\ \frac{m}{s}$

Weniger Δv im Perigäum $\rightarrow$ Effizienter

Raumfahrt Übung 5

5.2

c) $r_{Erde} = 1{,}496\cdot10^{11}\,m$ $\quad r_{Mars} = 2{,}280\cdot10^{11}\,m$

$a = 1{,}888\cdot10^{11}\,m$

$v_{h,1} = 29{,}78\,\frac{km}{s}$

$\Delta v_{ch} = v_{h,1}\left(\sqrt{\frac{2r_M}{r_E+r_M}} - 1 + \sqrt{\frac{r_E}{r_M}}\left(1-\sqrt{\frac{2r_E}{r_E+r_M}}\right)\right)$

$= 55968\,\frac{m}{s}$

$t = \frac{1}{2}P = \pi\sqrt{\frac{a^3}{\mu_S}} = 258{,}9$ Tage

d) $v_{peri} = \sqrt{\frac{2\mu_M}{r_{peri}}} = 4886\,\frac{m}{s}$

$v_{h,200} = \sqrt{\frac{\mu}{a}} = 3455\,\frac{m}{s}$

$\Delta v = 1431\,\frac{m}{s}$

$m_s = 900\,kg \quad I_s = 250 \;; \quad c_e = 2453\,\frac{m}{s}$

$\Delta v = c_e \cdot \ln\left(\frac{m_s}{m_s - m_T}\right)$

$m_T = 397{,}8\,kg$

e) $\Delta v = c_e \cdot \ln\left(\frac{m_s}{m_s - 300kg}\right) = 994{,}6\,\frac{m}{s}$

$v_{peri,neu} = v_{peri} - \Delta v = 3891\,\frac{m}{s}$

$v_{peri} = \sqrt{\mu_M\left(\frac{2}{r_{peri}} - \frac{1}{a}\right)}$

$a = 4{,}898\cdot10^{6}\,m$

$a = \frac{r_{peri} + r_{apo}}{2}$

$r_{apo} = 6{,}211\cdot10^{6}\,m \quad r_{peri} = 3{,}585\cdot10^{6}\,m$

5.3

a) $c_e = 3000\,\frac{m}{s} \quad r_h = 6{,}578\cdot10^{6}\,m \quad F = 20\,N$

$\dot{m} = \frac{F}{c_e} = 6{,}667\cdot10^{-3}\,\frac{kg}{s}$

$m_0 = 10\,000\,kg \quad m_{T,verbr.} = 5000\,kg$

$t = \frac{5000}{\dot{m}} = 7{,}5\cdot10^{5}\,s = 8{,}68$ Tage

b) $\Delta v = c_e \ln\left(\frac{m_0}{\frac{1}{2}\cdot m_0}\right) = 2079\,\frac{m}{s}$

$v_{h,0} = \sqrt{\frac{\mu_E}{r}} = 7784\,\frac{m}{s}$

Aufspiralen: $\Delta v_{ch} = v_{h,0} - v_h = \sqrt{\mu_E}\left(\sqrt{\frac{1}{r_0}} - \sqrt{\frac{1}{r}}\right)$

$r_{neu} = 1{,}225\cdot10^{7}\,m \rightarrow H_{neu} = 5900\,km$

c) ~~$v_{u,500}$~~ $a = \frac{r_{Peri} + r_{Apo}}{2} = 9{,}414 \cdot 10^6\,m$ ↯

$\Delta t = \frac{\pi}{\sqrt{\mu_E}} \left(\frac{a}{2}\right)^{3/2} = 1{,}607 \cdot 10^3\,s$ ↯ Bei der Höhe des Apogäum vertippt

$= 4544{,}1\,s$

d) $\Delta V_{ch} = V_{u,1}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1 + \sqrt{\frac{r_1}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)\right)$

$r_1 = 6{,}578 \cdot 10^6\,m \qquad r_2 = 1{,}225 \cdot 10^7\,m$

$V_{u,1} = 7784\,\frac{m}{s} \qquad V_{u,2} = 5704\,\frac{m}{s}$

$\Delta V_{ch} = 2024\,\frac{m}{s}$

$\Delta V_{ch} = c_e \cdot \ln\left(\frac{m_0}{m_0 - m_T}\right)$

$m_T = 4{,}918\,t$

etwas geringer Treibstoffverbrauch wie beim Aufspiralen

Raumfahrt Übung 6

6.1

a) $v_1^2 = \mu_S\left(\frac{2}{r_m} - \frac{1}{a}\right) \rightarrow a = 6{,}003 \cdot 10^{12}\ m$

$v_2^2 = \mu_S\left(\frac{2}{r_j} - \frac{1}{a}\right) \rightarrow v_2 = 17{,}86\ \frac{km}{s}$

b) $\Delta\varepsilon_{max} = \underset{\uparrow\ v_j}{v_p}\ \underset{\uparrow\ v_2}{v_{3,max}} = v_j \sqrt{\frac{\mu_j}{r_{peri}}}$; $v_j = \sqrt{\frac{\mu_S}{r_j}} = 13{,}06\ \frac{km}{s}$

$\Delta\varepsilon_{max} = 2{,}333 \cdot 10^8\ \frac{J}{kg}$ $\qquad \beta_1 = 120^\circ$; $x = 1$

$r_{peri} = 3{,}972 \cdot 10^8\ m$

$v_3^2 = v_2^2 + \underset{\uparrow\ v_j}{v_p^2} - 2 v_2 v_p \cos(\gamma_1)$

$v_2^2 = v_3^2 + v_j^2 + 2 v_3 v_j \cos(\beta_1)$

$v_3 = 20{,}35\ \frac{km}{s}$

$\gamma_1 = 80{,}7^\circ = 1{,}408\ rad$

c) $v_5^2 = v_2^2 - 2\frac{h_1}{h_2^2} + \frac{4 v_3 v_j}{h_2}\sqrt{1 - \frac{1}{h_2^2}}\sqrt{1 - \left(\frac{h_1}{2 v_j v_3}\right)^2}$

$h_1 = v_2^2 - v_j^2 - v_3^2 = -3{,}657 \cdot 10^8$

$h_2 = 1 + \frac{r_{peri}}{\mu_j} v_3^2 = 2{,}298 = 1 + x^2 = 2$

$v_5 = 29760\ \frac{m}{s}$

$\Delta v = v_2 - v_5 = 11{,}3\ \frac{km}{s}$

$\Delta\gamma = \gamma_2 - \gamma_1$

$v_4 = v_3 \rightarrow v_4^2 = v_5^2 + v_j^2 - 2 v_j v_5 \cos(\gamma_2)$

$\gamma_2 = 37{,}18^\circ = 0{,}649\ rad$

$\Delta\gamma = -43{,}53^\circ = -0{,}76\ rad$

d) Bahnenergie: vorher: $\varepsilon_1 = \frac{v_2^2}{2} - \frac{\mu_S}{r_j} = -\frac{\mu_S}{2a_1} = -1{,}705 \cdot 10^7\ \frac{J}{kg} < 0$

$\rightarrow$ Ellipse

nachher: $\varepsilon_2 = -\frac{\mu_S}{2 \cdot a_2} = \frac{v_3^2}{2} - \frac{\mu_S}{r_j} = 2546 \cdot 10^8\ \frac{J}{kg} > 0$

$\rightarrow$ Hyperbel

6.2

a) $r_{peri} = 100000 \text{ km} = 100000000 \text{ m}$

$v_j = 13100 \frac{m}{s}$

$M_j = 1{,}9 \cdot 10^{27} \text{ kg}$

$v_2 = 10\,000 \frac{m}{s}$

$\gamma_1 = 40°$

$v_3^2 = v_2^2 + v_j^2 - 2 v_2 v_j \cos(\gamma_1)$

$v_3 = 8820 \frac{m}{s}$

$v_2^2 = v_3^2 + v_j^2 + 2 v_3 v_j \cos(\beta_1)$

$\beta_1 = 133{,}2°$

b)

$\mu_j = \gamma \cdot M_j = 1{,}268 \cdot 10^{17}$

$\Delta \varepsilon_{max} = v_j \sqrt{\frac{\mu_j}{r_{peri}}}$

$= 4{,}878 \cdot 10^{8} \frac{J}{kg}$

$\Delta \varepsilon = 2 v_j \sqrt{\frac{\mu_j}{r_{peri}}} \left(\frac{x^2 - \sqrt{2 + x^2}}{(1+x^2)^2} \sin(\beta_1) - \frac{x}{(1+x^2)^2} \cos(\beta_1) \right)$

$x = v_3 \sqrt{\frac{r_{peri}}{\mu_j}} = 0{,}2477$

$\Delta \varepsilon = 2{,}025 \cdot 10^{8} \frac{J}{kg}$

Es werden ~~ka~~ nur 41,51% der möglichen Energieänderung erreicht.

$v_5^2 = v_2^2 - 2 \frac{k_1}{k_2^2} + \frac{4 v_3 v_j}{k_2} \sqrt{1 - \frac{1}{k_2^2}} \sqrt{1 - \left(\frac{k_1}{2 v_j v_3}\right)^2}$

$k_1 = v_2^2 - v_j^2 - v_3^2 = -1{,}655 \cdot 10^{8}$; $k_2 = 1 + \frac{r_{peri}}{\mu_j} v_3^2 = 1$

$v_5 = 20760 \frac{m}{s}$

$v_{k3,s} = \sqrt{\frac{2 \mu_S}{r_j}} = 18470 \frac{m}{s} < v_5 \rightarrow$ Fluchtgeschw. wird erreicht!

c) $v_2 = 10000 \frac{m}{s}$ $v_3 = 8820 \frac{m}{s}$

$\beta_1 = 2{,}325$ rad $\Delta E = 2{,}0257 \cdot 10^8 \frac{m^3}{s^2}$

$a = \frac{r_{peri} + r_{apo}}{2} =$

$\frac{v_1^2}{2} - \frac{\mu_S}{r_E} = \frac{v_2^2}{2} - \frac{\mu_S}{r_J}$

$v_1 = 38773 \frac{m}{s}$

$\cos(\gamma_0) = \frac{r_P v_2}{r_E v_1} \cos(\gamma_0) \rightarrow \gamma_0 = 20{,}74° = 0{,}362$ rad

d) $H = 400km \rightarrow r_u = 6778$ km

$v_u = \sqrt{\frac{\mu_E}{r_u}} = 7668 \frac{m}{s}$

$v_\infty^2 = v_S^2 + v_P^2 - 2 v_P v_S \cos(\gamma_2)$ $\gamma_2 = 0{,}362$ rad $v_S = 38773 \frac{m}{s}$

$v_\infty = 15770 \frac{m}{s}$ $v_P = \sqrt{\frac{\mu_S}{r_E}} = 29780 \frac{m}{s}$

$\frac{1}{2} v_\infty^2 = \frac{1}{2} v_{start}^2 - \sqrt{\frac{\mu_E}{r_u}}^2$

$v_{start} = 18623 \frac{m}{s}$

$\Delta v = 10954 \frac{m}{s}$ $(= v_{start} - v_u)$

6.3

a) Manöver 1:

$v_{u,2} = \sqrt{\frac{\mu}{r}} = 3075 \frac{m}{s}$

$a_{ellips} = \frac{r_2 + r_1}{2} = 2{,}45 \cdot 10^7$ m

$v_{E,Apo} = 1624 \frac{m}{s}$ $\rightarrow \Delta v_1 = \sqrt{v_{u,2}^2 + v_{E,Apo}^2 - 2 v_{u,2} \cdot v_{E,Apo} \cdot \cos(28{,}5°)}$

$\Delta v_1 = 1821 \frac{m}{s}$

Manöver 2:

$v_{E,peri} = 10010 \frac{m}{s}$

$v_{u,1} = 7634 \frac{m}{s}$ $\rightarrow \Delta v_2 = 2376 \frac{m}{s}$

$\Delta v_{ges} = 4197 \frac{m}{s}$

b) $V_{E,Apo} = 7576\,\frac{m}{s}$ $\qquad \varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$

$\Delta V_1 = 1559\,\frac{m}{s}$ $\qquad a = 6{,}528 \cdot 10^6\,m$

$V_{Apo,neu} = 7449\,\frac{m}{s}$

$V_{4,1} = 7634\,\frac{m}{s}$

$\Delta V_2 = 184{,}9\,\frac{m}{s}$

$\Delta V_{ges} = 1744\,\frac{m}{s}$

Raumfahrt Übung 7

7.1

a) $R = \frac{R_M}{M} = 837{,}4 \; \frac{J}{kg\,K}$; $h_0 = \eta \varepsilon_T = 1{,}317 \cdot 10^7 \; \frac{J}{kg}$

~~[illegible]~~ ; $c_p = R \frac{\kappa}{\kappa - 1} = 4{,}024 \cdot 10^3 \; \left[\frac{J}{kg\,K}\right]$

$$W_e = \sqrt{2h_0}\sqrt{1 - 1^{\frac{\kappa-1}{\kappa}}}$$

$$W_{e,\alpha=0} = \sqrt{2(h_0 - c_p T_e)} = 4496 \; \frac{m}{s}$$

$$\lambda = \frac{1}{2}(1 + \cos^2(\alpha)) = 0{,}9903$$

$$W_{e,real} = W_{e,\alpha=0} \cdot \lambda = 4453 \; \frac{m}{s} = c_e$$

b) $\eta_i = \frac{c_e^2}{2\varepsilon_T} = 0{,}7376$

7.2

a) $\dot{m} = 127 \; \frac{kg}{s}$; $d_e = 700\,mm = 0{,}7\,m$

$p_0 = 15{,}2\;bar$; $p_e = 0{,}84\;bar$

$R = 438 \; \frac{J}{kg\,K}$; $\kappa = 1{,}26$; $\Gamma = 0{,}66$

$$\varepsilon = \frac{A_e}{A_t} = \Gamma \left(\frac{p_e}{p_0}\right)^{-\frac{1}{\kappa}} \left(\frac{2\kappa}{\kappa-1}\left(1 - \left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right)\right)^{-\frac{1}{2}}$$

$$\varepsilon = 3{,}747$$

b) $H_2 = 22\,km$ $H_1 = 0\,m$ $A_e = \pi \cdot \left(\frac{d_e}{2}\right)^2 = 0{,}3848\,m^2$

$p_a = 0{,}063\,bar$ $p_a = 1\,bar$ $A_t = 0{,}1223\,m^2$

$$F_2 = p_0 A_t \left(\Gamma \sqrt{\frac{2\kappa}{\kappa-1}}\sqrt{1 - \left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}} + \frac{A_e}{A_t}\left(\frac{p_e - p_a}{p_0}\right)\right)$$

$$F_2 = 2{,}867 \cdot 10^5\,N$$

$$F_1 = 2{,}500 \cdot 10^5\,N$$

c) ~~[illegible]~~ $W_e = \sqrt{2h_0}\sqrt{1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}}$

$$= \sqrt{\frac{2\kappa}{\kappa-1}}\sqrt{RT_0}\sqrt{1 - \left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}}$$

$$\dot{m} = \frac{p_0 A_t \Gamma}{\sqrt{RT_0}} \rightarrow T_{0,akt} = \cancel{[illegible]}\; 2737\,K$$

~~[illegible]~~

$$c_{e,0\,km} = \frac{F_0}{\dot{m}} = 1969 \; \frac{m}{s}$$

$$c_{e,22\,km} = \frac{F_{22}}{\dot{m}} = 2253 \; \frac{m}{s}$$

14,4% Erhöhung der effektive Austrittsgeschw.

2

d) $\frac{p_0}{p_e} = \left(\frac{T_0}{T_e}\right)^{\frac{k}{k-1}}$

$\cancel{T_0 = T_e + \frac{w_e^2}{2c_p}}$

$\dot{m} = \frac{p_0 A_T \Gamma}{\sqrt{RT_0}}$

$T_0 = 2737\,K$

7.3 $p_0 = 30\,bar$ $A_e = 1\,m^2$

$T_0 = 3000\,K$ $p_a = 0{,}5\,bar$

$Ma_e = 3$ $k = 1{,}26$ $R = 437{,}6\,\frac{kJ}{kg\,K}$

a) $F = \dot{m} w_e + (p_e - p_a) A_e$

$\dot{m} = \frac{p_0 A_t \Gamma}{\sqrt{RT_0}}$; $\Gamma = \sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = 0{,}66$

$\varepsilon = \frac{A_e}{A_t} = \Gamma\left(\frac{p_e}{p_0}\right)^{-\frac{1}{k}}\left(\frac{2k}{k-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{k-1}{k}}\right)\right)^{-\frac{1}{2}} =$

$A_t = 0{,}176\,m^2$

$\frac{p_0}{p_e} = \left(1+\frac{k-1}{2}Ma_e^2\right)^{\frac{k}{k-1}}$ $\rightarrow p_e = 0{,}1024\,bar$

$\dot{m} = 304{,}1\,\frac{kg}{s}$

$F = p_0 A_t\left(\Gamma\sqrt{\frac{2k}{k-1}}\sqrt{1-\left(\frac{p_e}{p_0}\right)^{\frac{k-1}{k}}} + \frac{A_e}{A_t}\left(\frac{p_e - p_a}{p_0}\right)\right)$

$F = 8{,}759 \cdot 10^5\,N$

Raumfahrt Übung 8

8.1 $p_a = 1\,bar$; $p_e = 0{,}266\,bar$; $p_0 = 25\,bar$; $F = 20000\,N$; $w_t = 1155\,\frac{m}{s}$

a) $R = \frac{R_M}{M} = 437{,}6\,\frac{J}{kgK}$; $\Gamma = 0{,}66$; $k = 1{,}26$

$$\varepsilon = \frac{A_e}{A_t} = \Gamma\left(\frac{p_e}{p_0}\right)^{-\frac{1}{k}}\left(\frac{2k}{k-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{k-1}{k}}\right)\right)^{-\frac{1}{2}} = 10 = \frac{A_e}{A_t}$$

$$C_F = \frac{F}{p_0 A_t} = \Gamma\sqrt{\frac{2k}{k-1}}\sqrt{1-\left(\frac{p_e}{p_0}\right)^{\frac{k-1}{k}}} + \varepsilon\left(\frac{p_e-p_a}{p_0}\right)$$

$C_F = 1{,}309$; $A_t = 6{,}112\cdot10^{-3}\,m^2$

$A_e = 6{,}112\cdot10^{-2}\,m^2$

$d_t = 4{,}411\cdot10^{-2}\,m\cdot2 = 8{,}822\cdot10^{-2}\,m$

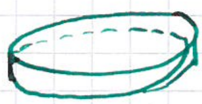
$$\dot m = \frac{p_0 A_t \Gamma}{\sqrt{R T_0}}$$

$w_t = \sqrt{k R T_t}$

$T_t = 2419\,K \rightarrow \frac{T_t}{T_0} = \frac{2}{k+1} \rightarrow T_0 = 2733\,K$

$\rightarrow \dot m = 9{,}222\,\frac{kg}{s}$

b)



Δr

c)

$d_t(70s) = d_t(0s) + 2\cdot\dot{r}\cdot t_b = 0{,}1102\,m$

$A_t(70s) = \pi\cdot\left(\frac{d_t(70)}{2}\right)^2 = 9{,}538\cdot10^{-3}\,m^2$

$\dot{m} = \frac{p_0 A_t(70)\Gamma}{\sqrt{RT_0}} \longrightarrow \dot{m} = 74{,}39\,\frac{kg}{s}$

$p_e = 0{,}4937\,bar$

$F(70) = p_0 A_t(70)\left(\Gamma\sqrt{\frac{2k}{k-1}}\sqrt{1-\left(\frac{p_e}{p_0}\right)^{\frac{k-1}{k}}} + \frac{A_e(70)}{A_t(70)}\left(\frac{p_e-p_a}{p_0}\right)\right)$; A_e konst

$F(70) = 33470\,N$

$m_T(70) = \int_0^{70}\dot{m}(t)\,dt$

$= \int_0^{70}\frac{p_0\left(\pi\cdot\left(\frac{(d_t(0)+2\cdot\dot{r}\cdot t_b)}{2}\right)^2\right)\Gamma}{\sqrt{RT_0}}\,dt$; $\dot{r} = 0{,}757\cdot10^{-3}\,\frac{m}{s}$

$m_T(70) = 879{,}8\,kg$

8.1

a) $H_0 = 5{,}5\,km$; $p_{a,0} = 0{,}5\,bar$

$L = 4{,}0\,m$; $d = 0{,}43\,m$; $\rho_B = 1690\,\frac{kg}{m^3}$; $R = \frac{R_m}{\bar{M}} = 361{,}5\,\frac{J}{kg\,K}$

$T_0 = 3100\,K$; $W_e = 2532\,\frac{m}{s}$; $k = 1{,}26$; $\Gamma = 0{,}66$; $\bar{M} = 23\,\frac{kg}{kmol}$

$t_B = 39\,s$; $H_B = 10\,km$; $p_{B,a} = 5{,}54\cdot10^{-4}\,bar$

$W_e = \sqrt{2(h_0 - c_p T_e)}$; $c_p = \frac{R_m}{\bar{M}}\cdot\frac{k}{k-1} = 1752$

$T_0 = T_e + \frac{W_e^2}{2c_p} \longrightarrow T_e = 1270\,K$

$\varepsilon = \frac{A_e}{A_t} = \Gamma\left(\frac{p_e}{p_0}\right)^{-\frac{1}{k}}\left(\frac{2k}{k-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{k-1}{k}}\right)\right)^{-\frac{1}{2}} =$

$p_e = \rho_e R T_e \Rightarrow \rho_e = 0{,}1089\,\frac{kg}{m^3}$; $p_e = p_{0,a} = 5{,}5\,bar$

$\dot{m} = \rho_e W_e A_e$

$\dot{m} = \frac{m_T}{t_B}$; $m_T = \rho_B\cdot L\cdot\pi\cdot\left(\frac{d}{2}\right)^2 = 987{,}7\,kg$

$\dot{m} = 25{,}17\,\frac{kg}{s}$

$A_e = 0{,}09129\,m^2$

b) $H_0 = 5{,}5\,km$

$F_0 = \dot{m} W_e + (p_e - p_a) A_e = 63730\,N$

$c_{e0} = W_e = 2532 \frac{m}{s}$ (Angepasst auf H_0)

$I_{s0} = \frac{c_e}{g_0} = 258{,}15$

$H_{B70} = 70\,km$

$F_{70} = \dot{m} W_e + (\overset{5.5}{p_e} - \overset{5.54\cdot 10^{-4}}{p_a}) A_e = 6{,}829 \cdot 10^4\,N$

$c_{e70} = \frac{F}{\dot{m}} = 2773 \frac{m}{s}$

$I_{s70} = 276{,}6\,s$

c) $H_0 = 5{,}5\,km$ $\quad p_0 = 45\,bar$

$F_0 = p_0 A_t \left(\Gamma \sqrt{\frac{2k}{k-1}} \sqrt{1 - \left(\frac{p_e}{p_0}\right)^{\frac{k-1}{k}}} + \frac{A_e}{A_t} \left(\frac{p_e - p_a}{p_0}\right)\right)$

$\varepsilon = \frac{A_e}{A_t} = 9{,}694 \rightarrow$ ~~A_t 8,9447 $\cdot 10^{-3}$ m²~~

~~F_0 ... $\cdot 10^4$ N~~

$\dot{m} = \frac{p_0 A_t \Gamma}{\sqrt{R T_0}} \rightarrow A_t = 8{,}977 \cdot 10^{-3}\,m^2$

$\hookrightarrow A_e = 8{,}697 \cdot 10^{-2}\,m^2$

$F_0 = 64570\,N$

$F_{70} = 68860\,N$

$p_0 A_t = \frac{\dot{m} \sqrt{R T_0}}{\Gamma}$ $\quad$ mehr Schub mit höherer Temp in Brennkammer oder höherem Massenstrom oder kleinerem A_t.

Kein Druckverlust bis zum Düsenhals -> Verbesserungen im Divergenten Teil der Düse möglich.

8.3 $\quad k = 1{,}2 \quad\quad k = 1{,}33 \quad\quad k = 1{,}26$

$\vdots \quad\quad \vdots \quad\quad \vdots$

[illegible]

Raumfahrt Übung 9 $\boxed{1}$

9.1 $p_a = 1{,}013\,bar$; $F = 20\,kN$; $T_e = 1250\,K$;

$\varepsilon_T = 13442 \frac{kJ}{kg}$; $\bar{M} = 10 \frac{g}{mol}$; $\kappa = 1{,}26$; $R = 831{,}4 \frac{J}{kgK}$

a) Ideales, verlustfreies Triebwerk → $\eta = 1$

$h_0 = \eta\, \varepsilon_T = 13442 \frac{kJ}{kg}$; $c_p = R \frac{\kappa}{\kappa-1} = 4029$

$T_0 = T_e + \frac{w_e^2}{2c_p}$; $w_e = \sqrt{2(h_0 - c_p T_e)} = 4100 \frac{m}{s}$

$T_0 = 3336\,K$

$T_t = T_0 \left(\frac{2}{\kappa+1}\right) = 2952\,K$

$w_t = \sqrt{\kappa R T_t} = 1759 \frac{m}{s}$

$w_e = 4100 \frac{m}{s}$

$\frac{T_0}{T_e} = 1 + \frac{\kappa-1}{2} Ma_e^2$

$Ma_e = 3{,}583$

b) $\cancel{\frac{p_0}{p_e} = \left(\frac{T_0}{T_e}\right)^{\frac{\kappa}{\kappa-1}} = 64{,}16}$ $\dot{m} = 6{,}1 \frac{kg}{s}$; $p_0 = 25\,bar$

$\cancel{\frac{p_e}{p_0} = 0{,}01559}$

$\dot{m} = \rho \cdot w \cdot A = const = \frac{p_0 A_t \Gamma}{\sqrt{R T_0}}$ → $A_t = 6{,}157 \cdot 10^{-3}\,m^2$

(1) $F = \dot{m} w_e + (p_e - p_a) A_e$

(2) $\dot{m} = \rho_e w_e A_e$

(3) $p_e = \rho_e R T_e$

$A_e p_e = \cancel{\ldots}\ 1546\,N$

$A_e = 6{,}473 \cdot 10^{-2}\,m^2$

$\frac{p_e}{p_0} = 9{,}555 \cdot 10^{-3}$

$\cancel{\ldots = 0{,}01 \frac{kg}{s}}$ ✓

$\varepsilon = \frac{A_e}{A_t} = 10{,}51$

$F_{max} = \dot{m} \cdot \sqrt{2 \varepsilon_T} = 31630\,N$

$\eta_i = \left(\frac{F}{F_{max}}\right)^2 = 0{,}3998$

Flöhle
Franz

~~[illegible]~~

~~[illegible]~~

~~[illegible]~~

~~[illegible]~~

~~[illegible]~~

~~a)~~
c) $\ddot{m} = 0{,}07 \frac{kg}{s^2} t \rightarrow \dot{m}(t) = 0{,}07 t \frac{kg}{s} t + C$

$\dot{m}(t) = \frac{p_0 A_t \Gamma}{\sqrt{R T_0}}$

~~[illegible]~~

$A_t = 7{,}065 \cdot 10^{-5} (t + 14{,}29 C)$

$A_t(t=0) \stackrel{!}{=} A_t \Rightarrow C = 6{,}\cancel{[illegible]}1$

$A_t(t) = 7{,}065 \cdot 10^{-5} (t + 9{,}139 \cdot 10^{1})$

$A_t(t) = \pi \cdot \left(\frac{d_t(t)}{2}\right)^2$

$d_t(t) = 9{,}485 \cdot 10^{-3} \sqrt{t + 9{,}139 \cdot 10^{1}}$

$d_t(100) = 0{,}013112\, m$

d) $\dot{m}(t=100) = \int_0^{100} \dot{m}\, dt = \left[\frac{1}{2} t^2 \cdot 0{,}07\right]_0^{100} = 960\, kg$

9.2 $d_t = 10\, cm$; $p_0 = 1{,}013\, bar$; $T_e = 950\, K$; $R = 831{,}4$

$\varepsilon_T = 13{,}442 \frac{MJ}{kg}$

a) $c_p = 4030 \frac{J}{kg K} = R \frac{k}{k-1} \rightarrow$

$k = 1{,}26$

$w_e = \sqrt{2(h_0 - c_p T_e)}$; $h_0 = \varepsilon_T \eta_v$; $\eta = 1$

$w_e = 4385 \frac{m}{s}$

b) $p_0 = 225\, bar$; $A_t = \pi \cdot \left(\frac{d_t}{2}\right)^2 = 0{,}007854\, m^2$

$\dot{m} = \frac{p_0 A_t \Gamma}{\sqrt{R T_0}}$; $T_0 = T_e + \frac{w_e^2}{2 c_p} = 3336\, K$

~~[illegible]~~ ; $\Gamma = \sqrt{k \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = 0{,}6599$

$\dot{m} = 10{,}03 \frac{kg}{s}$

$\frac{T_t}{T_0} = \frac{2}{k+1} \rightarrow T_t = \cancel{[illegible]}\ 2952\, K$

$w_t = \sqrt{k R T_t} = 1759 \frac{m}{s}$

9.2

c) $F = 275000\,N = \dot{m} w_e (p_e - p_a) A_e$ (1)

~~Pet~~

$\varepsilon = \frac{A_e}{A_t} = \Gamma \left(\frac{p_e}{p_0}\right)^{-\frac{1}{\kappa}} \left(\frac{2\kappa}{\kappa-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right)\right)^{-\frac{1}{2}} \Rightarrow A_e =$ (2)

$\dot{m} = \rho_e w_e A_e$; $\rho_e = \frac{p_e}{R T_e}$ (3), (4)

$p_e = \frac{1{,}267 \cdot 10^4}{A_e}$

$\cancel{A_e = \frac{1{,}267 \cdot 10^4 \cdot w_e}{R T_0}}$

$F = \dot{m} w_e + \underbrace{p_e A_e}_{\text{bekannt}} - \cancel{\ldots}\, p_a A_e$

$A_e = 0{,}4472\ m^2$

$p_e = 2{,}8602\,bar$

d) $p(H) = p(H_0)\, e^{-0{,}000\ldots}$

Auf welcher Höhe ist $p_e = p(H)$

$H(p = p_e) = 10850\ m$

$F(H = 10850\,m) = \dot{m} w_e = \cancel{\ldots}\ 307100\,N$

e) Schub im Vakuum größer, weil $p_e - p_a = p_e \rightarrow$ Schub wird im Vakuum um $p_e A_e$ N größer gegenüber der angepassten Höhe

Raumfahrt Übung 10

10.1

a) $\dot{\varphi} = 4\,\frac{rad}{s}$ nutationsfrei ~~[illegible]~~ $\dot{\varphi} = 229{,}2\,\frac{°}{s}$

um 20° gedreht

Düse mit 1N, $I_s = 225\,s$, $l = 0{,}75\,m$ 200ms Pulse

a) 1200 Nms

$\Delta l = F\tau n$ ~~[illegible]~~ $= \frac{\delta \Theta_z \omega_z}{L}\frac{\alpha}{\sin(\alpha)}$ (rad!)

$\alpha = \frac{\omega_z \tau_{puls}}{2}$ ~~[illegible]~~ 0,4

$n = 2868$ Impulse

b) Ein Puls pro Umdrehung, mit Frequenz $\frac{\omega_z}{2\pi} = \frac{2}{\pi}$ $\rightarrow t = \frac{n}{f} = 4505\,s$

c) Düse läuft insgesamt für 573,6 s

$I_s = \frac{F}{g_0 \dot{m}_t}$ $\rightarrow \dot{m}_t = 4{,}531 \cdot 10^{-4}\,kg/s$

$m_{t,ges} = 0{,}2599\,kg$

$F\tau n = \frac{\delta D}{L}\frac{\alpha}{\sin\alpha}$ mit $n=1$

$\delta_{min} = 6{,}973 \cdot 10^{-3}\,°$

Genauigkeit von $\frac{\delta_{min}}{2} = 3{,}486 \cdot 10^{-3}\,°$

10.2

$\omega = \pi/2\ \frac{rad}{s}$ $\Theta_z = 236\,kg\,m^2$

15N Schub, $I_s = 274\,s$, $\tau = 1\,s$, $L = 1{,}1\,m$

a) $\delta = 160°$ mit zwei Düsen $= \frac{8}{9}\pi$

$\Delta l = nF\tau \cdot 2 = \frac{\delta \Theta_z \omega_z}{L}\frac{\alpha}{\sin\alpha}$

$\alpha = \frac{\omega_z \tau}{2} = \frac{\pi}{4} \longrightarrow n = 34{,}84$

35 ~~Pk~~ Pulse nötig (von beiden Düsen)

~~[illegible]~~ Bei 15 Umdrehungen/min

$t_{gesamt} \approx 140\,s$

b) $\Delta I = \frac{\delta \Theta_z W_z}{L} \frac{\alpha}{\sin(\alpha)} = 1633\ Ns$ $\qquad 250° = \frac{25}{18}\pi$ rad

$\frac{\pi}{180}$ rad pro Tag

$\Delta I_{vorher} = n \tau F \cdot 2 = 1045\ Ns$

$\Delta I_{ges} = 1633 + 1045 = 2678\ Ns$ (in 250 Tagen)

$\Delta I_{ges} = n \cdot F \cdot \tau \cdot 2 \rightarrow n = 89{,}28$ Impulse

89 Impulse insgesamt → Laufzeit von 89 s beider Düsen

Eine Düse mit 178,6 s

$I_s = \frac{F}{g_0 \dot{m}_t} \rightarrow \dot{m}_t = 0{,}007145\ \frac{kg}{s}$

$M_t = 1{,}276\ kg$

c) $\tau_{Min} = 0{,}1\ s$; $F = 2 \cdot 15\ N$

$\Delta I_{Min} = \tau_{Min} F_{Min} = 3\ Ns$

$\Delta I_{Min} = \frac{\delta \Theta W}{L} \frac{\alpha}{\sin\alpha}$; $\alpha = 0{,}07857$

$\delta_{min} = 4{,}446 \cdot 10^{-3}$ rad (für eine Düse)

$\delta_{genau} = 0{,}2547°$

10.3 $\qquad f_0 = 2{,}8\ Hz \qquad f_1 = 0{,}23\ Hz \qquad T = 560$ Tage

a) $W_0 = f_0 2\pi = 17{,}59\ \frac{rad}{s}$; $\omega_1 = f_1 2\pi = 1{,}445\ \frac{rad}{s}$

Störmoment $M_s \sim W(t) \rightarrow M_s = C\,\omega(t)$

$\dot{\omega} = \frac{M_{stör}}{\Theta} = \frac{C}{\Theta} \omega(t)$

$\frac{d\omega}{dt} = \frac{C}{\Theta} \omega(t)$

$d\omega = \frac{C}{\Theta} \omega(t)\, dt$

~~$\int d\omega = \int \frac{C}{\Theta} \omega(t)\, dt$~~

$\int_{\omega_0}^{\omega} \frac{1}{\omega} d\omega = \int_{t_0}^{t} \frac{C}{\Theta} dt$

$\ln(\omega) - \ln(\omega_0) = \frac{C}{\Theta} t$

$W(T) \stackrel{!}{=} \omega_1 \longrightarrow C = \frac{1}{T}(+\Theta(\ln(\omega_1) - \ln(\omega_0)))$

~~$\ln(\omega) = \frac{1}{T}\Theta \ln(\frac{\omega_1}{\omega_0})$~~

$\rightarrow \ln(\frac{W}{W_0}) = \frac{t\Theta}{\Theta T} \ln(\frac{\omega_1}{\omega_0})$

$W(t) = W_0 (\frac{\omega_1}{\omega_0})^{\frac{t}{T}} = f_0 2\pi (\frac{f_1}{f_0})^{\frac{t}{T}}$

b) $\Theta_z = 3\,kg\,m^2$

$M_{stör} = C \cdot \omega(t)$ mit $C = \frac{1}{560\,Tage}\,\Theta \ln\left(\frac{\omega_1}{\omega_0}\right)$ mit $t = 0$

$= -2{,}726 \cdot 10^{-6}\,Nm$

c) $I_s = 65\,s$, $l = 0{,}25$

$F = \left|\frac{M_s}{l}\right| = 1{,}09 \cdot 10^{-5}\,N$

$\Delta I = F \cdot T = 527{,}6\,Ns$

$I_s = \frac{F}{g_0 \dot{m}_T} \rightarrow \dot{m}_T = 1{,}709 \cdot 10^{-8}\,\frac{kg}{s}$

$m_T = 0{,}8277\,kg$

d) $\Delta\delta = 45° = \frac{\pi}{4}\,rad$ $\quad 2\alpha = 60°$, $\alpha = \pi/6\,rad$ $\quad F = 1N$

$\omega_z = \omega_0 = 17{,}59\,rad/s$

$\alpha = \frac{\omega_z \tau}{2} \rightarrow \tau = 0{,}05953\,s$

$\Delta I = \frac{\delta \Theta \omega_z}{l} \frac{\alpha}{\sin\alpha} = 773{,}6\,Ns$

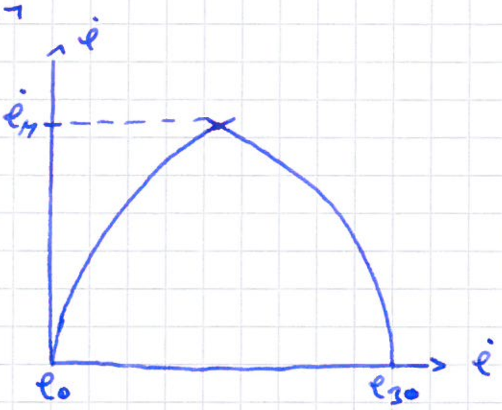
$\Delta I = F \tau n \rightarrow n = 2976$ Impulse

$t_{ges} = \frac{n}{f_0} = 1042\,s$ (Bei einem Puls pro Umdrehung)

Raumfahrt Übung 11

11.1

a)



b) $\Delta I = \frac{\delta \Theta \omega_z}{L} \frac{\alpha}{\sin\alpha}$; $\alpha = \frac{\omega_z \tau}{2}$

$M = F \cdot L = 1{,}091 \cdot 10^{5}\ Nm$

$\ddot{\varphi} = \frac{M}{\Theta_t} = 3{,}969 \cdot 10^{-3}\ \frac{rad}{s^2}$

$\dot{\varphi} = \int_{t_0}^{t} \frac{M}{\Theta}$

$\dot{\varphi} = \dots = \frac{M}{\Theta} t$

$\varphi = \frac{1}{2} \frac{M}{\Theta} t^2 + \underbrace{\cancel{\dot{\varphi}_0}}_{=0} t + \underbrace{\cancel{\varphi_0}}_{=0}$

$\rightarrow t = \sqrt{\frac{2\Theta\varphi}{M}}$

$\dot{\varphi} = \frac{M}{\Theta} \cdot \left(\sqrt{\frac{2\Theta\varphi}{M}} \right)$

$\dot{\varphi}$ maximal bei $\varphi = 15° = \frac{\pi}{12}\ rad$

$\dot{\varphi}_{max} = 2{,}613\ \frac{°}{s}$

c) $\dot{\varphi}_{max} = \frac{M}{\Theta} t \longrightarrow t_{max} = 11{,}49\ s$

$t_{ges} = 2 \cdot t_{max} = 22{,}98\ s$

$I_s = \frac{F}{g_0 \dot{m}_t}$

$\dot{m}_t = 2{,}73\ \frac{kg}{s}$

$m_{t,ges} = 62{,}49\ kg$

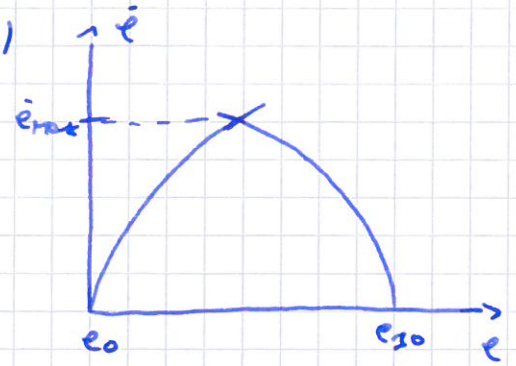
11.2

a) Zwei Triebwerke ; $F = 2 \cdot F_0 = 1\ N$

$\Delta I = \frac{\Theta \Delta\omega}{L} = 6{,}283\ Ns$

$F \Delta t = \Delta I \rightarrow \Delta t = 6{,}283\ s$

b)

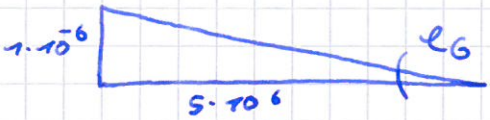


$M = F \cdot l = 1\,Nm$

$M_0 = \frac{M}{\Theta} = 1{,}389 \cdot 10^{-2}\ \frac{1}{s^2}$

$\dot\varphi_{max} = \frac{M}{\Theta}\,\frac{\Delta t}{2} = \sqrt{2 M_0 \frac{\varphi_2}{2}} = 2{,}697 \cdot 10^{-2}\ \frac{1}{s}$

c)



$\varphi_G = \tan^{-1}\left(\frac{1 \cdot 10^{-6}}{5 \cdot 10^{6}}\right) = 2 \cdot 10^{-13}\,°$

$\cdot 10^{-9}$ in ml

In Aufgabe mit anderen Werten gerechnet? $2 \cdot 10^{-16}$

$I_s = 10000\,s$, $L = 1\,m$ $F_{min} = 1 \cdot 10^{-6}\,N$

$T = 9$ Tage $\tau_{min} = 1 \cdot 10^{-4}\,s$

$\Delta I_{min} = \tau \cdot F = 1 \cdot 10^{-10}\,Ns$

$\Delta I = n \cdot \Delta I_{min}$

$n = T\,\frac{\Delta I_{min}\,L}{4\Theta\,\varphi_G} = 1{,}35 \cdot 10^{8}$ Impulse

$\Delta I = 1{,}35 \cdot 10^{-2}\,Ns$

$\Delta I_n = m_T\,g_0\,I_s = 2943\,Ns$

ΔI am Tag: $\frac{\Delta I}{9\,Tage} = 1{,}5 \cdot 10^{-3}\,Ns$

$T_{gesamt} = \frac{\Delta I_n}{\Delta I/Tag} = 1{,}962 \cdot 10^{6}$ Tage $= 5375$ Jahre

11.3

a) $2a = 60° = \frac{\pi}{3}$; $\delta = 45° = \frac{\pi}{4}$; $\Theta_z = 6800\ kgm^2$

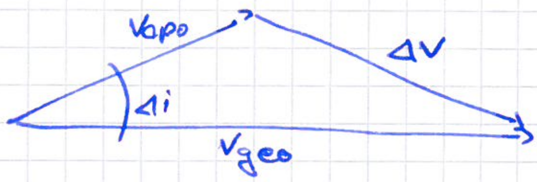
$\Delta I = \frac{\delta \Theta_z \omega_z}{L} \frac{a}{\sin a}$; $\omega_z = 2\pi f_z = \pi/2\ \frac{rad}{s}$

$\Delta I = 7327\ Ns$

$M = F \cdot L = 26{,}4\ Nm$

$\Delta I = S\, \Delta t \rightarrow \Delta t = 332{,}8\ s$

$t = \Delta t \cdot \frac{360}{60} = 1997\ s$



b) $V_{Geo} = 3075\ \frac{m}{s}$

$V_{E,apo} = 1613\ \frac{m}{s}$

$\Delta V = \sqrt{V_{Geo}^2 + V_{E,apo}^2 - 2 V_{Geo} V_{E,apo} \cos(5°)}$

$\Delta V = 1475\ \frac{m}{s}$

Dauer: $F_{Apo} = 1400\ N$, $I_s = 312\ s$ → $c_e = 3061\ \frac{m}{s}$

$\dot m = \frac{F}{c_e} = 0{,}4574\ \frac{kg}{s}$

$\Delta V \overset{!}{=} c_e \cdot \ln\left(\frac{m_0}{m_0 - \dot m \cdot t}\right)$; $m_0 = 3700\ kg$

$t = 3093\ s$

c) Zwei Tage = 172800 s

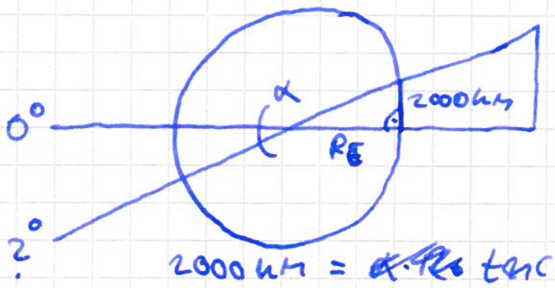
~~7000 km in 1 Tag~~

~~$\Delta V_{Geo} = 1{,}157\ \frac{m}{s}$~~

$\Delta V = \frac{2}{3} R_S \frac{\Delta \lambda}{\Delta t}$

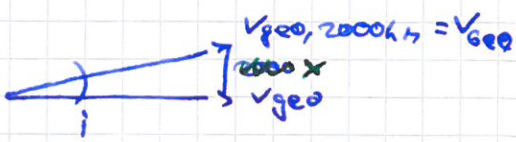
$= \frac{2}{3} \frac{\Delta s}{\Delta t}$

$= 7{,}776\ \frac{m}{s}$



$2000\ km = \tan(\alpha) \cdot R_E$

$\alpha = 17{,}47°$



d) $\Delta I = \frac{\Theta_x \omega_q}{L} \frac{\alpha}{\sin \alpha}$; $\Omega = \frac{\Theta_x - \Theta_z}{\Theta_x} \omega_z$; $\Theta_x = \Theta_{x0} \cdot \frac{m_0 - m_T}{m_0} = 6485\ kgm^2$

$\omega_q = \frac{\Theta_z \omega_z}{\Theta_x} \tan(\alpha) = 2{,}035 = 0{,}03552\ \frac{rad}{s}$

$= 232{,}7\ Ns$

?

Raumfahrt Übung 12 E1

~~[illegible]~~

12.1

a) $\varphi_0 = 0°$ $\quad \varphi_1 = 25°$

$c_e = 2400 \frac{m}{s}$ $\quad L = 1{,}7\,m$ $\quad \Theta = 1500\,kg\,m^2$

$\Delta t = 20\,s$

$$\dot{\varphi} - \cancel{\dot{\varphi}_0}^{=0} = 2 \frac{M}{\Theta} (\varphi - \cancel{\varphi_0})^{=0}$$

$M = F \cdot L$; $\quad \dot{\varphi} = 0$ für $\varphi = 0$ und $\varphi = 25$

~~[illegible]~~ $\frac{\varphi}{2} = \frac{1}{2} \frac{M}{\Theta} \left(\frac{t}{2}\right)^2$

$\varphi = \frac{1}{2} \frac{M}{\Theta} t^2$ $\qquad \varphi = 25°$ für $t = 20\,s$

$F = 3{,}85\,N$ $\qquad$ In Lösung wird erst halbe Zeit und halber Winkel eingesetzt

$M = 6{,}545\,Nm$

b) ~~[illegible]~~

$\frac{c_e}{g_0} = \frac{F}{g_0 \dot{m}_t} \longrightarrow \dot{m}_T = 1{,}604 \cdot 10^{-3}$

$m_{ges} = \Delta t \cdot \dot{m}_t = 3{,}208 \cdot 10^{-2}\,kg$

c) $m_{ges,2} = 8{,}021 \cdot 10^{-3}\,kg$

~~[illegible]~~

$\Delta t_{angetrieben} = (\dot{m}_T / m_{ges,2})^{-1} = 5\,s$

~~[illegible]~~ $\frac{\varphi}{2} = \frac{1}{2} \frac{M}{\Theta} \left(\frac{\Delta t}{2}\right)^2$

$\varphi_{angetrieben} = 0{,}02727\,rad$ $\qquad$ (beschl. und abbremsen zusammen)

Δt, nicht angetrieben:

$\varphi_{rest} = \frac{5}{36}\pi - 0{,}02727 = 0{,}4091\,rad$

~~[illegible]~~

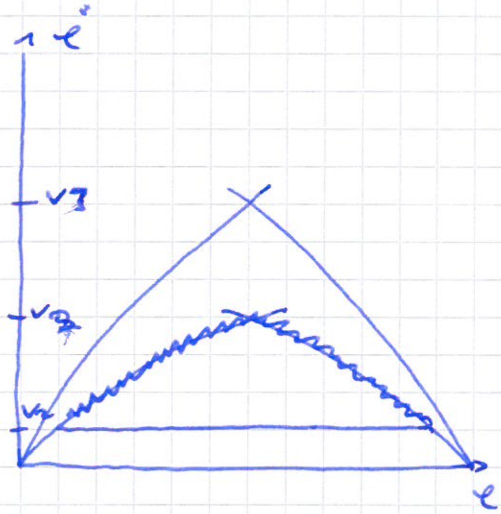
~~[illegible]~~ $\dot{\varphi} - \dot{\varphi}_0^2 = 2 \frac{M}{\Theta} (\varphi - \varphi_0)$

$\dot{\varphi} \cdot 2 = \frac{M}{\Theta} t + \cancel{\dot{\varphi}_0}^{=0}$

$\dot{\varphi} = 1{,}011 \cdot 10^{-2} \frac{rad}{s}$

$\Delta t_{rest} = (\dot{\varphi} / \varphi_{rest})^{-1} = 37{,}5\,s$

$\Delta t_{ges} = 42{,}5\,s$



d) $\Delta t = 10\,s$

~~Mittelgeschwindigkeit~~ Neuer Schub F_1:

$$\frac{e}{2} = \frac{M}{2\Theta}\left(\frac{t}{2}\right)^2$$

$$M = F \cdot L = 2{,}678 \quad ; \quad F_1 = 15{,}4\,N$$

$$M_T = \dot{m}_1 \Delta t_1 = \frac{F_1}{c_e}\Delta t_1 = 0{,}06477\,kg$$

Mittlere Geschwindigkeiten: $\dfrac{\frac{25°/2}{5}}{\frac{25°/2}{10}} = 2 = \dfrac{\dot{e}_1}{e_1}$

12.2

a) $\Delta I = \dfrac{\Theta_x \omega_q}{L} \dfrac{\alpha}{\sin(\alpha)}$

$$\omega_q = \omega_N \tan(\vartheta_N) = \frac{\Theta_z \omega_z}{\Theta_x} \tan(\vartheta_N) = \cancel{\ldots} \; 0{,}7657\,\frac{rad}{s}$$

$$\vartheta_N = 6° = \frac{\pi}{30} \qquad \Theta_x = \Theta_y = 370\,kgm^2 \qquad \Theta_z = 600\,kgm^2$$

$$\alpha = 5° = \frac{\pi}{36} \qquad \omega_{zN} = 0{,}25\,\frac{U}{min} = 0{,}02618\,\frac{rad}{s}$$

$$\Delta I_x = 60{,}29\,Ns$$

$$\Delta I_y = 60{,}29\,Ns$$

1) drei Booster / 1 Booster / Oberstufe

130 s 110 s

a) Masse eines Boosters: $M_{T,B} + \cancel{\ldots}$ $M_{S,B}$

$M_S = M_T + \sigma_B \cdot M_B$

$M_B = 154{,}1\,t$

$M_0 = 4 \cdot M_B + M_{OS}$

$M_{OS} = M_0 \cdot \mu_{OS} \quad \rightarrow M_0 = 655{,}2t \quad ; M_{OS} = 38t$

b) $\dot{m}_B = \frac{M_{T,B}}{t_B} = 1038 \frac{kg}{s}$

$I_s = \frac{Ce}{g_0} \rightarrow Ce_B = 2708 \frac{m}{s}$

$F_B = \dot{m}_B \cdot Ce = 2{,}8107 \cdot 10^6\,N$ (Pro Booster)

$F_1 = 3 \cdot F_B = 8{,}433 \cdot 10^6\,N$

$a_{eff} = \frac{F_1}{M_0} - g_0 = 3{,}061 \frac{m}{s^2}$

c) $V_{end} = 9{,}5 \frac{km}{s}$; $\Delta V_{verlust} = 1{,}5\,km/s$

$\Delta V_1 + \Delta V_2 + \Delta V_3 \overset{!}{=} 9{,}5\,km/s + 1{,}5\,km/s$

Erste Stufe wird „komplett" genutzt, zweite auch

$\Delta V_1 = \bar{Ce} \ln\left(\frac{M_0}{M_b^*}\right) \quad ; M_b^* = M_0 - 3 \cdot M_{T,B}$

$\bar{Ce} = \frac{(\dot{m}_B\, Ce_B) \cdot 3}{3 \cdot \dot{m}_B} = 2708$

$\Delta V_1 = 2{,}607 \frac{km}{s}$

$\Delta V_2 = Ce_B \ln\left(\frac{M_{0,2}}{M^*_{b,2}}\right) \quad ; M^*_{b,2} = (M_b + M_{OS}) - M_{T,B}$

$\Delta V_2 = 3{,}156 \frac{km}{s}$ ← Hier leicht anderes Ergebnis wie in ML ...

$\Delta V_1 + \Delta V_2 = 5763 \frac{m}{s} \rightarrow \Delta V_3 \overset{!}{=} 5237 \frac{m}{s}$

$\Delta V_3 = 5237 \frac{m}{s} = Ce \ln\left(\frac{M_{0,OS}}{M^*_{b,OS}}\right)$

$M_{0,OS} = 38t \quad ; M^*_{b,OS} = 38t - M_{T,OS} \quad ; Ce_{OS} = I_{s,OS} \cdot g_0 = 3778 \frac{m}{s}$

$M_{T,OS} = 30{,}69t \quad ; \quad M_{S,OS} = 4t$

$M_L = M_{OS} - M_{T,OS} - M_{S,OS} = 3{,}37t$

21

d) $\cancel{\mu_{3,opt} = A_3 + \sqrt{A_3^2 + B_3}}$

$\cancel{A_3 = \frac{\mu_4}{\sigma_3} \frac{c_{e,3} - c_{e,2}}{2 c_{e,2}}}$

$\cancel{B_3 = \frac{\mu_4}{\sigma_3} \frac{c_{e,3}}{c_{e,2}} \sigma_2 \mu_2}$

$\mu_{2,opt} = A_2 + \sqrt{A_2^2 + B_2}$

$A_2 = \frac{\mu_3}{\sigma_2} \frac{c_{e,2} - c_{e,1}}{2 c_{e,1}}$ $\quad B_2 = \frac{\mu_3}{\sigma_2} \frac{c_{e,2}}{c_{e,1}} \sigma_1 \mu_1$

$\mu_3 = \mu_{0,S} = 0{,}058$

$\sigma_2 = \sigma_{FB} = 0{,}125$

$c_{e,2} = c_{e,1} \rightarrow A_2 = 0$

$\sigma_1 = \sigma_2$

$\mu_1 = 1$

$\rightarrow \mu_{2,opt} = 0{,}2408$

$\mu_{2,opt} = \frac{M_{0,2}}{M_0} \rightarrow M_{0,2} = 157{,}8\,t$

$M_2 = M_{0,2} - M_{0,3} = 119{,}8\,t$

$M_{2,S} = 79{,}3\,t$

$\Delta V_{2,opt} = c_{e,2} \ln\left(\frac{M_{0,2}}{M_{0,2}'}\right)$; $M_{0,2}' = M_{0,2} - M_2 + M_{2,S}$

$\Delta V_{2,opt} = 2743\,\frac{m}{s}$ ↯ kleiner als ΔV_{normal} ?

(2) Kreisbahn, 500 km Höhe

a) 1000 × 15000 km Ellipse

$V_{peri} = \sqrt{\mu \left(\frac{2}{r_{peri}} - \frac{1}{a}\right)}$ ... keine Musterlösungen mehr für diese Aufgabe...

Raumfahrt klausur 2011 B

1

a) $m_{S,1} = m_1 \cdot \sigma_1$

$m_1 = m_{S,1} + m_{T,1}$

$m_1 = 92,84\,t$ $\quad m_{S,1} = 4,84\,t$

$m_2 = m_{T,2} + m_{S,2} = 26\,t$

$m_3 = 10,9\,t$

$\dot{m}_{T,4} = \frac{F_4}{I_{s,4} \cdot g_0} = 0,7928\,\frac{kg}{s}$

$m_{T,4} = \dot{m}_{E,4} \cdot t_4 = 523,3\,kg$

$m_4 = m_{S,4} + m_{T,4} = 943,3\,kg$

$\underbrace{(m_1 + m_2 + m_3 + m_4 + m_L)}_{m_0} \cdot \mu_L = m_L$

$m_L = 1,454\,t$

$m_0 = 132,2\,t$ In Lösung: 134,3 t

b) $\Delta v_{ges} = \Delta v_1 + \Delta v_2 + \Delta v_3 + \Delta v_4$

$\Delta v_1 = c_{e,1} \ln\left(\frac{m_0}{m^*_{b,1}}\right)$; $m^*_{b,1} = m_0 - m_{T,1}$

$\Delta v_1 = 3010\,\frac{m}{s}$ eig 2925 $\frac{m}{s}$

$\Delta v_2 = c_{e,2} \ln\left(\frac{m_{0,2}}{m^*_{b,2}}\right)$; $m_{0,2} = m_2 + m_3 + m_4 + m_L$; $m^*_{b,2} = m_{0,2} - m_{T,2}$

$\Delta v_2 = 3349\,\frac{m}{s}$

$m_{T,2} = \dot{m}_{T,2} \cdot \Delta t_2 \rightarrow \dot{m}_{b,2} = 0,338\,\frac{t}{s}$

$c_{e,2} = \frac{F_2}{\dot{m}_{T,2}} = 3550\,\frac{m}{s}$

$\Delta v_3 = c_{e,3} \ln\left(\frac{m_{0,3}}{m^*_{b,3}}\right)$

$c_{e,3} = I_{s,3} \cdot g_0 = 2884\,\frac{m}{s}$

$m_{0,3} = m_3 + m_4 + m_L$; $m^*_{b,3} = m_{0,3} - m_{T,3}$

$\Delta v_3 = 4110\,\frac{m}{s}$

$\Delta v_4 = c_{e,4} \cdot \ln\left(\frac{m_{0,4}}{m^*_{b,4}}\right)$; $m_{0,4} = m_4 + m_L$

$m_{T,4} = m_4 - m_{S,4} \rightarrow m^*_{b,4} = m_{0,4} - m_{T,4}$

$c_{e,4} = I_{s,4} \cdot g_0 = 3090\,\frac{m}{s}$

$\Delta v_4 = 767\,\frac{m}{s}$

$\Delta v_{ges} = 11230\,\frac{m}{s} > 11,2\,\frac{km}{s}$ eig $11145\,\frac{m}{s} < 11,2\,\frac{km}{s}$

Ohne Verluste durch Luftwiderstand reicht das Antriebsvermögen knapp aus, in Wirklichkeit jedoch nicht.

$\boxed{2}$

c) $M_L = 2t$

$l_{s,1} = 340s$

$M_{0,neu} = M_{0,alt} - M_{L,alt} + M_{L,neu}$

$M_0 = 132,7\,t \quad = 134,8\,t$ ab hier damit weitergerechnet...

d) $\mu_{3,opt} = A_3 + \sqrt{A_3^2 + B_3}$

$A_3 = \frac{\mu_4}{\sigma_3} \frac{c_{e,1} - c_{e,2}}{2 \cdot c_{e,2}} \qquad B_3 = \frac{\mu_4}{\sigma_3} \frac{c_{e,1}}{c_{e,2}} \sigma_2 \mu_2$

$\mu_4 = \mu_L = 0,017 \qquad \mu_2 = \frac{M_{0,2}}{M_0} = 0,2956$

$M_{S,1} = M_{S,3,alt} + M_{S,4,alt}$

$\sigma_2 = \frac{M_{S,2}}{M_{0,2}} = 0,0502$

$c_{e,1} = l_{s,1} \cdot g_0 = 3335 \frac{m}{s}$

$\sigma_1 = \frac{90 \cdot \frac{M_{T,1}}{1000}}{M_{0,1}} \stackrel{!}{=} \frac{90}{1000+90} = 0,08257$

$M_{S,1} = \frac{\sigma_1}{\sigma_1 - 1} \cdot M_{T,1} \; ; \; M_{S,1} = 90 \cdot \frac{M_{T,1}}{1000}$

$A_3 = -4,034 \cdot 10^{-3} \qquad B_3 = 1,887 \cdot 10^{-3}$ ↯ Andere Wert für μ_2 oder σ_2 ???

$\mu_{3,opt} = 0,003959$

$\mu_{3,opt} = \frac{M_{0,3}}{M_0} \quad ; \; M_{0,3} = 73,84\,t$

$M_0 =$